1977 (to be published); J. Friedrich *et al.*, to be published.
[5]J. Heisenberg and S. Krewald, to be published.
[6]P. Ring and J. Speth, Nucl. Phys. A235, 315 (1974); J. Speth, E. Werner, and W. Wild, Phys. Rep. 33C, 127 (1977).
[7]J. W. Negele and D. Vautherin, Phys. Rev. C 5, 1472 (1972).
[8]E. Weber, W. Knupfer, E. Grecksch, and M. G. Huber, Phys. Lett. 65B, 189 (1976).
[9]H. Zarek *et al.*, Phys. Rev. Lett. 38, 570 (1977); R. A. Lindgren, C. F. Williamson, and S. Kowalski, Phys. Rev. Lett. 40, 504 (1978).
[10]T.-S. H. Lee and S. Pittel, Phys. Rev. C 11, 607 (1975); J. S. Dehesa, J. Speth, and A. Faessler, Phys. Rev. Lett. 38, 208 (1977).

t-dN and t-d^*N Vertices via Spectator Functions

V. S. Bhasin and A. N. Mitra
Department of Physics and Astrophysics, University of Delhi, Delhi-110007, India
(Received 30 December 1977)

We calculate the coupling constants $C^2(^3\text{H-}Nd)$ and $C^2(^3\text{H-}Nd^*)$ by a simple method recently proposed to simulate three-body vertices through spectator functions which require effective two-body normalization. The values obtained (3.2 and 4.8) are in good agreement with the recent determinations of these quantities by Plattner *et al.* from $p+d$ and $p+{}^3\text{He}$ scattering data.

While the concept of few-nucleon vertices is quite old,[1] and its usefulness as a practical tool for analyzing transfer reactions[2] is generally recognized, it has acquired renewed significance of late as a basic theoretical ingredient in the context of rapid developments in three-body techniques.[3] In particular, the determination of the three-nucleon (^{3}He-dp, ^{3}H-dn) couplings from the data has come up for considerable attention in current literature.[4] The recent experimental determination of the analogous ^{3}He-d^*p coupling has opened up the interesting possibility of treating d and d^* on the same footing for analyzing few-body scattering amplitudes in terms of these composites looked upon as single entities despite their vastly different structures (bound versus antibound states). From the theoretical point of view it is a more difficult task to calculate the t-d^*N coupling ($t={}^3$He, ^{3}H; $N=p,n$) than t-dN because of the importance of the cut contribution in the d^* case,[5] a feature which tends to make the t-d^*N vertex more sensitive to input parameters[6] than the t-dN vertex. In particular, a consensus still seems to be lacking on a reliable theoretical estimate of t-d^*N coupling, unlike that of t-dN.

We report here a very simple calculation of t-dN and t-d^*N coupling constants C^2, C^{*2} whose values, viz., 3.15 and 4.77, are in rather good agreement with their recent determination[7] (3.4 ± 0.20, 5.85 ± 0.25) from the analysis of p-d and p-^{3}He scattering data. Our method, which is based on extrapolation to the triton pole of the (off-shell) t-dN and t-d^*N form factors derived directly from the N-d and N-d^* spectator functions corresponding to the triton *bound* state, is expected to be much less sensitive to the "cut" contribution (especially the d^* cut) than one based on scattering amplitudes. The latter method tends to make the t-d^*N calculation less reliable than t-dN, while the present method, which works directly from the *bound*-state wave functions, keeps a safe distance from the cut regions for both the Nd and Nd^* cases, thus facilitating a parallel treatment of comparable accuracy for both.

Now the usual price needed for a standard bound-state calculation of vertices as overlap integrals of the respective cluster wave functions[8] is a knowledge of the normalization constants of the relevant bound-state clusters (t, d, etc.). The present method differs from the conventional bound-state treatment in two respects, viz., (i) we use only the N-d and N-d^* spectator functions[9-11] as a means for providing an effective two-body treatment to an essentially three-body problem, and (ii) we use a *reduced* combinatorial factor with respect to the standard treatment[8] to compensate for "overestimation" of the overlap integral through a two-body description.

