Mechanism for the ω_{pe} Radiation in Tokamaks

I. H. Hutchinson, Kim Molvig, and S. Y. Yuen
Plasma Fusion Center and Francis Bitter National Magnet Laboratory, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
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The emission of a narrow line of electromagnetic radiation at the central plasma frequency, observed on several Tokamaks, is explained in terms of the scattering of enhanced plasma oscillations from thermal ion acoustic fluctuations. The emission and induced absorption rates calculated for the plasma fluctuation levels expected are considerable and the radiation is self-absorbed to an effective black-body level. The frequency, power, and polarization then predicted are in agreement with the experiment.

It has been known for some time that tokamaks, in the intermediate-density regime ($\omega_{pe}<\Omega_{ce}$, $E/E_R \ll 1$) emit radiation at the plasma frequency.[1] The characteristics of this radiation have been observed on several tokamaks.[1-3] In brief there is a narrow line ($\Delta\omega/\omega_p \lesssim 0.2$) at the central plasma frequency with the emission power near or somewhat above the black-body level corresponding to the central electron temperature. Recent measurements[4] indicate more emission in the extraordinary polarization than the ordinary. The total radiated power does not represent a substantial energy loss. However, its properties provide diagnostic information, for example, on runaway electrons, which can themselves be an important component of the plasma.

This emission is clearly an anomaly in the sense that it cannot result from discreteness or wave-particle processes, since the electromagnetic waves near ω_{pe} which can propagate out of the plasma do not reasonantly interact with electrons. In general, wave-particle processes, whether thermal or nonthermal, satisfy $\omega+n\Omega_{ce} - v_{\parallel}k_{\parallel}=0$, with $v_{\parallel} \ll c$ and n a small integer, possibly zero. Since $\omega/k_{\parallel} \gtrsim c$ for electromagnetic plasma waves, satisfaction of the resonance condition usually requires $\omega \simeq n\Omega_{ce}$, which is why the thermal emission lines occur at the cyclotron harmonics.[5]

In this Letter, we examine the production of electromagnetic waves by the scattering of enhanced electrostatic plasma oscillations from thermal-level ion acoustic fluctuations.[6] The enhanced plasma spectrum has been predicted to accompany runaway electrons and play a role in determining the self-consistent distribution function.[7] The present theory shows that for the levels of electrostatic energy density expected, the electromagnetic emission is so strong that (induced) self-absorption of the radiation is significant and results in a black-body–type emission intensity that is nearly independent of the energy density of the enhanced plasma spectrum. This property renders the present results somewhat independent of the mechanism causing the enhanced plasma spectrum. However, one aspect of the underlying theory which is necessary to obtain agreement with experiment is the frequency spectrum of the enhanced electrostatic waves. The concentration of runaway electrons and associated plasma waves at the column axis as predicted in Ref. 7 thus accounts rather well for the observed frequency of the ω_{pe} line. This is in contrast to runaway models based on radial diffusion[8] which lead to enhanced plasma oscillations at the column edge.[9]

The self-absorbed quality of the radiation is important in understanding the experimental observations for two reasons. The first involves the question of polarization. The coupling to electromagnetic waves propagating perpendicular to the magnetic field is stronger for the ordinary polarization, since the electrostatic fluctuations and the associated nonlinear currents are aligned with the magnetic field. This inherent polarization of the emission process can be altered if both polarizations are self-absorbed to some effective black-body level. Secondly, the level of electrostatic fluctuations is expected to be an extremely sensitive function of the discharge parameters, proportional to the density of the runaway tail which is itself a very strong function of E/E_R. (Where $E_R \equiv m\nu_e\nu_{ei}/e$ is the runaway field and E the Ohmic-heating field.) The ω_{pe} radiation, on the other hand, has experimentally only a rather weak dependence on E/E_R, a fact which the present theory explains since the self-absorbed emission power has no direct dependence on plasma fluctuation level.

In the following, for simplicity, we present first the calculation for the ordinary mode in perpendicular propagation, ignoring reflections from the chamber walls. The question of polarization, for which the effect of reflections must be included, is then discussed to demonstrate consistency with experimental observation.

