for the mechanism here discussed, if the plasma frequency spectral density were much smaller than Ref. 7 predicts [Eq. (5)], as would occur if simple $T_{\parallel}/T_e$ enhancement were the generating mechanism, then the optical depth would be inadequate to explain the experimental radiation level.

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[9]While this aspect of radial loss models is not widely recognized, it follows directly from the prototypical equation $eD\partial f_{\parallel}/\partial p_{\parallel}=D(p_{\parallel},r)\partial^2 f_{\parallel}/\partial r^2$, using the runaway source function, that $f_{\parallel}$ has a positive slope in $p_{\parallel}$ at outer radii and this will lead to enhanced plasma fluctuations there.

[10]The method of calculating $\mathcal{E}_k{}^{P_{\parallel}}$ was indicated in Ref. 7 above. The slope of the distribution function is $\partial f_{\parallel}/\partial p_{\parallel} =\gamma^2\nu_{ei}/\pi\omega_{pe}p_{\parallel}{}^2$, determined by marginal stability. The turbulent diffusion coefficient necessary to maintain this slope then follows from the particle kinetic equation, $(\partial/\partial p_{\parallel})D_w\partial f_{\parallel}/\partial p_{\parallel}\cong eE\partial f_{\parallel}/\partial p_{\parallel}$ or in the steady state $D_w\partial f_{\parallel}/\partial p_{\parallel}=eEf_{\parallel}$. We define $f_{\parallel}\equiv n_T/p_e$, where $n_T/n=0.35(E_R/E)^{11/8}\exp[-E_R/4E-(2E_R/E)^{1/2}$ (with $E_R =\nu_{ei}p_e/e$) is known from the classical runaway problem. Knowing D_w, Eq. (5) follows directly from the formula for the quasilinear diffusion coefficient, $D_w =8\pi^2e^2\int d^3k\,\mathcal{E}_{\vec{k}}(k_{\parallel}{}^2/k^2)\delta(\omega_{pe}-k_{\parallel}p_{\parallel}/m\gamma)$.

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Resonant Absorption of Laser Light by a Magnetized Plasma

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Theory and a particle-simulation code have been used to investigate resonance absorption of obliquely incident, p-polarized laser light by a magnetized inhomogeneous plasma. The absorption is increased (decreased) by the addition of the external magnetic field with the same (opposite) sign as the angle of incidence. The theoretical results from the two-dimensional particle simulations are in excellent agreement.

Laser light which is p polarized and obliquely incident onto a plasma drives electron plasma waves at the critical surface, resulting in resonant absorption.[1] On the other hand, even normally incident laser light drives upper-hybrid waves[2] if a dc magnetic field is applied perpendicular to the ac electric field, again resulting in resonant absorption. We report here the first theoretical and computational investigation of the combination of the two effects, namely, resonant absorption of p-polarized light which is obliquely incident onto a plasma with an external magnetic field. The absorption must be calculated self-consistently because dc magnetic fields are generated[3,4] by resonant absorption. The maximum absorption is increased to 99%. We find excellent agreement between theoretical predictions and two-dimensional particle simulation calculations.

The high-frequency electron fluid velocity is given by

$$\partial \vec{u}_h/\partial t = -(e/m)\vec{E}_h - (\gamma_e kT_e/mn_l)\nabla n_h - (e/mc)\vec{u}_h\times\vec{B}_0 + \vec{s}_h, \tag{1}$$

where $\vec{s}_h = -(e/mc)(\vec{u}_h\times\vec{B}_l + \vec{u}_l\times\vec{B}_h) - (\vec{u}_h\cdot\nabla)\vec{u}_l - (u_l\cdot\nabla)\vec{u}_h \simeq (-ie/m\omega)\nabla(\vec{u}_l\cdot\vec{E}_h)$, to the third order in the high-frequency quantities; the subscript h (l) designates high- (low-) frequency quantities; $\vec{B}_0(\vec{B}_l)$ is the externally imposed (induced) dc magnetic field; and $\vec{u}_l$ is the low-frequency electron fluid velocity. We have used $\partial/\partial t \to i\omega$ and[4]

$$\nabla\times\vec{u}_l = (e/mc)\vec{B}_l. \tag{2}$$

By combining the high-frequency Faraday's and Ampere's laws, we obtain $(-k_0^2 + \nabla\times\nabla\times)\vec{E}_h = (ik_0 4\pi e/c)(n_l\vec{u}_h + n_h\vec{u}_l)$. This equation becomes [n_h is obtained from $\nabla\cdot\vec{E}_h = -4\pi e n_h$ and $\vec{u}_h$ is obtained from Eq. (1)]:

$$\begin{aligned}&\nabla\times\nabla\times\vec{E}_h - (\gamma_e kT_e/mc^2)\nabla(\nabla\cdot\vec{E}_h) - k_0^2\epsilon\vec{E}_h\\ &\quad= -(ie/mc)\vec{B}_0\times(-k_0^2 + \nabla\times\nabla\times)\vec{E}_h - (ik_0^2/\omega)[\vec{u}_l\nabla\cdot\vec{E}_h + (\omega_{pe}^2/\omega^2)\nabla(\vec{u}_l\cdot\vec{E}_h)],\end{aligned} \tag{3}$$

where $\omega_{pe}^2 = 4\pi n_l e^2/m$ and $\epsilon = 1 - \omega_{pe}^2/\omega^2$.

Also, we use the low-frequency Ampere's law to eliminate $\vec{B}_l$ from Eq. (2) and obtain[4]

$$\nabla\times\nabla\times\vec{u}_l = (-\omega_{pe}^2/c^2)[\vec{u}_l + (e^2/2m^2\omega^3)\,\mathrm{Im}(\vec{E}_h\nabla\cdot\vec{E}_h^*)]. \tag{4}$$

Equations (3) and (4) completely specify the high-frequency $\vec{E}_h$ and the low-frequency $\vec{u}_l$ in terms of $\vec{B}_0$. The induced magnetic field $\vec{B}_l$ can be found from Eq. (2). We now apply these equations to resonant absorption of p-polarized laser light incident obliquely onto an inhomogeneous plasma [the plane of incidence is the x-y plane, $n_l = (x/L)n_c$, where n_c is the critical density, and $\vec{B}_0 = B_0\hat{z}$]. The high-frequency electric field can be written as $\vec{E}_h = [\hat{x}E_x(x) + \hat{y}E_y(x)]\exp[i(\omega t - k_0\alpha_0 y)]$, where $\alpha_0 = \sin\theta$ and θ is the angle of incidence; thus, all the low-frequency quantities are functions of x only. By defining $\vec{u}_l = \hat{x}u_x + \hat{y}u_y$, Eqs. (3) and (4) can be written as

$$\beta^2 d^2E_x/dx^2 + a_x dE_x/dx - (ik_0 n_l/cn_c)[d(u_xE_x + u_yE_y)/dx] + b_xE_x + i\alpha_x dE_y/dx - \beta_xE_y = 0, \tag{5}$$

$$d^2E_y/dx^2 + a_y dE_y/dx + b_yE_y + i\alpha_y dE_x/dx - \beta_yE_x = 0, \tag{6}$$

$$d^2u_y/dx^2 = k_0^2(n_l/n_c)(u_y - v_y), \tag{7}$$

where

$$u_x = (e^2/2m^2\omega^3)[\mathrm{Im}(E_x dE_x^*/dx) + k_0\alpha_0\mathrm{Re}(E_xE_y^*)],$$

$$v_y = (-e^2/2m^2\omega^3)[\mathrm{Im}(E_y dE_x^*/dx) + k_0\alpha_0|E_y|^2], \quad a_x = k_0[b_0(u_y/c + \alpha_0\beta^2) - iu_x/c],$$

$$a_y = -k_0\alpha_0 b_0, \quad b_x = k_0^2[\epsilon - \alpha_0^2 - b_0^2(1-\alpha_0^2) - i\alpha_0 b_0 n_l u_x/n_c c],$$

$$b_y = k_0^2[\epsilon - \alpha_0^2\beta^2 - (1 + n_l/n_c)\alpha_0 u_y/c], \quad \alpha_x = k_0\alpha_0(1 - \beta^2 - b_0^2), \quad \alpha_y = k_0[\alpha_0(1-\beta^2) - u_y/c],$$

$$\beta_x = k_0^2\{\alpha_0 u_x/c + ib_0[n_l/n_c + \alpha_0^2\beta^2 + (1 + n_l/n_c)\alpha_0 u_y/c]\}, \quad \beta_y = k_0^2[\alpha_0 n_l u_x/n_c c - ib_0(1-\alpha_0^2)],$$

$$b_0 = eB_0/mc\omega, \quad \text{and } \beta^2 = \gamma_e kT_e/mc^2.$$

Equations (5) and (6) are the generalized coupled equations for obliquely incident light. The coupling of the induced magnetic field and electron flow is included. These wave equations can be solved by the usual Gaussian elimination method. The boundary conditions are freely outgoing waves at the vacuum side and evanesent waves at the high-density side. The damping of the electrostatic wave propagating down the density gradient is modeled phenomenologically[1,3] such that the absorption due to the generation of the electrostatic wave is treated correctly. An iterative procedure is used to solve these nonlinear equations.