This method of calculation of the t-dN vertex, which we shall term as the method of spectator functions (SF), was recently proposed[12] in the context of a model three-boson problem, and was also checked against the standard bound-state calculation[8] of the same quantity as an overlap integral of the t and d wave functions. The method was found to work within 10% accuracy over a wide range of momentum transfers,[12] including the value at the pole. A particular advantage of the SF method is that it does not require any separate knowledge of the d, d^*, and t normalizations. Not only does this fact simplify the calculations enormously but the irrelevance of the d^* normalization in this treatment is also a safeguard against the conflicting claims in the literature on the status of d^*.[13,14]

The calculational features of the SF method, which is basically structured in the separable potential approach, are adequately described in the existing literature[9-11] up to sufficiently realistic potentials. Without attempting to explain the method anew,[9] we shall merely give a self-contained set of definitions and normalizations of the relevant quantities involved. Instead of using the original spectator functions $G(P)$, $F(P)$, etc.,[9-11] which are more convenient for the scattering problem, we first define, in the ideal three-boson case,[12] a *modified* SF, viz. $\tilde{G}(P)=W(P)G(P)$, in terms of the quantity [see, e.g., Eq. (3.18) of Ref. 9]

$$W^2(P)=[\lambda^{-1}-h(P)](P^2+\alpha_0^2)^{-1},$$
$$=\int d^3q\,g^2(q)(q^2+\tfrac{3}{4}P^2+\alpha_t^2)^{-1}(q^2+\alpha_d^2)^{-1}, \quad (1)$$

where $\frac{3}{4}\alpha_0^2M^{-1}=(\alpha_t^2-\alpha_d^2)M^{-1}$ represents the difference between the triton and deuteron binding energies. The introduction of $\tilde{G}(P)$ is motivated by the fact that it satisfies an integral equation with a more explicitly *symmetrical* kernel[12] than does $G(P)$.[9-11] The *Ansatz* of MS[12] now consists in relating the t-dN vertex function $\gamma(P^2)$, for the three-boson case, to $\tilde{G}(P)$ as

$$\gamma(P)=\frac{(2\pi)^{3/2}}{2\mu_{Nd}}N_{Nd}\sqrt{\tfrac{3}{2}}\,(P^2+\alpha_0^2)\tilde{G}(P), \quad (2)$$

$$\tilde{G}(P)=W(P)G(P),\quad N_{Nd}^{-2}=\int d^3qG^2(q). \quad (3)$$

$\gamma(P^2)$ is so normalized that the triton-pole contribution to the S-channel N-d scattering cross section is

$$(d\sigma/d\Omega)_t=\mu_{Nd}^4\pi^{-2}\gamma^4(-\alpha_0^2)(\hbar^2+\alpha_0^2)^{-2}. \quad (4)$$

As explained in MS, the reduced combinational factor $\sqrt{\frac{3}{2}}$ in (2) (instead of the usual[8] $\sqrt{3}$), represents a sort of compensation for the overestimation of the t-d overlap integral by the modified spectator function $\tilde{G}(P)$—something analogous to the so-called Z-factor effect.[8] Such a treatment for obtaining form factors in terms of theoretical wave functions is no doubt approximate,[12] but the extreme simplicity of the normalization, Eq. (3), compared with conventional treatments should progressively bring out its differential advantages over the latter where the numerical complexities of normalization of the *entire* wave function increase rapidly with $N\geqslant 3$.

For the more realistic case of two SF's, viz. $G(P)$ and $F(P)$,[10] representing the N-d and N-d^* channels, we can define the corresponding quantities $\tilde{G}(P)$ and $\tilde{F}(P)$, exactly as in Eq. (3), where the function $W(P)$, Eq. (1), can be defined in either case unambiguously down to the limit $\alpha_d^2=0+\epsilon$ (note the factor q^2 arising from d^3q). This circumstance is particularly relevent to the N-d^* channel since the function $W_{d^*}(P)$ tends to be insensitive to the precise pole position of d^* away from the origin, as long as the magnitude of $|\alpha_{d^*}|$ ($\approx 0.15\alpha_d$) remains close to zero, irrespective of the "sheet" on which it lies. Our approximation merely consists in ignoring a small cut contribution to $W_{d^*}(P)$—the correction is at most of order $x\ln x$, $x=|\alpha_{d^*}|/\alpha_t$—arising out of a little excursion of $\alpha_{d^*}^2$ from the origin into the complex α^2 plane. This fact ensures the validity of a parallel treatment for both the functions $\tilde{G}(P)$ and $\tilde{F}(P)$ in their respective bound-state regions with comparable binding energies [$(\alpha_t^2-\alpha_d^2)/M$ vs $\sim\alpha_t^2/M$]. This is a nontrivial gain for the d^* case whose normalization no longer appears in this mode of calculation of the t-d^*N vertex, in parallel with the t-dN vertex.