We consider, then, the restrictions imposed by the frequency and wave-vector selection rules. With the subscripts t, p, and s denoting transverse (electromagnetic), plasma, and scatterer, respectively, these are

$$\omega_t = \omega_p + \omega_s, \quad \vec{k}_t = \vec{k}_p + \vec{k}_s, \tag{1}$$

and $\omega_s \ll \omega_p$. Since the plasma spectrum has $k_{p\perp} \simeq 0$, and $k_{t\perp} < \omega_t/c \simeq \omega_p/c$, we must have $k_{s\perp} < \omega_p/c$. Also, $-k_{s\parallel} = k_{p\parallel} > \omega_p/c$ for the plasma wave implies $|k_{s\parallel}| > \omega_p/c$. Thus the scattering wave has a low phase velocity, $\omega_s/k_s \ll c$, and propagates close to the magnetic field direction, $k_{s\parallel} > k_{s\perp}$. The magnetized plasma wave, $\omega = \omega_p k_\parallel/k$, with $\omega \ll \omega_p$ is thereby ruled out as a scatterer. However, ion waves, specifically acoustic waves, can easily satisfy condition (1).

In order to determine the expected emission power by the scattering of plasma waves from sound waves as described by Eqs. (1), we express the nonlinear current fluctuation in the direction of the polarization vector of the transverse mode as

$$J_{\vec{k},\omega}{}^{\mathrm{nl}} = \int d^3k'\, d\omega'\, \mu(\vec{k}, \vec{k}'; \omega, \omega') \tilde{E}_{\vec{k}-\vec{k}', \omega-\omega'} \tilde{E}_{\vec{k}', \omega'}, \tag{2}$$

where the $\tilde{E}$'s are electric field magnitudes of plasma and sound waves. The coupling coefficient for this process, μ, is

$$\mu = e\omega_p / 4\pi T_e k_\parallel. \tag{3}$$

To give an elementary derivation of this, we write J^{nl} in the form

$$J^{\mathrm{nl}} = -e\tilde{n}_p \tilde{v}_s - e\tilde{n}_s \tilde{v}_p - en\tilde{v}^{\mathrm{nl}}, \tag{4}$$

where the subscripts p and s refer to the linear mode, and vector components in (4) are along the magnetic field for the ordinary mode. We require the density and velocity fluctuations to be expressed in terms of the fields. Since the electron velocity fluctuation associated with the sound wave is very small and since $\tilde{v}^{\mathrm{nl}}$ is proportional to $\tilde{v}_s$, the middle term in (4) is dominant. Using $\tilde{n}_s = ne\tilde{\varphi}_s/T_e = i(ne/k_\parallel T_e)\tilde{E}_s$, and $\tilde{v}_p = -i(e/m\omega_p)\tilde{E}_p$, Eq. (3) follows.

The plasma and sound wave fluctuations are a result of wave-particle interactions and, for the most part, are only weakly affected by the wave-wave process responsible for the radiation. These spectral densities (for field plus particle energies) are known from the runaway theory[10] for the plasma waves:

$$\mathcal{E}_{k_\parallel}{}^P = \int d^2k_\perp \mathcal{E}_{\vec{k}}{}^P = \frac{1}{8\pi}\frac{m^2}{e^2}\frac{\omega_p{}^4}{k_\parallel{}^3}\frac{E}{E_R}\frac{n_T}{n}, \tag{5}$$

where n_T is the density of the runaway tail, and from the equipartition theorem for the acoustic fluctuations

$$\mathcal{E}_{\vec{k}}{}^S = T_S/2(2\pi)^3, \tag{6}$$

where T_S denotes their effective temperature, $T_i < T_S < T_e$.

The kinetic equation is steady state for the transverse-mode spectral density in our convention is[11]

$$\nabla\cdot\vec{v}_g \mathcal{E}_{\vec{k}}{}^t = \int d^3k'\, d^3k''\, |\mu'|^2 \delta(\vec{k}-\vec{k}'-\vec{k}'')\,\delta(\omega_t(\vec{k}) - \omega_p(\vec{k}') - \omega_s(\vec{k}''))$$
$$\times\left[\mathcal{E}_{\vec{k}'}{}^P \mathcal{E}_{\vec{k}''}{}^S - (\omega_s/\omega_p)\mathcal{E}_{\vec{k}}{}^t \mathcal{E}_{\vec{k}'}{}^P - \mathcal{E}_{\vec{k}}{}^t \mathcal{E}_{\vec{k}''}{}^S\right], \tag{7}$$

where $|\mu'|^2 = (4\pi)^4|\mu|^2(\omega_t\,\partial\epsilon/\partial\omega_t)^{-1}(\omega_p\,\partial\epsilon/\partial\omega_p)^{-1}(\omega_s\,\partial\epsilon/\partial\omega_s)^{-1}$, $\omega_t \simeq \omega_p$ has been used, and, as mentioned previously wave-particle interactions are absent from the transverse mode. Emission originates with the first term in (7). The last two terms describe induced absorption of the transverse wave energy.

