Depression of Superfluid Density by Velocity in Liquid ^{4}He

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The dependence of the superfluid density ρ_s on velocity v_s (with $v_n \simeq 0$) has been measured for the first time by observing the shift with amplitude of the resonant frequency of a superfluid Helmholtz resonator. The results agree within experimental and calibration error with the theoretical prediction of Khalatnikov.

The possibility of nondissipative currents in liquid helium II, characteristic of superfluidity, requires the inclusion of a third independent variable in a complete thermodynamic description. An appropriate variable is the square of the Galilean-invarinat velocity, $w = v_s - v_n$, the difference between the superfluid and normal-fluid velocities.[1] If pressure and temperature are selected as the other independent variables, the remaining thermodynamic functions, such as specific entropy s, total density ρ, and normal-fluid density ρ_n (or superfluid density $\rho_s = \rho - \rho_n$) are functions of p, T, and w^2. The w^2 dependence of s and ρ are connected by Maxwell relations to the temperature and pressure derivatives of ρ_n/ρ, which are well known, but the dependence of ρ_n on w^2 is a separate equation of state which cannot be determined from measurements on the classical variables.[1,2]

Khalatnikov[3] has calculated $\rho_n(\rho, T, w^2)$ on the basis of the Landau quasiparticle model. For the temperature range above about 1 K, in which rotons make the dominant contribution to ρ_n, his result is

$$\rho_n(w^2) = \rho_n(0)[1 + \tfrac{1}{10}(p_0 w/kT)^2 + O(w^4)], \qquad (1)$$

where p_0 is the momentum of the roton minimum and k is Boltzmann's constant.

There are no past experiments which give convincing evidence for a dependence of ρ_s on w^2. The most sophistocated experiment is that of Kojima *et al.*[4,5] using the technique of Doppler-shifted fourth sound in a powder-packed annulus, which gave a null result consistent with Eq. (1). A previous report[6] of an effect two orders of magnitude larger than Eq. (1) is in conflict with Ref. 4 and probably should be attributed to presence of superfluid vortices. Experiments looking for a shift in second-sound velocity in (turbulent) counterflow have given a null effect[7] and an incompatibly large positive effect attributed to ρ_s depression[8] or to vortices.[9] In view of recent work in rotating helium,[10,11] the last explanation is credible.

The apparatus used in the present experiment (Fig. 1) has been described previously.[12] Volumes V_1 and V_2 are connected by a 9.4-μm-diam orifice O. A flexible diaphragm C with a capacitive displacement sensor measures the differential pressure. The system is filled with liquid helium II through long powder-packed capillary tubes, and is then isolated by heating a central portion H of each capillary above T_λ, blocking superfluid flow. Resonant oscillation of superfluid between the two volumes is driven thermomechanically by a heater in one chamber, or mechanically by flexing one wall of V_1 with a piezoelectric transducer. The resonant frequency (~11–45 Hz) varied with temperature as predicted by theory, including the effect of compliance of the diaphragm. The Q ranged from 28 to 613.

The resonant frequency depends on ρ_s through the mass current. If $\rho_s = \rho_{s0}(1 - \epsilon v_s^2)$, the resulting frequency shift to order v_s^2 is

$$\frac{\Delta f}{f} = -\tfrac{3}{8}\gamma\epsilon V_s^2 = -\tfrac{3}{80}\gamma\frac{\rho_n}{\rho_s}\left(\frac{p_0 V_s}{kT}\right)^2, \qquad (2)$$

where V_s is the peak value of the velocity averaged over the minimum cross section of the orifice, and the last expression incorporates the Khalatnikov result. The geometry-dependent constant γ equals unity for a long, uniform channel.

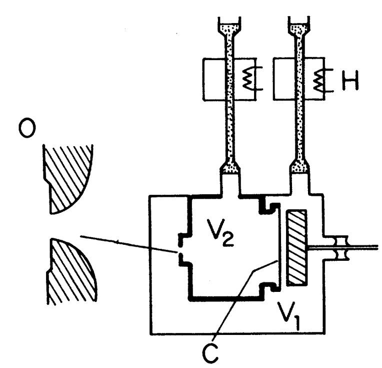

FIG. 1. Schematic diagram of the apparatus. The components are described in the text.

Measurements are made by following the center of the resonance, defined by phase shift. Initially, the half-power points are located at a low amplitude, and the phase adjustment of a two-channel synchronous detector[13] is set to put one channel in quadrature with the pressure signal at the center frequency of the resonance peak. Then at each amplitude the drive frequency is adjusted to maintain the null in the quadrature channel. The in-phase channel serves to measure the pressure amplitude, from which the velocity amplitude is calculated. Supplementary measurements show that the half-power points are displaced by only half as much as the peak, as expected on the basis of Eq. (2). There is no evidence of excess dissipation due to generation of superfluid vorticity up to an amplitude at which the oscillation undergoes sudden collapse, as reported earlier.[12]