With this background, we are in a position to work out the dimensionless coupling constants C^2 and C^{*2} commonly defined in the literature,[3-5] with the identifications

$$C^2=(3\pi)^{-1}\mu_{Nd}^2[\gamma_d^2(-\alpha_0^2)\alpha_0^{-1}],\quad C^{*2}=(3\pi)^{-1}\mu_{Nd}^2[\gamma_{d^*}^2(-\tfrac{4}{3}\alpha_t^2)\tfrac{1}{2}\sqrt{3}\,\alpha_t^{-1}], \quad (5)$$

where $\gamma_d(P^2)$ and $\gamma_{d^*}(P^2)$ are given by (2) and (3) with $\tilde{G}\to\tilde{G}$ and $\tilde{F}$ in turn. To calculate the G and F functions we have used Fig. 3 (p. 24) of SK[11] with the identifications $G(P)=a(P)$ and $F(P)=b(P)$, and worked out their respective two-body normalizations N_{Nd}, N_{d^*N} in accordance with Eq. (3). The resulting functions $\gamma_d(P^2)$, $\gamma_{d^*}(P^2)$ are shown in Fig. 1(a). It is seen that these functions admit of fairly smooth extrapolations to the respective poles $P^2=-\alpha_0^2$ and $-\frac{4}{3}\alpha_t^2$. With the help of Eq. (4) we finally obtain the

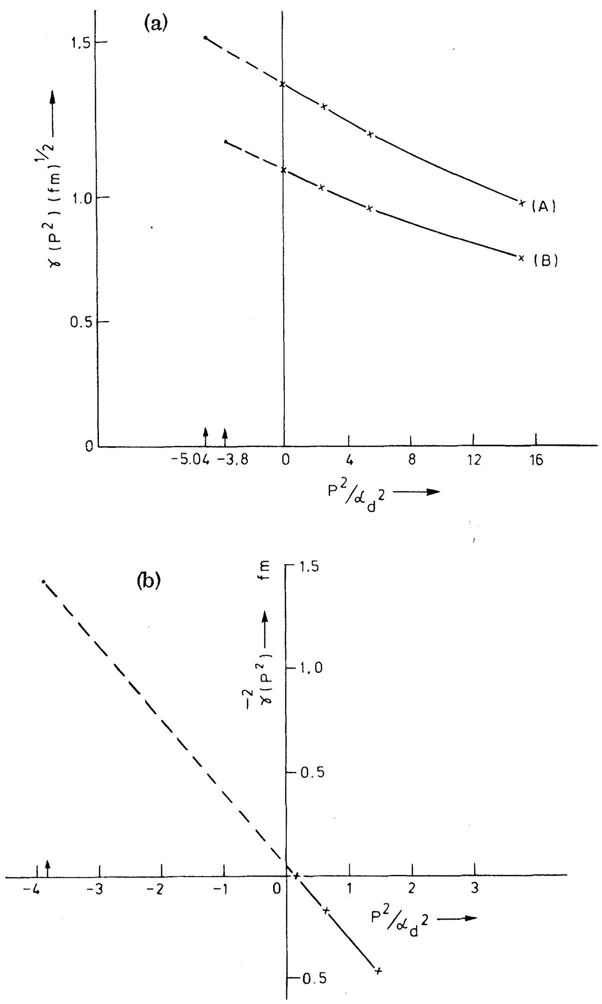

FIG. 1. (a) Vertex functions $\gamma(P^2)$ in fm$^{1/2}$ vs P^2 (in α_d^2 units). Curves A and B refer to $\gamma_{t\text{-}dN}(P^2)$ and $\gamma_{t\text{-}d^*N}(P^2)$ extrapolated to the corresponding pole positions $-\alpha_0^2$ and $-\frac{4}{3}\alpha_t^2$, respectively. (b) Plot of $\bar{\gamma}^2(P^2)$ in fm vs P^2 (in α_d^2 units), using Eq. (8) of text.

