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Unified Approach to Matter Coupling in Weyl and Einstein Supergravity

A. Das and M. Kaku
Department of Physics, City College of City University of New York, New York, New York 10031

and

P. K. Townsend
Institute for Theoretical Physics, State University of New York at Stony Brook, Stony Brook, New York 11794
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We present a new unified method of coupling matter to both Poincaré and conformal supergravity. As an example we construct the coupled Maxwell-Weyl and the coupled Maxwell-Einstein supergravity theories. We also discuss the close connection with a recent auxiliary-field formulation of supergravity.

There are essentially two classes of supergravity theories, Einstein supergravity which is based on the graded Poincaré groups, and Weyl supergravity which is based on the graded conformal groups. The former can accomodate O(N) internal symmetries and the latter U(N). The U(1) theory has recently been constructed[1-3] and its full superconformal invariance established.[3] Both Einstein and Weyl supergravity have merits and demerits, but this is not what we wish to discuss here. Rather, we wish to show that some of the results of Weyl supergravity can be used to give a unified treatment of matter coupling to *both* supergravity theories. In particular we first construct a new locally superconformal and Maxwell gauge invariant theory by coupling the $(1,\frac{1}{2})$ supermultiplet to Weyl supergravity. We then show that this result leads directly to the corresponding result for Einstein supergravity which is the locally supersymmetric Maxwell-Einstein system previously constructed.[4,5] The method is simple and we expect it to generalize to other massless supermultiplets.

The coupling of matter to Weyl supergravity is straightforward and has already been given for the scalar supermultiplet.[6] Every massless supermultiplet has three conserved currents, the energy-momentum tensor, $\theta_{\mu\nu}$, the spinor current, $j_\mu{}^Q$, and the chiral current, $j_\mu{}^A$.[7] These are coupled directly to the three gauge fields, $e_{a\mu}$, ψ_μ, and A_μ of Weyl supergravity. Additional terms must then be added to the action and matter field transformation laws to ensure complete superconformal invariance. This invariance requires, in addition to the usual Q supersymmetry, three new symmetries, Weyl invariance (D), chiral invariance (A), and S supersymmetry. The action is determined by the D, A, and S symmetries and low-order Q-supersymmetry variations. The construction is particularly simple because of the tight constraints imposed by the extra symmetries which are at the same time fairly easy to establish.

The essential point is now that since the gauge algebra of the transformations of $e_{a\mu}$, ψ_μ, A_μ closes off shell,[3] these transformation laws will not be changed by the presence of matter couplings,[6] a point which is explicitly verified. Because of this, the Weyl-supergravity action itself is *separately* invariant. We may therefore omit it from the coupled Maxwell-Weyl action without spoiling the superconformal invariance. By the same reasoning we may also add the action for the recently constructed[6] superconformal extension of Einstein supergravity. This action has the property that it is gauge equivalent to Einstein supergravity and, in fact, reduces exactly to the Einstein-supergravity action on appropriately fixing the extra D, A, and S gauges. The procedure for obtaining Maxwell-Einstein from Maxwell-Weyl supergravity will now be clear. Simply replace the Weyl-supergravity action by the superconformal extension of the Einstein-supergravity action. Fix the D, A, S gauges. Eliminate the auxiliary fields. The last step follows because without the Weyl-supergravity action the chiral gauge field, A_μ, is auxiliary and may be eliminated through its field equation. We carry out these steps and show that the previously constructed Maxwell-Einstein supergravity theory[4,5] is obtained.

This method has some interesting features in common with the recent auxiliary-field formulation of supergravity[8,9] for which the gauge algebra closes without the use of field equations. We comment on this connection in the conclusions.

Our conventions throughout are those of Refs. 3 and 6.