The experimental results are shown in Fig. 2. These data were taken in three runs, using two different orifices and two diaphragms, one of aluminum and one brass. The error bars indicate the precision of the frequency-shift measurements, which are limited by random excitation of the oscillation by building vibrations. The slopes of the fitted lines in Fig. 2 are plotted against temperature in Fig. 3. The dotted line represents Eq. (2) with $\gamma = 1$. There are certain other thermodynamic effects of order w^2, which can be classified as (a) terms arising from the transformation of Khalatnikov's result for ρ_n from constant volume to constant pressure, (b) the variation of ρ with w^2, and (c) effects of the Bernoulli pressure variation. The two-fluid model has two hydrodynamic equations and consequently two Bernoulli equations (see Ref. 2). If $v_n = 0$, these equations yield $\Delta p = -\frac{1}{2}\rho_s v_s^2$ and $\Delta T = 0$. After some thermodynamic manipulation, these constraints give

$$\Delta\rho_s = -\left(\frac{\partial\rho_n}{\partial w^2}\right)_{\rho,T} v_s^2 - \tfrac{1}{2}\rho\left(\frac{\partial p}{\partial\rho}\right)_T\left(\frac{\partial\rho_s}{\partial p}\right)_T^2 v_s^2. \quad (3)$$

Inclusion of the second term gives the solid line in Fig. 3.

In practice, the orifice is far from being a long uniform channel (see inset in Fig. 1). It is necessary to solve a boundary-value problem analogous to compressible flow. In general, there is a redistribution of the velocity field with increasing current in addition to the depression of ρ_s. For a long uniform channel with abrupt ends, the hydrodynamic complications appear in the end corrections, which are neglected. One other case which can be handled trivially is a gently tapered constriction, in which the velocity distribution can be approximated as uniform over each cross section. In this limit, the geometric constant appearing in Eq. (2) is $\gamma = \frac{1}{3}$ for a double exponential horn, $\gamma = \frac{3}{8}$ for a hyperbolic horn, or $\gamma = \frac{63}{128}$ for a

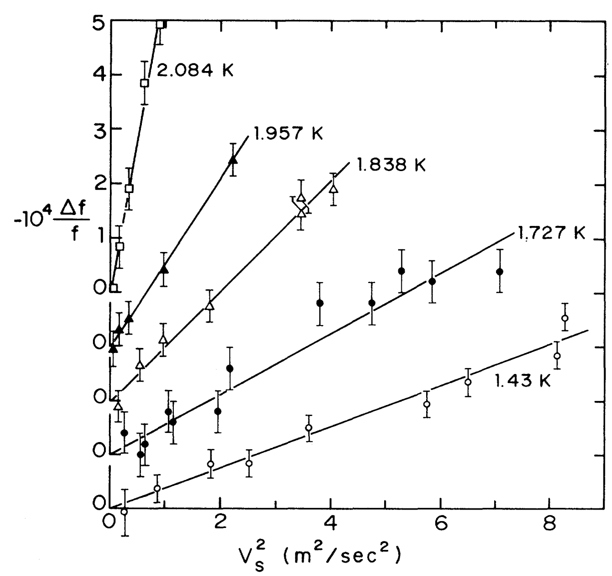

FIG. 2. Fractional change in resonant frequency as a function of the square of the superfluid-velocity amplitude, for several temperatures. The lines are least-square fits to the experimental points. The zeroes are offset.

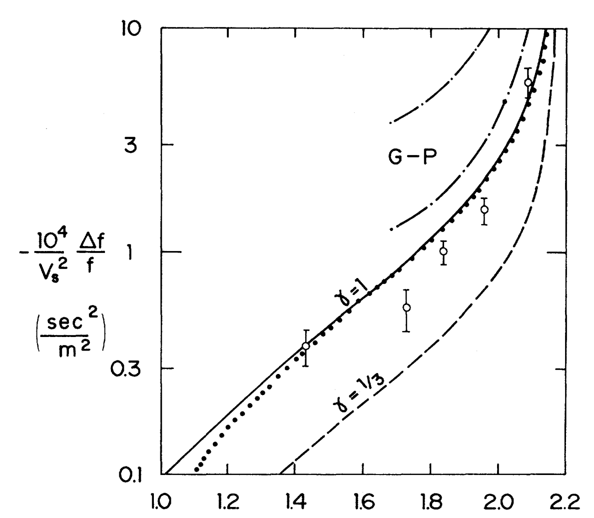

FIG. 3. Coefficient of the velocity effect as a function of temperature, in a semilogarithmic plot. The curves, which represent theoretical models, are discussed in the text.

parabolic horn. The theoretical prediction for $\gamma = \frac{1}{3}$ is shown by the dashed line in Fig. 3. It can be shown that velocity redistribution will increase γ, so these values are lower limits for the respective families of orifice shapes. We have not yet undertaken to solve a realistic boundary-value problem.