results

$$C^2 = 3.15 \pm 0.05, \quad C^{*2} = 4.77 \pm 0.10. \tag{6}$$

For a further check on our results we have considered an alternative method for estimating the value of C^2 based on a zero-energy N-d scattering amplitude, but off the energy shell. For this purpose we note that the function $G(P)$ for doublet N-d scattering satisfies the boundary condition[10,15]

$$G(\vec{P}) = (2\pi)^3\delta(\vec{P} - \vec{k}) + \frac{4\pi a_{1/2}(P)}{P^2 - k^2 - i\epsilon}, \tag{7}$$

where $a_{1/2}(k)$ is the on-shell doublet scattering amplitude so normalized that $|a_{1/2}|^2$ directly represents the differential cross section. On the other hand, the square root $\sqrt{a_{1/2}}(P)$ of the *off-shell amplitude* ($P^2 \neq k^2$) can be related to the vertex function $\gamma(P^2)$, Eq. (2), through the normalization ($\gamma \rightarrow \bar{\gamma}$)

$$\bar{\gamma}(P^2) = \mu_{Nd}^{-1}[\pi a_{1/2}(P)(P^2 + \alpha_0^2)]^{1/2}, \tag{8}$$

and its value at $P^2 = -\alpha_0^2$ represents the t-dN coupling constant. For the quantity $d_{1/2}(P)$ we have used Fig. 2 of Ref. 15 to plot the function $\bar{\gamma}(P^2)$, Eq. (8), in Fig. 1(b). Its extrapolation to the point $P^2 = -\alpha_0^2$ yields, via Eq. (5), the independent estimate $C^2 = 3.40 \pm 0.05$ in remarkably good agreement with experiment[7] as well as the determination (6). This result provides a quantitative check on the validity of the simple two-body method[12] which works for both t-dN and t-d^*N vertices while the applicability of the alternative (scattering) method is limited only to the t-dN case (because of the cut on the t-d^*N vertex). It also provides added confidence in the basic premises (including the combinatorial *Ansatz*) of the new approach which seems to work well beyond the model three-boson problem studied earlier.[12]

While a formal connection between the standard overlap-integral treatment[8] and the two-body SF method[9-12] employed here for evaluating three-body vertices still remains to be established, the results obtained in this paper are nevertheless encouraging enough to warrant further applications of this rather simple approach to a wider variety of physical amplitudes involving few-body vertex functions. In particular, the facility it provides for a parallel treatment of both d and d^* as ingredients of few-body vertices should make this method particularly welcome as a theoretical tool for analyzing breakup reactions which can be viewed in terms of production and exchange of such objects.[16] In the meantime, applications of this method to four-body vertices are in progress.

---

[1]See, e.g., R. Blankenbecler *et al.*, Nucl. Phys. 12, 1100 (1957).

[2]I. S. Shapiro, *Theory of Direct Nuclear Reactions* (Atomizdat, Moscow, 1963).

[3]See, e.g., Y. E. Kim and A. Tubis, Annu. Rev. Nucl. Sci. 24, 69 (1974).

[4]Some references are B. F. Gibson and D. R. Lehman, Phys. Rev. C 11, 29 (1975); L. S. Kisslinger and K. Nicols, Phys. Rev. C 12, 36 (1975); M. P. Locher, Nucl. Phys. A251, 493 (1975); T. K. Lim, Phys. Rev. Lett. 30, 709 (1973); M. Bolsterli and G. Hale, Phys. Rev.

