Collective Ion Acceleration through Temporal Modulation of Relativistic-Electron-Beam Energy

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A new method is proposed for collectively accelerating bunches of ions with an intense relativistic-electron beam. By varying the beam energy in time, acceleration of a slow Doppler-shifted cyclotron wave can be accomplished, without accompanying degradation of the beam equilibrium. Test-particle calculations employing this acceleration scheme indicate performance which is highly competitive with other collective-ion-acceleration mechanisms.

The possibility of using relativistic electron beams to collectively accelerate ions has sparked considerable interest recently.[1-7] Among the most promising concepts is that of a traveling-wave accelerator employing the self-fields of the beam itself.[1,2] In previous proposals, wave acceleration was induced by a weak spatial inhomogeneity in either the external guide fields or the geometry, with time-independent beam and field conditions. In contrast we propose to accelerate a slow Doppler-shifted cyclotron wave by temporally varying the beam energy in an axially homogeneous vacuum drift tube. The cyclotron wave is deemed most appropriate of the eight beam eigenmodes because its phase velocity can be varied from near zero to almost the speed of light and because of its moderately strong dependence on beam energy. Among the advantages for this type of acceleration are that it leads to less attenuation of the wave fields during the acceleration process, that the beam equilibrium is altered less by energy change than by either field or geometry variations, and that only simple cylindrical geometry is required.

All accelerator configurations must possess two general characteristics: large fields and the ability to move the fields so that synchronism with the particles is maintained. Conventional accelerators are limited to fields on the order $(4-5)\times 10^4$ V/cm. Fields of 10^6 V/cm or better, however, appear accessible with present relativistic electron beams. Development of higher-energy beams, moreover, will facilitate generation of even stronger fields. Limitations at these very high field strengths may come from field-induced breakdown of chamber materials rather than plasma phenomena. The central problem in using intense relativistic beams, therefore, is in devising controllable methods for moving the field.

Eigenmodes of the electron beam are particularly attractive for collective ion acceleration because of their known propagation characteristics, stability, and periodic field structure. The last of these tends to enforce synchronism of ions. Schemes which use zeroth-order fields, such as virtual cathodes[3,4,7] or localized pinches,[5] produce strong fields but are difficult to control. Previous traveling-wave collective-acceleration schemes[1,2] do employ eigenmodes, but have relied on weak spatial inhomogeneities to increase the phase velocity of the waves. Weak inhomogeneity is required so that the homogeneous dispersion relation is valid locally. The mathematical description of the waves is therefore derived from a WKB analysis. Since the conditions are all steady, the frequency is invariant. Wavelength variations are thus responsible for the increase in phase velocity, $V_{\text{ph}}=\omega/k$. The wave fields must remain large enough to trap the ions, however, and this field varies as $E_z\sim(k-\omega/c)\varphi$. Hence, the trapping field is decreased during the acceleration. To avoid losing ions from the potential well, the wave acceleration must be decreased and the length of the accelerator increased. Since the main economics from collective ion acceleration come from reductions in accelerator length, this is not desirable.

A conceptually different means of inducing wave acceleration is to modulate the beam energy temporally. If the modulation is introduced at the diode—the simplest way—the perturbations propagate with the beam at $v_0=c(\gamma^2-1)^{1/2}/\gamma$. The wave phase velocity is usually much less than v_0, however, so the wave "sees" an almost uniform change in the medium. For example, a plane-wave phase function is $\psi_0=k_0z-\omega_0t$. An infinitesimal variation of the wave number, $k=k_0-\delta k$, can give an arbitrarily large phase change as $z\to\infty$. The entire wave train has simply dilated. Near the front edge of the modulation, the phase velocity, which is the trajectory of a point of constant phase, can then approach arbitrarily close to v_0, as $z \to \infty$. The rate at which this wave velocity increases is governed by the rate of change of the beam energy, $\epsilon = m_0 c^2(\gamma - 1)$, and not by the magnitude of the change. Thus, in a sufficiently long system, one can envision ion acceleration to very high energies with only modest variations in γ.

The goal of collective ion acceleration is to reduce accelerator length, of course, and not to make it arbitrarily long. To quantify the above concepts, we must first derive an appropriate phase function. The main characteristics of the beam cyclotron wave can be derived by considering the r and θ components of the cold-fluid equations. The properties of cyclotron-wave propagation are easily calculated from the phase function.