The experimental data lie between the two extreme hydrodynamic models and are consistent with the expected temperature dependence of $\Delta\rho_s$, at least below 2.0 K. The Landau quasiparticle model fails to predict the correct $\rho_s(T)$ above about 1.8 K, unless values are used for the roton energy gap $\Delta(T)$ which differ from those measured by neutron scattering.[14-16] The status of the independent quasiparticle theory underlying Eq. (1) is therefore increasingly doubtful as the temperature approaches T_λ. An alternate doubtful approach near T_λ is the phenomenological theory of Ginzburg and Pitaevskii,[17] which uses specific heat and $\rho_s(T)$ as input data and predicts[18,19]

$$\Delta\rho_s/\rho_s = -(m/2\alpha)w^2, \tag{4}$$

with $(2\alpha/m)^{1/2} = 4010(\rho_s/\rho)$ cm/sec.

The predictions of this theory with the two extreme hydrodynamic models are shown by dot-dashed lines in Fig. 3. They lie substantially higher than the Khalatnikov predictions and have a different asymptotic temperature dependence approaching T_λ. Further experimental work in this region would appear to hold considerable interest.

There are other effects which could, in principle, contribute to the variation of resonant frequency with amplitude, including nonlinearity of the diaphragm, normal-fluid flow through the orifice, quantized-vortex generation in the orifice or in a parallel leak, and deflection of pinned vortices.

The maximum dynamic deflection of the diaphragm is about 4 nm. The stiffness of the diaphragm was found to be constant within 0.4% in static deflections up to 2 μm. The Q of the resonance is limited by normal-fluid flow through the orifice and by heat conduction through the chamber walls,[20] and provides an upper limit on the normal-fluid flow. In all cases this limit is less than 8 cm/sec, and at the higher three temperatures it is estimated that $v_n < 2$ cm/sec. The relation between pressure drop and normal-fluid velocity[21] includes a term of order $\rho_n v_n^2$, which can become a significant fraction of linear viscous force. This is the cause of a weak *increase* of Q with amplitude, which is observed at all temperatures except 1.43 K. The viscous term is at least 100 times larger than the normal-fluid inertial term (except somewhat less at 1.727 K). Therefore, the normal-fluid flow can be considered quasistationary, must be very nearly in quadrature with the superfluid flow, and cannot contribute significantly to the frequency shift.

The generation of superfluid vortices below the amplitude for collapse would be a dissipative process to first order, and would be expected to increase steeply with amplitude. The stringent limits on such an effect from measurements of the Q of the resonance rules out any appreciable frequency shift from this cause. Pinned vortices can store energy reversibly by deforming in the superfluid flow and the effect can be nonlinear. A vortex line pinned in the throat of the orifice would have to be shorter than about 40 nm in order not to be washed out at 300 cm/sec; such a vortex would increase the inertance of the orifice by about 1 part in 10^8, with some fractional nonlinearity. It is doubtful that this type of orifice, which appears smooth under both scanning and transmission electron microscopes, can provide sufficient pinning sites for this to be a problem.

A superfluid leakage path shunting the orifice or from either chamber to the bath could produce a frequency shift with amplitude, provided that it was driven to its critical velocity at a relativiely small resonator amplitude. This is analogous to the effect in film oscillations observed by Hammel, Keller, and Sherman[22] and explained by Campbell.[23] For several runs a leak existed in an electrical feedthrough between the outer chamber and the bath, and it caused a reduction of Q and a linear amplitude dependence of frequency. A leak between chambers would be conspicuous in the amplitude dependences of Q and f at the lowest temperatures (~1.2 K).

Well-known nonlinearities in orifice flow of air[24] or water[25] result from generation of vorticity, which does not occur in the superfluid in the velocity range used here.[12] Depression of ρ_s with velocity implies a maximum current density[18,19] which is comparable with the theoretical intrinsic critical velocity.[26] Depression of ρ_s has also been invoked in a model of the vortex core[27] which may be relevant near T_λ.

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Influence of Voids on the Linear Magnetoresistance of Indium

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The influence of the presence of cylindrical voids on the linear magnetoresistance of indium wires is studied experimentally. We show that the voids increase the linear magnetoresistance drastically in accordance with a theoretical model.

The question of linear magnetoresistance in simple metals has attracted a great deal of interest in recent years and seems to be one of the still not clearly settled problems of electron transport in metals. The theory of Lifshitz, Azbel', and Kaganov[1] predicts for uncompensated metals without open orbits a magnetoresistivity (MR) which saturates at high magnetic fields; however, in the alkali metals and in simple polyvalent metals, like indium and aluminium, a linear magnetoresistance (LMR) has been found experimentally. In this Letter we present a quantitative comparison of experiments and theories based on the presence of macroscopic imperfections. By introducing artificial voids in In samples, we show that the LMR increases drastically, and this increase is found to be in excellent agreement with theoretical predictions based on classical arguments.

Some of the observed LMR in the metals K,[2,3] Al,[4] and In[5,6] has been attributed to intrinsic effects like magnetic breakdown or field-dependent relaxation times.[7,8] However, the strong dependence of the LMR on sample handling and sample

