ing in that they yield, similarly to the trinucleon case,[16,17] a strong enhancement of the secondary maximum.[18-20]

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Ionization Energy Loss of Relativistic Electrons in Thin Silicon Detectors

W. Ogle, P. Goldstone, C. Gruhn, and C. Maggiore
University of California, Los Alamos Scientific Laboratory, Los Alamos, New Mexico 87545
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We have measured the ionization loss of electrons with Lorentz factors $\gamma = 2.91$ and $1.6\times10^4 \le \gamma \le 10^5$ passing through a (100.71 ± 0.15)-μm silicon detector. Our results are in agreement with accepted theory for $\gamma = 2.91$ and $\gamma = 1.6\times10^4$; however, for $\gamma \ge 3\times10^4$ our results are systematically $(7\pm2)\%$ below the theoretical value. We suggest an explanation in terms of the finite detector thickness and a relativistic effect.

The presently accepted Fermi-Sternheimer theory of ionization loss[1] predicts that for large Lorentz factors, $\gamma \gtrsim 10^3$, the most probable ionization loss will approach an asymptotic value, the Fermi plateau, independent of γ. The ionization loss proceeds through momentum transfers, q, greater than a minimum component parallel to the incident trajectory,

$$q_{\parallel\,\min} = \vec{q}\cdot\hat{p} \simeq (h\omega/\beta c)(1-\beta\sqrt{\epsilon}) \equiv \hbar/l_{\rm coh}, \qquad (1)$$

where $\hat{p}$ is the direction of the initial momentum, $\beta = v/c$, and $\epsilon = 1-\omega_0^2/\omega^2$ is the dielectric function of the detector. From the uncertainty relation, Eq. (1) defines a coherence length[2] over which the interaction takes place. The minimum of $q_\parallel$ is found for $\omega = \gamma\omega_0$, which results in a maximum coherence length $l_{\rm coh} = \gamma^2 c/\omega_0$. When the coherence length thus defined exceeds the path length, a, of the incident particle in the detector, the interaction time exceeds the transit time, and we may expect deviations from the accepted theory. To test the theory under these circumstances, we have made an absolute measurement of the ionization loss of ultrarelativistic electrons in a thin detector.

An ionization-loss theory based on considerations of transition radiation was proposed by Garibyan,[3] who predicted a $\ln(\gamma)$-dependent increase for $\gamma \gg (a\omega_0/c)^{1/2}$. Alikhanyan *et al.*[4] claim to have seen the predicted increase, but the effect has not been verified and no absolute measurement has yet been made. Garibyan's theory does not include multiple scattering and requires modification for $\gamma \ge \gamma_{\rm cr}^* = (mc^2/E_s)^2(L\omega_0/c)$, where $E_s = 21$ MeV and L is the radiation length. Since $\gamma_{\rm cr}^* = 9\times10^3$ for silicon, our experiment is not a test of this theory. However, our measurement may have some bearing on the radiative correction proposed by Tsytovich,[5] which would reduce the Fermi plateau value by 5% to 10% for $\gamma \gtrsim 200$. Although one group[6] claims to have seen this effect in certain emulsion data, evidence from other groups[7,8] suggests that the correction, if it exists, is smaller than 5%. Crispin and Fowler[8] argue that the effect of a strongly absorptive medium, neglected by Tsytovich, makes the correction inappropriate for ionization-loss phenomena.

The experiment was performed in a parasitic mode in the electron tagging arm of the tagged-photon beam at Fermi National Accelerator Laboratory. The most probable energy loss was measured by observing the charge signal from a (100.71 ± 0.15)-μm-thick ($\sim 10^{-3}$ radiation length) silicon surface-barrier detector. The tagging electrons were momentum dispersed in a vacuum pipe with less than 10^{-3} radiation lengths of material ahead of the detector at the highest two momenta and less than 2.2×10^{-2} radiation lengths at the two lower momenta. The trigger consisted of a smaller-area silicon detector, plastic scintillator hodoscope counters, and a total-energy lead-glass shower counter, all downstream. For calibration purposes data were also taken in the same geometry using a ^{207}Bi source and the internally converted electrons (0.976 and 1.051 MeV). The cost to produce an electron-hole pair was determined for ^{207}Bi electrons by simultaneously measuring absolute charge and absolute energy loss using a following stopping detector. The result was 3.80 ± 0.02 per electron-hole pair, which is in agreement with previous published values.[9]

The most probable energy loss was obtained by fitting a convolved spectrum plus background to the data. This convolved spectrum consisted of the measured electronic noise spectrum, calculated radiative corrections, and a calculated Landau distribution. The electronic noise spectrum was taken with a precision pulser and typically had an energy resolution of 5.5 keV (full width at half-maximum). The radiative corrections were made taking into account both material upstream and the detector. This correction typically reduced the most probable energy detected by 0.2 keV. The background spectrum was least at the highest momentum. This correction reduced the value of the most probable energy loss by 0.3 keV at $\gamma = 10^5$ and 1.0 keV at $\gamma = 1.6 \times 10^4$. No background correction was applied to the data at $\gamma = 2.91$.