Consider a cold cylindrical electron beam propagating along the z axis. For an axisymmetric wave ($m = 0$), neglect of diamagnetic effects leads to a suitable set of model equations for small-amplitude cyclotron waves,

$$\frac{\partial \tilde{p}_r}{\partial t} + v_0 \frac{\partial \tilde{p}_r}{\partial z} - \frac{v_\theta \tilde{p}_\theta}{r} = -\frac{e}{m} E_r - \frac{e}{mc\gamma} p_\theta B_z + \frac{e}{mc} v_0 B_\theta, \tag{1a}$$

$$\frac{\partial \tilde{p}_\theta}{\partial t} + v_0 \frac{\partial p_\theta}{\partial z} + \frac{v_\theta \tilde{p}_r}{r} \cong \frac{e}{mc\gamma} \tilde{p}_r B_z, \tag{1b}$$

where $p_i = \gamma v_i$, $V_r = 0$ and $\gamma \cong [1 - (v_0/c)^2]^{-1/2}$. Since we expect an equilibrium rotation, let $v_\theta = V_\theta^R + \tilde{v}_\theta$. The average rotational velocity is

$$V_\theta^R \cong -\frac{\Omega_0}{2\gamma}\left[1 \pm \left(1 - 2\frac{\omega_p^2}{\gamma \Omega_0^2}\right)^{1/2}\right] r, \tag{2}$$

where $\Omega_0 = eB_z/mc$, $\overline{\omega}_p^2 = 4\pi \overline{n}_e e^2/m$, and $\overline{n}_e = \int_0^R n_e \times dr/R$. In the slow rotation mode with $\gamma \gg 1$, the rotation frequency, $\omega_R = V_\theta^R/r$, is small compared to the relativistic gyrofrequency, Ω_0/γ. It furthermore becomes proportionately smaller with increasing γ. Its neglect makes the key physics more transparent without seriously affecting any results. Combining the remainder of (1a) and (1b) gives

$$\left(\frac{\partial^2}{\partial t^2} + 2v_0 \frac{\partial^2}{\partial t \partial z} + v_0^2 \frac{\partial^2}{\partial z^2}\right)\tilde{p}_r + \left(\frac{\Omega_0}{\gamma}\right)^2 \tilde{p}_r = 0, \tag{3}$$

where the assumption that $\gamma \gg 1$ makes v_0 such a weak function of γ that we treat it as a constant. For homogeneous steady conditions, (3) gives the dispersion relation

$$\omega_0 = k_0 v_0 \pm \Omega_0/\gamma_0. \tag{4}$$

A more accurate electromagnetic analysis gives[8]

$$\omega_0 = k_0 v_0 \pm \frac{\Omega_0}{\gamma_0} \frac{(k_0^2 + k_\perp^2)c^2}{(k_0^2 + k_\perp^2)c^2 + \omega_p^2/\gamma_0}, \tag{5}$$

but when $\gamma_0 \gg \omega_p^2/2k^2c^2$ (4) is an excellent approximation. Equation (3) is, therefore, a reasonably accurate equation for describing cyclotron-wave propagation. If we now express $\tilde{p}_r$ in terms of amplitude and phase functions such that

$$\tilde{p}_r = A(z,t) e^{i\psi(z,t)},$$

we can derive a simple equation for ψ in the slowly-varying–amplitude approximation

$$\left(\frac{\partial}{\partial t} + v_0 \frac{\partial}{\partial z}\right)\psi(z,t) = \pm \frac{\Omega_0}{\gamma}, \tag{6}$$

where the + sign is appropriate here. (This corresponds to $\omega = kv_0 - \Omega_0/\gamma$.) If γ is a function only of $\eta = t - z/v_0$, the transformation to (η, ξ) variables, $\xi = t + z/v_0$, permits a general solution to (6),

$$\psi(\eta, \xi) = \tfrac{1}{2}[\Omega_0/\gamma(\eta)]\xi + C(\eta), \tag{7}$$

provided that B_z is independent of ξ. To evaluate the function $C(\eta)$ we need boundary conditions based on physical considerations.

A particularly interesting boundary condition arises if a monochromatic wave of frequency ω_0 is launched at $z = 0$. This might be accomplished with a simple loop antenna, well removed from the beam diode. Then the condition at $z = 0$ is simply

$$\psi(0,t) = -\omega_0 t_0. \tag{8}$$

Without specifying $\gamma(t - z/v_0)$, we can evaluate (7), which leads to

$$\psi(z,t) = k_0 z - \omega_0 t - \frac{\Omega_0}{\gamma_0}\left(1 - \frac{\gamma_0}{\gamma}\right)\frac{z}{v_0}, \tag{9}$$

where we have used (4) as a formal definition of ω_0. The phase velocity corresponds to a point of constant phase ($d\psi/dt = 0$). Thus the general expression for phase velocity, with condition (8), is

$$V_{\rm ph} = \omega_0 - \frac{\Omega_0}{\gamma_0}\frac{z}{v_0}\frac{\partial}{\partial \eta}\left(\frac{\gamma_0}{\gamma(\eta)}\right)\left\{k_0 - \frac{\Omega_0}{\gamma_0 v_0}\left[1 - \frac{\gamma_0}{\gamma(\eta)} + \frac{z}{v_0}\frac{\partial}{\partial \eta}\left(\frac{\gamma_0}{\gamma(\eta)}\right)\right]\right\}^{-1}. \tag{10}$$

For $\partial\gamma^{-1}/\partial\eta<0$, (10) indicates that the phase velocity of a cyclotron wave can be increased, at a rate dependent on the pulse shape of the beam-energy modulation and the distance from the emitter.

