could, of course, lead to a better agreement between calculation and experiment, but it would be more natural to attribute the "missing" strengths to weakly excited or continuum states which are not identified experimentally. If we assume a rms radius in the range of 4.84 to 4.97 fm as predicted by HF calculations, then the "missing" strength becomes of the order of 50%. This is a particularly unreasonable result for neutron transfers on the $N=50$ closed-neutron-shell ^{88}Sr nucleus.

From the results for proton-transfer reactions also shown in Table II, it is observed that using the rms radius of 4.66 fm, the total sum slightly exceeds the upper limit, thus indicating that the rms radius of $1g_{9/2}$ protons may be slightly larger than that of the $1g_{9/2}$ neutrons. Any unobserved proton-transfer strength would further increase the deduced value of the rms radius.

In conclusion we have shown that, by analyzing existing data for various nucleon-transfer reactions on Sr isotopes, strong support is obtained for the result of Sick *et al.*[2] that the rms radius of $1g_{9/2}$ neutrons in ^{87}Sr is 4.66 ± 0.04 fm.

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Strength Functions for Isobaric Analog Resonances

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A new strength function continuous in energy and a smoothing parameter is constructed which arbitrarily closely approximates the discontinuous strength function for line broadening defined by Lane. The functional form of the Lane strength function is then derived for a fragmented isobaric analog resonance. Its non-Lorentzian form is explicitly and qualitatively different from that assumed in a recent analysis of isobaric analog resonances.

The Duke University high-resolution data on isobaric analog resonances (IAR) have been analyzed using a multilevel R matrix to extract the spin, parity, resonance energies, and widths of all the observed fine structure. Some of these data have been analyzed[1] using the asymmetric Lorentzian strength function (SF) which results[2,3] from using the Lorentz-weighted average of Brown[4] to evaluate $\langle \gamma_\lambda^2/D_\lambda \rangle$, where γ_λ^2 are the reduced widths and D_λ is the level spacing. But the most complete analysis[5] is based on an SF defined by Lane[6] which uses "box averaging," i.e., constant weight over the averaging interval, to evaluate $\langle \gamma_\lambda^2/D_\lambda \rangle$. In their analysis Bilpuch *et al.*[5]

assumed that this latter SF also has the shape of an asymmetric Lorentzian depending on the IAR parameters in the same way as the Lorentz-weighted average. However, consistent sets of IAR parameters could not be found by attempting to fit the Lane SF using this assumption.[7] This result has been attributed by Lane, Lynn, and Moses[8] to a general failure of the SF to describe spreading when the spreading width $\Gamma^{\downarrow}$ is comparable to the level spacing. Unfortunately, this is the case for almost all observed IAR.

The purpose of this Letter is to show that various difficulties enumerated by Lane, Lynn, and Moses arise from their use of box averaging to define an SF, and that their comments and conclusions simply do not apply to the Lorentz-averaged SF. Furthermore, the failure of Bilpuch *et al.*[5] to obtain consistent sets of IAR parameters using the Lane SF is explained by the fact that the energy dependence of the Lane SF is not an asymmetric Lorentzian but a more sharply peaked function which has a different functional dependence on the IAR parameters.

The basis for the discussion is the K-matrix formulation[2,3] of the theory of configuration broadening of a doorway state. The resonant part of the K matrix (or its reduced form obtained by factoring out the energy dependence due to penetrability of the incident particle) has exactly the same form as the R matrix although its parameters have a somewhat different significance.[9] The reduced widths and resonance energies of the IAR found from the Duke University multilevel fit can be identified with the corresponding parameters of the K matrix. In the ensuing discussion, I consider only the case in which the fine-structure states responsible for fragmentation of the IAR have zero intrinsic width. This case is easily generalized to the case of interference between acquired and intrinsic widths, and the figures in this Letter display fits which include the resulting asymmetry.

The resonant part of the reduced K matrix in the elastic channel (channel indices are suppressed) for an IAR at E_A of reduced width γ_A^2 fragmented by coupling through matrix elements M_i to fine-structure states at ϵ_i is

$$\hat{K}_R(E)=\gamma_A^2[E-E_A-\textstyle\sum_i M_i^2(E-\epsilon_i)]^{-1}. \quad (1)$$

The poles and residues of this reduced K matrix are the resonance energies E_λ and reduced widths γ_λ^2 determined from a multilevel fit to the Duke University high-resolution data. An identity relates these to γ_A^2 and the sets $\{\epsilon_i\}$ and $\{M_i^2\}$

$$\sum_\lambda \frac{\gamma_\lambda^2}{E-E_\lambda}=\gamma_A^2\left(E-E_A-\sum_i \frac{M_i^2}{E-\epsilon_i}\right)^{-1}. \quad (2)$$

For the picket-fence model in which all matrix elements are equal to M and the level spacing is a constant D, the envelope of the reduced widths is a Lorentzian

$$\gamma_\lambda^2=\frac{D\gamma_A^2\Gamma^{\downarrow}/2\pi}{(E_\lambda-E_A)^2+(W/2)^2}, \quad (3)$$

with parameters $\Gamma^{\downarrow}=2\pi M^2/D$ and $(W/2)^2=(\Gamma^{\downarrow}/2)^2+M^2$.[2] In this case the sum rule $\sum\gamma_\lambda^2=\gamma_A^2$ and the envelope determine all the parameters. In a real nucleus the M_i^2 fluctuate about some average value and levels are not equally spaced. With care "average" values of the parameters can still be extracted from the width distribution.[10] But for more reliable analysis a *precisely* defined averaging procedure must be used to generate a strength function (denoted SF), schematically defined as $S(E)=\langle\gamma_\gamma^2/D_\lambda\rangle$, whose shape is *completely* determined by a (small) set of average parameters characterizing the IAR and its spreading.