Maxwell-Weyl supergravity.—The Lagrangian is

$$\mathcal{L}^{\mathrm{M-W}} = \mathcal{L}^{\mathrm{W}} - \tfrac{1}{4} e g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} - \tfrac{1}{2} e \bar{\lambda}\gamma^\mu D_\mu \lambda$$
$$+ \tfrac{3}{8} i e \bar{\lambda}\gamma^\mu \gamma_5 \lambda A_\mu - \tfrac{1}{2} e \bar{\lambda}\gamma^\mu \sigma\cdot F \psi_\mu - \tfrac{1}{4}\bar{\psi}_\lambda \sigma^{\mu\nu}\gamma^\lambda \lambda \bar{\lambda}\gamma_\nu \psi_\mu - \tfrac{1}{32}\epsilon^{\mu\nu\rho\sigma}\bar{\lambda}\gamma_5\gamma_\nu \lambda \bar{\psi}_\mu \gamma_\rho \psi_\sigma , \quad (1)$$

where $F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ and D_μ is the local Lorentz-covariant derivative *without* torsion and is given explicitly in Ref. 6. B_μ is the spin-1 field and λ the Majorana spin-$\frac{1}{2}$ field of the $(1,\frac{1}{2})$ supermultiplet. The transformation rules for the D, A, S, and Q symmetries are given in Table I. $\mathcal{L}^{\mathrm{W}}$ is the Weyl-supergravity Lagrangian given in Ref. 3.

The construction is simple. After adding the A_μ and ψ_μ Noether couplings and covariantizing with $e_{a\mu}$, the $\lambda^2\psi_\mu{}^2$ terms are determined by S supersymmetry up to additions of $e\bar{\lambda}\lambda(\bar{\psi}_\mu\psi^\mu - \frac{1}{4}\bar{\psi}\cdot\gamma\gamma\cdot\psi)$ and $e\bar{\lambda}\gamma_5\lambda(\bar{\psi}_\mu\gamma_5\psi^\mu + \frac{1}{4}\bar{\psi}\cdot\gamma\gamma_5\gamma\cdot\psi)$ which are separately S invariant. This ambiguity is removed by requiring that all A_μ-dependent terms in $\delta_Q\mathcal{L}^{\mathrm{M-W}}$ cancel. The Lagrangian is then determined and it remains to check complete Q supersymmetry. In Table II we give a list of all the possible Q-supersymmetry variations and their sources. Some of these variations are identical to those of Maxwell-Einstein supergravity and are thus known to cancel. The remaining variations require the usual algebraic manipulations and Fierz rearrangements to show that they cancel also.

Maxwell-Einstein supergravity.—The superconformal extension of Einstein supergravity[6] has three fields A, B, χ of a scalar supermultiplet in addition to the gauge fields $e_{a\mu}$, A_μ, ψ_μ. These are "compensating" fields of superconformal invariance [just as φ is a "compensating" field of Weyl invariance in the well-known Weyl invariant theory $\mathcal{L} = \frac{1}{2}(\partial_\mu\varphi)^2 - \frac{1}{12}R\varphi^2$]. When the "extra" D, A, S gauges are fixed by choice of A, B, χ the Lagrangian reduces to that of Einstein supergravity but with A_μ an auxiliary field. The explicit form of this superconformal extension is given in Ref. 6 and we denote it by $\mathcal{L}^{\mathrm{SCE}}$. The gauge transformations of $e_{a\mu}$, A_μ, ψ_μ, in $\mathcal{L}^{\mathrm{SCE}}$ are also those of $\mathcal{L}^{\mathrm{M-W}}$ and hence given in Table I.

TABLE I. The D, A, S, and Q invariances of $L^{\mathrm{M-W}}$.

	S supersymmetry	A chiral invariance	D Weyl invariance	Q supersymmetry[a]
$\delta e_{a\mu}$	0	0	$-e_{a\mu}\omega_D$	$\frac{1}{2}\bar{\epsilon}_Q\gamma_a\psi_\mu$
$\delta\psi_\mu$	$-\gamma_\mu\epsilon_S$	$\frac{3}{4}i\gamma_5\psi_\mu\omega_A$	$-\frac{1}{2}\psi_\mu\omega_D$	$\mathcal{D}_\mu\epsilon_Q$
δA_μ	$i\bar{\epsilon}_S\gamma_5\psi_\mu$	$\partial_\mu\omega_A$	0	$-i\bar{\epsilon}_Q\gamma_5\varphi_\mu$
δB_μ	0	0	0	$\frac{1}{2}\bar{\epsilon}_Q\gamma_\mu\lambda$
$\delta\lambda$	0	$\frac{3}{8}i\gamma_5\lambda\omega_A$	$\frac{3}{2}\lambda\omega_D$	$-\frac{1}{2}\sigma\cdot F\epsilon_Q + \frac{1}{2}(\bar{\psi}_\mu\gamma_\nu\lambda)\sigma^{\mu\nu}\epsilon_Q$