For the high $\mathcal{E}^P$ levels we consider, the last term is negligible. Then if we take $\mathcal{E}^P$ and $\mathcal{E}^S$ as given, we can rewrite Eq. (7) as a linear transport equation,

$$\nabla\cdot\nabla_g \mathcal{E}_{\vec{k}}{}^t + \gamma\mathcal{E}_{\vec{k}}{}^t = P_{\vec{k}}{}^t, \tag{8}$$

where

$$\gamma = \int d^3k' |\mu'|^2 (\omega_s/\omega_p)\mathcal{E}_{\vec{k}'}{}^P \delta(\omega_t(\vec{k}) - \omega_p(\vec{k}') - \omega_s(\vec{k}-\vec{k}')), \tag{9}$$

$$P_{\vec{k}}{}^t = \int d^3k'\, d^3k''\, \delta(\vec{k}-\vec{k}'-\vec{k}'')\,\delta(\omega_t - \omega_p - \omega_s)\,|\mu'|^2 \mathcal{E}_{\vec{k}'}{}^P \mathcal{E}_{\vec{k}''}{}^S. \tag{10}$$

It remains to integrate Eq. (8) along the rays of the group velocity.

In the process, we distinguish a central region of depth L, containing the enhanced plasma spectrum and a vacuum-wall region.

The damping rate, γ, can be evaluated with Eq. (9) using Eqs. (3) and (5). At transverse frequencies corresponding to the largest $\mathcal{E}^P$, [that is such that $k_{\parallel}' \sim \omega_p/c$ in Eq. (5)], this becomes

$$\gamma \sim \tfrac{1}{4}\pi\omega_p (E/E_R)(n_T/n)(c/V_e)^2, \qquad (11)$$

when the central region is opaque, $\gamma L/v_g \gg 1$, the transverse wave develops a characteristically black-body spectrum, $\mathcal{E}_{\vec{k}}^t = P_{\vec{k}}^t/\gamma_{\vec{k}}$, where the induced emission and absorption balance locally. For example, when $n = 4\times10^{13}$ cm, $T = 1$ keV, and $E = 10^{-2}$ V/cm, so that $E_R/E = 17$ and $n_T/n = 7.2\times10^{-4}$, we find $c/\gamma \sim 3$ cm, so that the central region is "black" even without accounting for the reduced group velocity of the ordinary polarization.

Having established opacity, we can obtain the radiation field directly from the integrand in Eq. (7). Since $(\omega_s/\omega_p)\mathcal{E}^P \gg \mathcal{E}^S$, the effective temperature of the radiation is independent of $\mathcal{E}_{\vec{k}}^P$ and is given by $T_{\rm eff} = (\omega_p/\omega_s)\langle T_S\rangle$. The weighted average, $\langle T_S\rangle$, is characteristic of the edge of the central region, the "gray" layer.

The linewidth resulting from the scattering, $\Delta\omega_t \sim \omega_s$, is extremely narrow, well below the resolution of the experiments so far reported. We therefore compare the total power in the feature $I_t \propto (\omega_p/\omega_s)\langle T_S\rangle \Delta\omega_t \sim \omega_p\langle T_S\rangle$, to that of a black body with the observed linewidth, $I_{bb} \propto T_e\Delta\omega$. For $T_i \sim T_e/3$, $\langle T_S\rangle \simeq T_i$,[12] and thus the ratio

$$I_t/I_{bb} = \omega_p\langle T_S\rangle/T_e\Delta\omega \qquad (12)$$

is about 3, consistent with the observed power level.

Finally, we consider the polarization, determined primarily by the effects of transmission into the vacuum and reflections from the wall. The coupling to the extraordinary wave is negligibly small for perpendicular propagation. At oblique angles, however, the extraordinary wave has a parallel electric-field component and coupling of the order calculated above can occur. At some angle, β, the coupling is sufficient for the extraordinary mode to be "black." If, as is typical, the acceptance angle of the detector is less than $\frac{1}{2}\pi - \beta$, then tracing back extraordinary-mode rays from the detector one finds the plasma optically thin on the first pass. If the rays are randomized in angle on reflection by imperfections in the walls or, for example, bellows construction, then this process continues until the ray is scattered into an angle at which the plasma is optically thick.

For the ordinary mode, a different effect also comes into play. The effective black-body emission level is reduced in the plasma by a factor of ϵ, the local dielectric constant (generally $\ll 1$); however, in propagating out to the vacuum, the solid angle subtended by a ray bundle is focused by the same factor[5] and so the vacuum black-body level is recovered at the detector. This leaves some angles with zero radiation and rays incident on the plasma at these angles from the outside are totally internally reflected, emerging without encountering the emitting layer. If we assume that such rays are "randomized" on reflection from the walls, a proportion, ϵ, will return at angles such as to be absorbed.