The wave acceleration described by (10) differs fundamentally from that induced by spatial modulation. With the latter, the wavelength of a constant frequency disturbance dilates as a function of axial displacement. Since $V_{\rm ph}=\omega/k$, the phase velocity increases proportionately, but the trapping field, $E_{\rm tr}\propto k-\omega/c$, suffers a corresponding decrease. By energy modulating the electron beam, on the other hand, phase changes propagated down the wave train at v_0 alter both the local wavelength and frequency. This ameliorates the degradation of the trapping fields. So long as $v_0 \gg V_{\rm ph}$, the result is a nearly uniform wave-train dilation. Although a lag exists because of the finite propagation velocity of the modulations, the basic picture remains valid. A small change in beam energy can result in a very large velocity change at large displacements. In both spatial and temporal modulation, self-consistent acceleration of traveling waves must degrade the trapping fields. By increasing the frequency as well as the wavelength, however, it is easy to show that temporal modulation leads to less degradation.

The requirement that particles remain trapped implies that the trapping field must be at least as large as the accelerating field. This criterion can be translated into a condition for maximizing the effective acceleration,

$$|a|=\frac{q}{M_i}\left(k-\frac{\omega}{c}\right)\varphi_0, \tag{11}$$

where $k\equiv\partial\psi/\partial z$, $\omega\equiv-\partial\psi/\partial t$, and a, the wave acceleration, is derived from the condition $d^2\psi/dt^2=0$. The functional dependence of γ for maximum average acceleration comes from the solution of (11), which, however, is an exceedingly nonlinear equation. Numerical solutions corresponding to (11), to constant acceleration, and to various analytic forms for γ will be presented in a more complete article. For the present purposes, we will exhibit only a simple numerical example to illustrate the parameters one might expect in an accelerator based on this method.

Despite arguments based on comparison of wave field to wave acceleration, one must ultimately solve the ion equation of motion,

$$\frac{d\bar{p}_i}{dt}=\frac{Ze}{M_i}E_0\sin\psi, \tag{12}$$

where $\bar{p}_i=\gamma_i\bar{v}_i$, and $E_0=(k-\omega/c)\varphi_0$. We have solved (12) numerically for an ensemble of test ions. The test particles were given the same initial velocity and uniformly spaced over one cyclotron wavelength. The wave was assumed to maintain a constant potential, although there is uncertainty about this point. Higher beam energy would facilitate growth to larger potential, but there is at present no good method for gauging the magnitude of potential change with γ variation. For an initial wave field of 10^6 V/cm, and a linear modulation of the energy, $\gamma=\gamma_0(1+\eta/\tau)$, protons initially at 8.5 MeV were accelerated to over 150 MeV in a distance of 380 cm when the beam energy was increased from 3 to 18 MeV. This corresponds to an average accelerating field of over 3.8×10^5 V/cm, which is comparable with the expectations of other traveling-wave schemes and almost an order of magnitude better than conventional accelerators. Velocity distributions of the test particles at $t=0$, τ, and 5τ are shown in Fig. 1. Although significant detrapping has occurred by $t=5\tau$, most particles with $v>0.5c$ were accelerated later to $v>0.60c$. Optimized energy forms will permit further reductions in accelerator length.

Most of the requirements needed for this method of ion acceleration are either within current technology or within foreseeable extensions of it. Intense relativistic electron beams of 3–5 MeV, for instance, are commonplace, while intense beams of 30–50 MeV are not. The fact that $\gamma=20$ machines have been built, however, suggests that higher-energy machines might be feasible, if sufficient incentive were available.[9] The abil-