[a] We define $\varphi_\mu = \frac{1}{3}\gamma^\nu(\mathcal{D}_\nu\psi_\mu - \mathcal{D}_\mu\psi_\nu) + \frac{1}{6}\gamma_\mu\gamma_5\mathcal{D}_\sigma\psi_\rho\epsilon^{\lambda\nu\rho\sigma}g_{\lambda\mu}$, where $\mathcal{D}_\mu = D_\mu - \frac{3}{4}iA_\mu\gamma_5 - \frac{1}{8}(\bar{\psi}_\mu\gamma_\rho\psi_\lambda - \bar{\psi}_\mu\gamma_\lambda\psi_\rho - \bar{\psi}_\lambda\gamma_\mu\psi_\rho)\sigma^{\lambda\rho}$.

Now, as explained earlier, we may simply replace $\mathcal{L}^{W}$ in (1) by $\mathcal{L}^{SCE}$ *without spoiling the superconformal invariance*. This follows from the fact that the gauge algebra closes on the fields which couple to matter, $e_{a\mu}$, A_μ, ψ_μ. (The gauge algebra of $\mathcal{L}^{SCE}$ does not close on χ since one needs also the auxiliary scalar F and pseudoscalar G of the scalar supermultiplet, but complete closure is not necessary for our purposes.) We now fix the D, A, S gauges by choosing $A=\sqrt{6}\,\kappa^{-1}$, $B=\chi=0$ as in Ref. 6. Thus, for present purposes, we need only know the form of $\mathcal{L}^{SCE}$ after this gauge choice, and this is given by the first two terms in (2.1) below. This procedure provides the following Lagrangian

$$\mathcal{L}^{M-E}=\mathcal{L}_{\text{supergravity}}+(3/4K^2)eA_\mu A^\mu-\tfrac{3}{8}ie\bar{\lambda}\gamma^\mu\gamma_5\lambda A_\mu$$
$$-\tfrac{1}{4}eg^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma}-\tfrac{1}{2}e\bar{\lambda}\gamma^\mu D_\mu\lambda-\tfrac{1}{2}e\bar{\lambda}\gamma^\mu\sigma\cdot F\psi_\mu-\tfrac{1}{4}e\bar{\psi}_\lambda\sigma^{\mu\nu}\gamma^\lambda\lambda\,\bar{\lambda}\gamma_\nu\psi_\mu-\tfrac{1}{32}\bar{\lambda}\gamma_5\gamma_\nu\lambda\bar{\psi}_\mu\gamma_\rho\psi_\sigma\epsilon^{\mu\nu\rho\sigma}. \quad (2.1)$$

Eliminating A_μ gives rise to $(\bar{\lambda}\lambda)^2$ terms. Then if we take $\lambda\to-\lambda$, (2.1) is identical to the Lagrangian of Ref. 4 upon Fierz rearrangement.

Fixing A, B, χ as above to arrive at (2.1) is not a consistent truncation of either Q or S supersymmetry. But it is a consistent truncation of a linear combination. In fact,

$$\delta(\epsilon_Q)=\delta_Q(\epsilon_Q)+\delta_S\left(\epsilon_S=-\frac{i}{4}A\!\!\!/\gamma_5\epsilon_Q-\frac{g\sqrt{6}}{2\kappa}G_Q\right), \quad (2.2)$$

is a consistent truncation of $\mathcal{L}^{SCE}$ and matter field transformation laws. g in (2.2) is proportional to the cosmological constant. In the case $g=0$, this particular combination of Q and S supersymmetry is already known to reproduce the correct transformation laws of Maxwell-Einstein supergravity.[9] Our approach provides an explanation of this fact.