The resultant absorption as a function of external dc magnetic field (angle of incidence) with the angle of incidence (magnetic field) as a parameter (neglecting the induced dc magnetic field) are given in Fig. 1 (2) (dashed lines). The two familiar curves ($\alpha_0 = 0$ and $b_0 = 0$) have been investigated extensively. The $b_0 = 0$ curve is consistent with the numerical results of Forslund *et al.*[1] White and Chen[2] have obtained a maximum absorption of 30% for the $\alpha_0 = 0$ curve by using a Whittaker-form wave equation and a very specific transmitting density profile (maximum density is n_c) instead of a linear profile. Kruer and Estabrook[2] and Grebogi, Liu, and Tripathi,[2] predicted 40% and 70%, respectively, by first estimating the solutions for the electromagnetic drivers which drive the electron plasma waves. Our self-consistent numerical solution gives a maximum of 60% absorption. If $b_0 \neq 0$ and $\alpha_0 \neq 0$, the peak absorption occurs at a lower magnetic field (angle of incidence), and the peak absorption is higher (up to 99%). Also there is zero absorption even for substantial angles of incidence. Just as in previous work, we find that the absorption is almost independent of plasma temperature.

Contours of constant absorption as a function of external dc magnetic field and angle of incidence are shown in Fig. 3 (the induced field is neglected). We see that resonant absorption is increased (decreased) if the external magnetic field and the angle of incidence have the same (different) sign(s). We can understand these results by considering the linearized version ($T_e = 0$) of Eq. (5) (with Faraday's law) as follows:

$$(\epsilon - b_0^2)E_x = -\alpha_0(1-b_0^2)B_h + ib_0(n_l/n_c)E_y. \qquad (8)$$

We see that E_x is resonant at the upper-hybrid

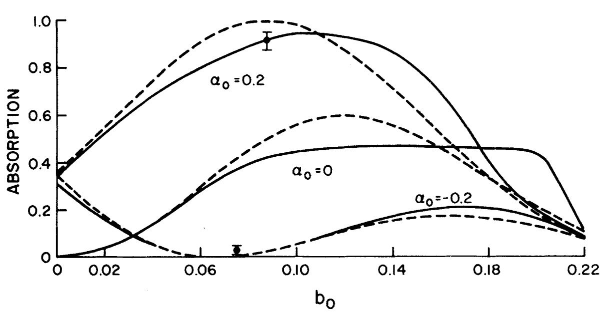

FIG. 1. Absorption as a function of external dc magnetic field ($b_0 \equiv eB_0/m\omega c$). The dashed lines are for infinitesmal incident power, and the solid lines are $v_0/c = 0.05$ for $\alpha_0 \equiv \sin\theta = 0$ and $\alpha_0 = -0.2$, and $v_0/c = 0.035$ for $\alpha_0 = 0.2$, where $v_0 = eE_0/m\omega$. E_0 is the peak electric field of the incident light. The solid circle is obtained from a two-dimensional simulation. The parameters are $v_e/c = 0.1$, $k_0L = 10$ for all figures, and $v_e = (kT_e/m)^{1/2}$.

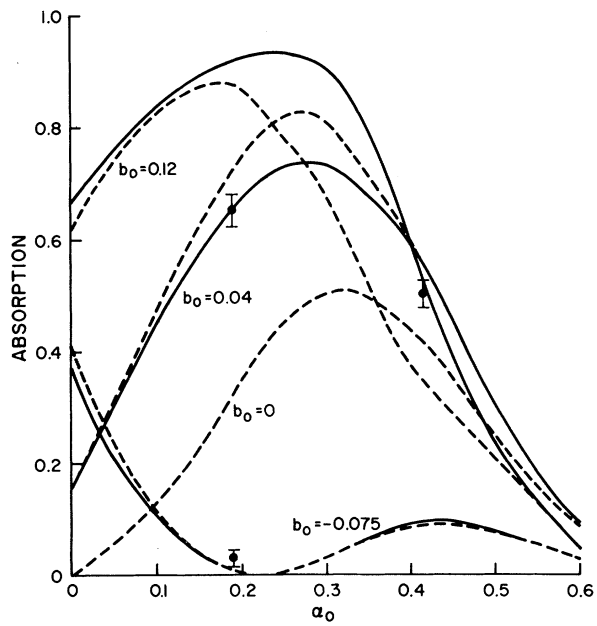

FIG. 2. Absorption as a function of angle of incidence. Same interpretation and parameters as in Fig. 1, except that $v_0/c = 0.035$ for $b_0 = 0.04$ and $b_0 = 0.12$, and that $v_0/c = 0.05$ for $b_0 = -0.075$.