It is easy to see that under the assumption of randomization, if $\epsilon \ll \sin\beta$, as is generally the case, the emission in the extraordinary mode will dominate the ordinary. The polarization ratio will then be determined by the wall reflection properties. It will be approximately the same as that associated with the cyclotron radiation, in agreement with experiment.[4]

The theory is thus able to explain the occurrence of a narrow line at the central plasma frequency, its approximate power level, and its polarization.

The experimentally observed variation in power level is perhaps greater than might be anticipated from Eq. (12) by identifying T_S with T_i. Some variation can be accounted for by accurately integrating over the line shape. This can be shown to give a logarithmic dependence of the power on the runaway rate, albeit of the order indicated in Eq. (12). Also the sound-wave fluctuation could be enhanced $(T_S > T_i)$ either by the scattering mechanism here considered or some other effect. In such cases, T_S might become a weak function of discharge parameters. Alternatively, if a significant frequency spread exists in the plasma frequency fluctuations then "thermal" radiation can occur over a wider frequency band and can lead to increased total power in the feature.

The possibility of scattering from other waves, for example from other branches of the ion spectrum, satisfying Eqs. (1) is not ruled out. However, if their coupling coefficients are likewise strong enough to produce self-absorbed radiation then Eq. (12) will apply and the results will be virtually identical. If they are not self-absorbed, then their contribution will be negligible. Equally, for the mechanism here discussed, if the plasma frequency spectral density were much smaller than Ref. 7 predicts [Eq. (5)], as would occur if simple $T_{\parallel}/T_e$ enhancement were the generating mechanism, then the optical depth would be inadequate to explain the experimental radiation level.

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[9]While this aspect of radial loss models is not widely recognized, it follows directly from the prototypical equation $eD\partial f_{\parallel}/\partial p_{\parallel}=D(p_{\parallel},r)\partial^2 f_{\parallel}/\partial r^2$, using the runaway source function, that $f_{\parallel}$ has a positive slope in $p_{\parallel}$ at outer radii and this will lead to enhanced plasma fluctuations there.

[10]The method of calculating $\mathcal{E}_k{}^P{}_{\parallel}$ was indicated in Ref. 7 above. The slope of the distribution function is $\partial f_{\parallel}/\partial p_{\parallel} =\gamma^2\nu_{ei}/\pi\omega_{pe}p_{\parallel}{}^2$, determined by marginal stability. The turbulent diffusion coefficient necessary to maintain this slope then follows from the particle kinetic equation, $(\partial/\partial p_{\parallel})D_w\partial f_{\parallel}/\partial p_{\parallel} \cong eE\partial f_{\parallel}/\partial p_{\parallel}$ or in the steady state $D_w\partial f_{\parallel}/\partial p_{\parallel}=eEf_{\parallel}$. We define $f_{\parallel}\equiv n_T/p_e$, where $n_T/n=0.35(E_R/E)^{11/8}\exp[-E_R/4E-(2E_R/E)^{1/2}$ (with $E_R =\nu_{ei}p_e/e$) is known from the classical runaway problem. Knowing D_w, Eq. (5) follows directly from the formula for the quasilinear diffusion coefficient, $D_w =8\pi^2e^2\int d^3k\,\mathcal{E}_{\vec{k}}(k_{\parallel}{}^2/k^2)\delta(\omega_{pe}-k_{\parallel}p_{\parallel}/m\gamma)$.

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[12]N. A. Krall and A. W. Trivelpiece, *Principles of Plasma Physics* (McGraw-Hill, New York, 1973) (e.g., 11.3.2), noting our $\mathcal{E}_k$ is total (electric field and particle) energy, equal to $W_k/k^2\lambda_D{}^2(2\pi)^3$ in their notation.

Resonant Absorption of Laser Light by a Magnetized Plasma

Wee Woo
Department of Applied Science, University of California, Davis, California 95616

and

Kent Estabrook
Lawrence Livermore Laboratory, University of California, Livermore, California 94550

and

J. S. DeGroot
Department of Applied Science, University of California, Davis, California 95616
(Received 13 February 1978)

Theory and a particle-simulation code have been used to investigate resonance absorption of obliquely incident, p-polarized laser light by a magnetized inhomogeneous plasma. The absorption is increased (decreased) by the addition of the external magnetic field with the same (opposite) sign as the angle of incidence. The theoretical results from the two-dimensional particle simulations are in excellent agreement.

Laser light which is p polarized and obliquely incident onto a plasma drives electron plasma waves at the critical surface, resulting in resonant absorption.[1] On the other hand, even normally incident laser light drives upper-hybrid waves[2] if a dc magnetic field is applied perpendicular to the ac electric