We have given a simple method of coupling matter to both Weyl and Einstein supergravity in a unified way. This is made possible by the closure of the gauge algebra in Weyl supergravity[3] and the existence of a superconformal extension of Einstein supergravity.[6] We illustrated our method with the Maxwell $(1,\frac{1}{2})$ supermultiplet but we believe that it is generally applicable. A key feature is the ability to add and subtract invariant actions to obtain another invariant action. This was not possible in the original formulation of supergravity. Presumably it will be possible in the new formulation in which three auxiliary fields, A_μ, P, and S are introduced to close the algebra off shell.[8,9] There are some interesting similarities between this work and ours. In particular we can give a simple understanding of why these three fields are needed to close the algebra. In the superconformal extension of Einstein supergravity with the scalar supermultiplet $(A, B, \chi, F, G,)$ the gauge transformations are expected to close off shell on *all* fields.[6] Since A, B, and χ are fixed by a D, A, S gauge choice, this leaves the auxiliary fields A_μ, F, G. Clearly the auxiliary fields P and S of Ref. 9 are to be identified with the F and G fields, respectively. Preliminary calculations confirm this conjecture.

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TABLE II. Variations and their sources in $\delta_Q L^{M-W}$.

Type of variation	Source
$\bar{\epsilon}\psi F^2$	(i) $\delta e_{a\mu}\sim\bar{\epsilon}\gamma_a\psi_\mu$ in F^2
	(ii) $\delta\lambda\sim\sigma\cdot F\epsilon$ in $\bar{\psi}\lambda F$
$\bar{\epsilon}\psi\lambda^2$	(i) $\delta e_{a\mu}\sim\bar{\epsilon}\gamma_a\psi_\mu$ in $\bar{\lambda}\lambda$
	(ii) $\delta B_\mu\sim\bar{\epsilon}\gamma_\mu\lambda$ in $\bar{\lambda}\psi F$
	(iii) $\delta A_\mu\sim\bar{\epsilon}\psi$ in $\bar{\lambda}\lambda A_\mu$
	(iv) $\delta\psi_\mu\sim\partial_\mu\epsilon$ in $\bar{\psi}\psi\bar{\lambda}\lambda$
	(v) $\delta\lambda\sim\bar{\psi}\lambda\epsilon$ in $\bar{\lambda}\lambda$
$\bar{\epsilon}\lambda F A_\mu$	(i) $\delta\lambda\sim\sigma\cdot F\epsilon$ in $\bar{\lambda}\lambda A_\mu$
	(ii) $\delta\psi_\mu\sim A_\mu\epsilon$ in $\bar{\lambda}\psi F$
$\bar{\epsilon}\lambda F$	(i) $\delta\lambda\sim\sigma\cdot F\epsilon$ in $\bar{\lambda}\lambda$
	(ii) $\delta B_\mu\sim\bar{\epsilon}\gamma_\mu\lambda$ in F^2
	(iii) $\delta\psi_\mu\sim\partial_\mu\epsilon$ in $\bar{\lambda}\psi F$
$\bar{\epsilon}\psi\lambda^2 A_\mu$	(i) $\delta e_{a\mu}\sim\bar{\epsilon}\gamma_a\psi_\mu$ in $\bar{\lambda}\lambda A_\mu$
	(ii) $\delta A_\mu\sim\bar{\epsilon}\psi$ in $\bar{\lambda}\lambda A_\mu$
	(iii) $\delta\psi_\mu\sim A_\mu\epsilon$ in $\psi^2\lambda^2$
	(iv) $\delta\lambda\sim\bar{\psi}\lambda\epsilon$ in $\bar{\lambda}\lambda A_\mu$
$\bar{\epsilon}\lambda\psi^2 F$	(i) $\delta e_{a\mu}\sim\bar{\epsilon}\gamma_a\psi_\mu$ in $\bar{\lambda}\psi F$
	(ii) $\delta\psi\sim\bar{\psi}\psi\epsilon$ in $\bar{\lambda}\psi F$
	(iii) $\delta\lambda\sim\bar{\psi}\lambda\epsilon$ in $\bar{\lambda}\psi F$
	(iv) $\delta\lambda\sim\sigma\cdot F\epsilon$ in $\psi^2\lambda^2$
$\bar{\epsilon}\lambda^2\psi^3$	(i) $\delta A_\mu\sim\psi^3\epsilon$ in $\bar{\lambda}\lambda A_\mu$
	(ii) $\delta\psi\sim\psi^2\epsilon$ in $\psi^2\lambda^2$
	(iii) $\delta\lambda\sim\bar{\psi}\lambda\epsilon$ in $\psi^2\lambda^2$
	(iv) $\delta e_{a\mu}\sim\bar{\epsilon}\gamma_a\psi_\mu$ in $\psi^2\lambda^2$