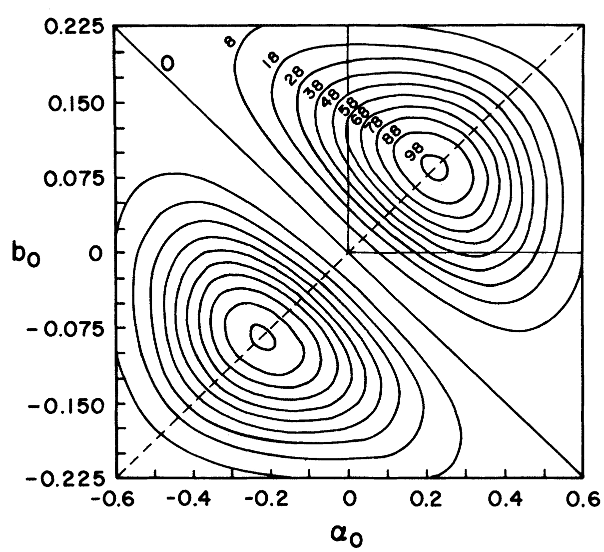

FIG. 3. Absorption contours as functions of α_0 and b_0. The drivers [see discussion following Eq. (10)] are in phase along the dashed line.

resonant point, $\epsilon = b_0^2$ and, additionally, E_x is driven by the sum of the two terms. The first (second) term is due to the component of the light ($\vec{u}_h \times \vec{B}_0$ force) which is parallel with the density gradient. The two effects are additive or do not depend on the phase difference between B_h and E_y. We calculate this phase difference by investigating the difference between the square roots of the effective dielectric functions obtained from the linearized versions of Eqs. (5) and (6). We find that the drivers are in (out of) phase if $k_0Lb_0 \approx (\pm)\pi\alpha_0$ ($3.8\alpha_0$ is obtained from Fig. 3). Thus, the external magnetic field and the angle of incidence must have the same (different) sign(s) for maximum (minimum) absorption. The absorption is an absolute maximum if the two drivers are in phase ($k_0Lb_0 \approx \pi\alpha_0$) and the drivers are maximized[1] $[(k_0L)^{2/3}\epsilon_t \approx 0.5$, where ϵ_t is the effective dielectric function at the turning point of the electromagnetic wave, $\epsilon_t = \frac{1}{2}\{\alpha_0^4 + 4b_0^2(1 - \alpha_0^2)]^{1/2}\}$, obtained from the linearized versions of Eqs. (5) and (6)].

The absorption is not drastically changed for low powers when the induced dc magnetic field [nonlinear terms in Eqs. (5) and (6)] is included (Figs. 1 and 2, solid lines). The main effect is to shift the external magnetic field (or angle of incidence) at which the absorption is a maximum. We see from Fig. 1 that the peak absorption is shifted to a larger external magnetic field. The reason for this is that the induced magnetic field is opposite to the external field near the turning point. Thus, a larger external field is required to match the conditions for maximum absorption.

We have used a two-dimensional particle simulation code (Estabrook, Valeo, and Kruer[1]) with fixed ions to investigate the validity of our results. We find excellent agreement for the absorption at low power ($v_0/v_e = 0.35$, Figs. 1 and 2, solid circles). We could not compare results at high power because our iterative procedure did not converge. Other sources for the induced field than resonant absorption also exist in the simulation (dominantly in the low-density region). However, as would be expected, only the induced field (which is small at low powers) in the resonant region affects the absorption. We verified this by using the total field from the simulation calculation as the external field in Eqs. (5) and (6). The change in absorption is insignificant.

In our numerical solutions of Eqs. (2) and (5) through (8), we found that the maximum induced magnetic field near the resonant point increases as incident power for low powers. It can be understood directly from these equations that $B_l \sim E_0^2$, if the nonlinear terms are neglected. For high incident power, the induced field increases slower than E_0^2. This kind of variation of B_l with incident power has been observed experimentally by Luhmann *et al.*[5]

In summary, we have shown that the resonant absorption of obliquely incident light is increased (decreased) by the addition of an external dc magnetic field with the same (opposite) sign as the angle of incidence. The maximum absorption is about 99%.

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