In Table I we list the measured values of the most probable energy loss along with the relative uncertainty. A scale uncertainty of 0.3 keV was estimated. The results are plotted in Fig. 1 together with the predictions of Sternheimer (solid line), Tsytovich (shaded region), Garibyan[10] (dash-dotted line), and our calculation (dashed line) discussed below. With consideration of multiple scattering, Garibyan's prediction would coincide with Sternheimer's. Our measurement agrees quite well with theory for $\gamma = 2.91$, which serves as a test of our experimental technique. The measurement at $\gamma = 1.6 \times 10^4$ also agrees within the experimental uncertainty and tends to confirm the results of Aitken, Lakin, and Zulliger.[7] The points at higher energy, however, are systematically (7 ± 2)% below the Sternheimer prediction.

TABLE I. Measured most probable ionization loss with relative errors for relativistic electrons.

γ	$\Delta\gamma$	ϵ_p (keV)	$\Delta\epsilon_p$ (keV)
2.91	0.15	24.8	0.5
1.6×10^4	1.6×10^3	25.6	0.6
3.0×10^4	2.0×10^3	24.1	0.6
4.8×10^4	4.0×10^3	24.3	0.6
1.0×10^5	1.0×10^4	24.3	0.3
1.0×10^5	1.0×10^4	33.3	0.8[a]

[a] 45° ± 3° incidence.

We suggest that the deviation of our experimental results from the Sternheimer theory can be understood from two considerations: (1) The thickness, a, of the detector acts as a screening distance, and momentum transfer occurs only for $l_{\mathrm{coh}} \leq a$; (2) in deriving Eq. (1) the accurate

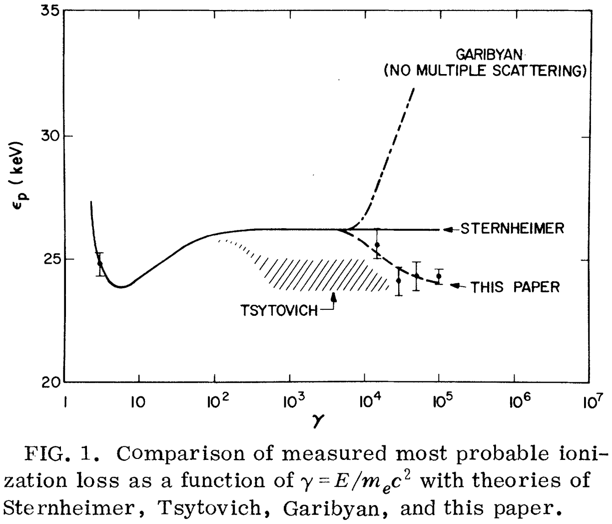

FIG. 1. Comparison of measured most probable ionization loss as a function of $\gamma = E/m_e c^2$ with theories of Sternheimer, Tsytovich, Garibyan, and this paper.

relativistic relation $q^2/2m = Q(1+Q/2mc^2)$ should be used for the momentum $\vec{q}$ of a free electron with energy $Q = \hbar\omega$. We find that this is equivalent to the replacements $\omega_0{}^2 \rightarrow \omega_0{}^2 + \omega_c{}^2$, where $\omega_c = 2\gamma c/a$, and $\omega \rightarrow \omega(1+\hbar\omega/2mc^2)$ in the logarithmic density-effect term. The correction to the Sternheimer value for large γ becomes

$$\Delta\epsilon_p = 2At\ln(\gamma\omega_0/l_c), \qquad (2)$$

where the frequency l_c is determined from

$$l_c(1+\hbar l_c/2mc^2) = \gamma(\omega_0{}^2 + \omega_c{}^2)^{1/2}, \qquad (3)$$

and the coefficient, At, is as defined by Sternheimer.[1] The effect of this correction is shown by the dashed line in Fig. 1. It vanishes for energies $\gamma \ll \gamma_c = a\omega_0/2c$ and $\gamma \ll 2mc^2/h\omega_0$.

We have also considered the possible importance of multiple scattering and electron-position pair production. The coherence length has a directional dependence proportional to $\gamma^2\theta^2$, where the angle θ between the incident and scattered trajectories is zero for the maximum coherence length. Over the coherence lengths of interest, both multiple scattering and pair production may lead to average scattering angles such that $\gamma^2\overline{\theta}^2 \gtrsim 1$. However, the minimum scattering angle remains zero, and therefore these processes do not change the coherence length.

With this experiment, we have measured an absolute ionization loss (7 ± 2)% below the Sternheimer value for three out of four high-energy points. We feel that the discrepancy is real. Our data are consistent with either the Tsytovich theory[5] or the modification to Sternheimer theory proposed in Eqs. (2) and (3), but not both. If we believe the results of experiments[7,8] done for $\gamma \lesssim 10^3$, which we feel are not definitely conclusive, then we must reject the Tsytovich theory. We shall attempt to resolve this question by making measurements over a range of energies from $\gamma \approx 100$ to $\gamma \gg \gamma_c$, with detectors of varying thicknesses.

It is a pleasure for us to express our appreciation to Fermi National Accelerator Laboratory and in particular to Dr. P. Garbincius, Dr. R. Morrison, and Dr. T. Nash for their cooperation and assistance. This work was performed under the auspices of the U. S. Department of Energy.

Note added.—Our conclusions concerning multiple scattering are in agreement with M. L. Ter-Mikaelyan, Zh. Eksp. Teor. Fiz. 38, 895 (1960) [Sov. Phys. JETP 11, 646 (1960)], and private communication.

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