Lett. 28, 1285 (1972); Y. E. Kim and A. Tubis, Phys. Rev. Lett. 29, 1019 (1972); J. A. Hendry and A. C. Phillips, Nucl. Phys. A211, 533 (1973).
[5]V. B. Belyaev *et al.*, Yad. Fiz. 24, 44 (1976).
[6]A. G. Baryshnikov *et al.*, Phys. Lett. 51B, 377 (1974).
[7]G. R. Plattner *et al*; Phys. Rev. Lett. 39, 127 (1977).
[8]L. P. Kok and A. S. Rinat, Nucl. Phys. A156, 593 (1970); A. Rinat, L. P. Kok, and M. Stingl, Nucl. Phys. A190, 328 (1972).
[9]For details on SF, see A. N. Mitra, in *Advances in Nuclear Physics*, edited by M. Baranger and E. Vogt (Plenum, New York, 1969), Vol. 3.
[10]A. N. Mitra, Nucl. Phys. 32, 529 (1962); A. N. Mitra and V. S. Bhasin, Phys. Rev. 131, 1265 (1963).
[11]A. G. Sitenko and V. F. Kharchenko, Nucl. Phys. 49, 15 (1963), referred to as SK.
[12]A. N. Mitra and V. K. Sharma, in *Few Body Dynamics*, edited by A. N. Mitra, I. Slaus, V. S. Bhasin, and V. K. Gupta (North-Holland, Amsterdam, 1976).
[13]See, e.g., R. R. Roy and B. P. Nigam, *Nuclear Physics Theory and Experiment* (Wiley, New York, 1967), p. 62.
[14]M. Nomoto, Nucl. Phys. 81, 180 (1966).
[15]V. S. Bhasin, G. L. Schrenk, and A. N. Mitra, Phys. Rev. 137, B398 (1965).
[16]D. I. Bonbright *et al.*, Phys. Rev. C 8, 114 (1973).

Impact-Parameter Dependence of Ar *K* X-Ray Excitation in Slow Ar-Ar Collisions

H. O. Lutz,[(a)] W. R. McMurray, R. Pretorius, R. J. van Reenen, and I. J. van Heerden
Southern Universities Nuclear Institute, Faure, Cape Province, South Africa
(Received 12 December 1977)

The impact-parameter dependence of Ar *K* x-ray excitation in 2.5-, 4.5-, and 8-MeV Ar-Ar collisions has been measured in a photon–scattered-ion coincidence experiment. The excitation probability has been found to increase sharply at small impact parameters; this effect cannot be explained by current theoretical models.

In recent years, considerable insight has been gained in the field of inner-shell excitation in slow ion-atom encounters. In particular, the *K*-vacancy production in slow collisions lent itself to a quantitative theoretical treatment[1] based on the Fano-Lichten model.[2] The basic mechanism is found to be transfer of a $2p$ vacancy via the $2p\sigma$-$2p\pi$ rotational coupling into the separated-atom *K* shell [process (i)]. Even in cases where no $2p\pi$ vacancy exists prior to collision, the basic nature of this excitation process remains since a $2p$ vacancy may be created on the incoming part of the trajectory independently of process (i) by a long-range interaction [process (ii)]. Both mechanisms have been identified experimentally from measurements of the impact-parameter–dependent *K* excitation.[3,4] In asymmetric collision systems, $2p\sigma$-$1s\sigma$ vacancy sharing on the outgoing part of the trajectory transfers vacancies into the *K* shell of the heavier atom; this transition was not a subject of the investigation to be reported in this paper. A more detailed discussion of the processes which may be involved can be found elsewhere; see, e.g., Meyerhof *et al.*[5] and Meyerhof, Anholt, and Saylor.[6] Because of the fundamental nature of the $2p\sigma$-$2p\pi$ rotational coupling mechanism it is essential to obtain information on the range of applicability of the corresponding theory.[1] There are processes other than (i) and (ii) which have not yet been considered quantitatively. An example may be direct coupling of the molecular $2p$ states at small internuclear separations to higher vacant states. The search for such processes is complicated by the fact that in total-cross-section experiments mechanisms (i) and (ii) unseparably contribute to the measured values. As a result, such experiments allow estimates only.[5,6] Preliminary evidence of processes other than (i) and (ii) has been found in a study of the impact-parameter–dependent *K* excitation in Ar-Ar collisions near threshold.[7] Because of the filled and strongly bound Ar $2p$ shell, the mechanisms (i) and (ii) are effectively suppressed at low ion energies, and other processes may become clearly visible.

This Letter reports an experimental demonstration of the existence of an excitation mechanism which cannot be explained within the framework of processes (i) and (ii). Continuing previously reported work,[7] a photon–scattered-ion coincidence experiment was performed in a study of the Ar-Ar collision system at ion energies of 2.5, 4.5, and 8 MeV. The experiment yields the impact-parameter dependence of Ar *K* excitation, $P(\rho)$. Ar^{3+} ion beams of 2.5, 4.5, and 8 MeV energy from the Southern Universities Nuclear Insti-