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Sum Rules and Moments for Lepton-Pair Production

Rudolph C. Hwa[(a)]
Rutherford Laboratory, Chilton, Didcot, Oxon OX11 0QX, England
(Received 3 February 1978)

Sum rules on lepton-pair production cross sections are derived on the bases of the Drell-Yan formula and the known sum rules in leptoproduction. I also obtain exact relationships between the square of the average transverse momenta of the valence quarks and moments of the dilepton cross sections.

Unlike the case of leptoproduction, no sum rules have thus far been derived for lepton-pair production in hadron collisions. Theoretical understanding of the lepton-pair production processes is not on a firm ground because the usual short-distance and light-cone ideas are not applicable. The Drell-Yan mechanism[1] of quark-antiquark annihilation has been found to be quite successful,[2] even though the reason for the dominance of the annihilation term is not totally clear. Recently, progress has been made[3,4] in studying the correction to the naive parton result in the context of QCD (quantum chromodynamics) and relating it to the scaling violation of the parton distribution functions. In particular, Sachrajda[4] has found that when the gluon contributions are included in low-order perturbation-theory calculations in QCD, the Drell-Yan formula remains valid. As the Drell-Yan picture gains credibility phenomenologically and theoretically, it becomes interesting to investigate the general implications of that picture independent of the details of the input. I derive here sum rules based on that picture which can be tested experimentally. Moreover, I give explicit expressions for the transverse momenta of the valence quarks in hadrons.

In the Drell-Yan picture[1] the cross section for dimuon production in high-energy collisions between two hadrons, h_1 and h_2, is

$$d\sigma(h_1h_2)/dM^2 = (C/M^2)\sum_f e_f^2 \int [S_{h_1}{}^f S_{h_2}{}^{\bar{f}} + S_{h_1}{}^{\bar{f}} S_{h_2}{}^f]\,\delta((k_1+k_2)^2 - M^2)(k_1{}^0 k_2{}^0)^{-1} d^3k_1 d^2k_2, \tag{1}$$

where $C = 4\pi\alpha^2/9$, M is the dimuon mass, the sum is over flavor, e_f is the charge of a quark of flavor f in units of e, and k_i is the parton momentum in hadron h_i. $S_{h_i}{}^{f,\bar{f}}$ is the invariant parton distribution function of quark f (antiquark $\bar{f}$) in the hadron h_i and depends on the invariants formed out of the momenta of the hadron P, parton k_i, and photon q.[5] The "structure" functions S satisfy the constraints (for proton)

$$N_p^u - N_p^{\bar{u}} = 2, \quad N_p^d - N_p^{\bar{d}} = 1, \quad N_p^s - N_p^{\bar{s}} = 0, \tag{2}$$

where

$$N_h^f = \int S_h^f(P,k,q) d^3k/k^0. \tag{3}$$

In addition to the quark-antiquark annihilation term (1), there are in principle many other terms involving gluons. In the leading-logarithm–approximation, low-order calculations[3,4] they can be recast in the form of (1) with a concomitant change of the S functions compatible with scaling violation of the structure functions in deep inelastic scattering. Thus, with enhanced confidence in the dominance of