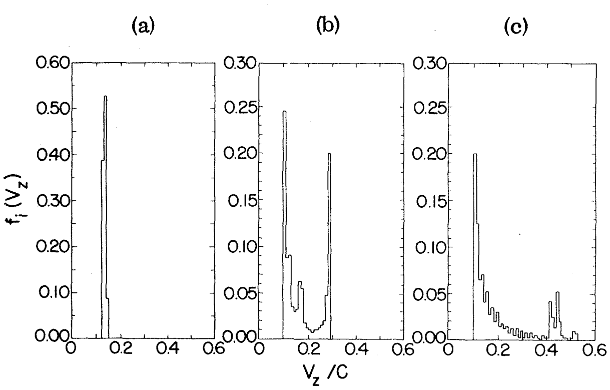

FIG. 1. Velocity distributions of 400 test particles initially at $v=0.13c$. Beam energy was varied as $\gamma=\gamma_0\times[1+(t-z/v_0)/\tau]$. Distributions plotted at (a) $t=0$, (b) $t=\tau$, (c) $t=5\tau$, with $\omega_p\tau=400$. Peak energy at $t=5\tau$ was 164 MeV.

ity to control the energy of the beam in time is not the simplest requirement on an electron-beam machine, but may be feasible through design of a multielement accelerator or decelerator downstream from the ordinary diode.[10]

A more fundamental limitation of this method concerns acceleration to velocities close to c. As mentioned above, there is a lag between introduction of a modulation and the time it reaches the particles, because of finite propagation speed. For $V_{ph} \gtrsim 0.5c$, one can imagine the waves almost outrunning the modulation. Of course, the "kick" from the modulation eventually does catch up, but the additional distance traversed by the particles in the interim has the effect of weakening the average accelerating field. For these high velocities, therefore, it may be more efficient to use other collective-acceleration schemes, some of which, in fact, are inefficient at low velocity.[11] In other applications, ion energies of 100–200 MeV may be acceptable as they are. Finally, even though high velocities may be inaccessible, the use of higher-atomic-number ions, such as in heavy-ion fusion, would still permit generation of significant ion energies, 10–20 GeV for $A = 200$, in very-modest-length accelerators.

There are still many unanswered questions concerning the effect of energy modulations on collective plasma effects. Preliminary studies indicate that the eigenmodes of the cyclotron waves are not seriously affected by this, but these matters require more detailed analytic and numerical studies. These studies are in progress, as well as studies into the effect of energy modulation on other beam modes.

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[1]M. L. Sloan and W. E. Drummond, Phys. Rev. Lett. 31, 1234 (1973).

[2]P. Sprangle, W. M. Manheimer, and A. T. Drobot, Phys. Rev. Lett. 36, 1180 (1976).

[3]C. L. Olson, Phys. Rev. A 11, 2631 (1974).

[4]C. L. Olson, Part. Accel. 6, 197 (1975).

[5]S. Putnam, Phys. Rev. Lett. 25, 1129 (1970).

[6]A. V. Agafonov, A. A. Kolomenskii, and I. I. Logochev, Pis'ma Zh. Eksp. Teor. Fiz. 22, 478 (1975) [JETP Lett. 22, 231 (1975)].

[7]R. B. Miller, R. J. Faehl, T. C. Genoni, and W. A. Proctor, IEEE Trans. Nucl. Sci. 24, 1948 (1977).

[8]W. E. Drummond, G. I. Bourianoff, D. E. Hasti, W. W. Rienstra, M. L. Sloan, and J. R. Thompson, Air Force Weapons Laboratory Report No. AFWL-TR-74-343, 1975 (unpublished).

[9]D. C. Straw, private communication.

[10]A. I. Pavlovskii, V. S. Bosarnykin, G. D. Kuleshov, A. I. Gerasimov, V. A. Tananakin, and A. P. Klement'ev Dokl. Akad. Nauk SSSR 222, 817 (1975) [Sov. Phys. Dokl. 20, 441 (1975)].

[11]B. B. Godfrey, to be published.

Stabilization of Drift Waves by Lower-Hybrid Fields

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Stabilization of drift waves by external rf fields at frequencies near the lower-hybrid frequency has been observed experimentally in a Q machine in the collisionless regime with $(\omega_{pe}/\omega_{ce})^2 \ll 1$. Stabilization is due to a resonant ponderomotive force which increases the drift frequency, thus enhancing the electron Landau damping. The observations are in general accord with analyses in the literature when finite-geometry effects appropriate to the present experiment are taken into account.

The use of rf power at the lower-hybrid frequency has frequently been considered as a means of supplementary plasma heating for fusion purposes.[1] It is therefore of interest to examine the effect of lower-hybrid fields on low-frequency modes such as the drift instabilities, which are sometimes thought to play an undesirable role in experimental plasma devices.

In this Letter we present experimental results which indicate that the drift instability in a collisionless plasma can be stabilized by the application of rf fields near the lower-hybrid frequency. A novel feature of the work is the experimental demonstration of the effect of a *resonant* ponderomotive force which is amplified and changes sign when the frequency of the applied field is