In connection with the S matrix, Brown developed a remarkably useful procedure for performing Lorentz-weighted averaging of dispersion sums by evaluating them at a complex energy.[4] The method can be applied to the K matrix by defining the SF and Lorentz averages as follows[2,3]:

$$S(E;I)\equiv\langle\gamma_\lambda^2/D_\lambda\rangle_I\equiv\sum_\lambda\frac{(I/\pi)\gamma_\lambda^2}{(E-E_\lambda)^2+I^2},$$
$$\langle M_i^2/D_i\rangle_I\equiv\sum_i\frac{(I/\pi)M_i^2}{(E-\epsilon_i)^2+I^2}. \quad (4)$$

Now observe that $S(E;I)=-\mathrm{Im}K(E+iI)/\pi$ and use the fact that Eq. (2) is an identity in E. The parametric form of S for a fragmented IAR follows directly:

$$S^{\mathrm{IAR}}(E;I)=\frac{\gamma_A^2\Gamma_S/2\pi}{(E-\bar{E}_A)^2+(\Gamma_S/2)^2}. \quad (5)$$

The parameters in this SF are $\Gamma_S=\Gamma_I^{\downarrow}+2I$, $\Gamma_I^{\downarrow}=2\pi\langle M_i^2/D_i\rangle_I$, $\bar{E}_A=E_A+\Delta_I$, and $\Delta_I=\sum(E-\epsilon_i)M_i^2/[(E-\epsilon_i)^2+I^2]$. *This result is actually valid whatever the properties of the* $\{\epsilon_i\}$ *and* $\{M_i^2\}$, but it is *useful* only if the averaging interval is taken sufficiently large for $\Gamma_I^{\downarrow}$ and Δ_I to be constant. For the picket-fence model these quantities can be evaluated analytically; for them to be constant requires only that I be chosen so that $\exp(-2I\pi/D)\ll 1$! This condition is well satisfied for $I\geq D/$

2.

It must be noted that Lorentz averaging is remarkably suited to the definition of the SF for a fragmented doorway state. No other averaging procedure can be expected to yield an exact relation between $\langle\gamma_\lambda^2/D_\lambda\rangle$ and $\langle M_i^2/D_i\rangle$. This can easily be seen by considering the general problem of relating $\langle f\rangle$ and $\langle g\rangle$ when a relation, such as $f=(1+g)^{-1}$, holds.

Note from Eq. (4) that this SF is well defined for weak, or even zero, coupling. In the latter case the SF is a Lorentzian of width $\Gamma_S=2I$ and height $\gamma_A^2/I\pi$ centered at E_A. For weak coupling the statistical significance of the spreading width $\Gamma^\downarrow$ is compromised more by errors in the experimental determination of small widths than by the small spreading widths. The crucial point here is that the *range* of energy over which M_i^2/D_i exhibits only statistical fluctuations is of the order of MeV compared to spreading widths of tens of keV. Analysis of the variance of the $\Gamma^\downarrow$ shows that the fractional variance is of order $0.2(I/D)^{-1/2}$.[11] The limitation on increasing I/D so that $\Gamma^\downarrow$ yields the average $\langle M_i^2/D_i\rangle$ for the full "range" of M_i^2/D_i is imposed by the larger *experimental* uncertainty in determining small widths in the wings.

Turn now to consideration of SF defined by Lane using box averaging[6]:

$$S_L(E;I)=\frac{1}{2I}\sum_{(E-E_\lambda)^2<I^2}\gamma_\lambda^2. \tag{6}$$

First observe that S_L is a discontinuous function of energy for any choice of I; its shape is not specified by a small set of average parameters. Moreover, when the strength of the IAR is spread over a small number of levels, even the ratio of the discontinuities to the maximum cannot be reduced by increasing I. Therefore, a procedure for smoothing is an essential part of the definition of the strength function when $\Gamma^\downarrow/D$ is of order unity. I next specify such a procedure in constructing a continuous function of energy and a smoothing parameter ϵ which approximates S_L arbitrarily closely for small ϵ.

Begin by introducing a contour integral into Eq. (6) in order to extend the sum over all $\{\gamma_\lambda^2\}$:

$$S_I(E;I)=\frac{1}{4\pi Ii}\oint dz\sum_\lambda\frac{\gamma_\lambda^2}{z-E_\lambda}. \tag{7}$$

The contour consists of traversal in a positive sense of a rectangle of width 2ϵ with sides parallel to the real axis, a distance ϵ above and below the real axis, extending from $E-I$ to $E+I$. For $\epsilon\ll D,I$ one can neglect the contributions from the ends and obtain a continuous approximation to $S_L(E;I)$:

$$\hat{S}_L(E;I,\epsilon)=\frac{1}{2\pi I}\sum_\lambda\gamma_\lambda^2\left(\tan^{-1}\frac{E+I-E_\lambda}{\epsilon}-\tan^{-1}\frac{E-I-E_\lambda}{\epsilon}\right). \tag{8}$$

It can be shown that both S_L and $\hat{S}_L$ integrate to the area $\gamma_A^2=\sum_\lambda\gamma_\lambda^2$, so their difference is a fluctuating function of zero average value. In Fig. 1 are shown S_L and $\hat{S}_L$ for the $\frac{3}{2}^-$ level of ^{41}K with $\epsilon=0.01D=0.1$ keV and $I=2D=20$ keV; the two curves can only be distinguished at corners.

To find the parametric form of $\hat{S}_L$ for a fragmented IAR, insert Eq. (2) into Eq. (8) and again evaluate the contributions to the contour integral from the sides of the rectangle parallel to the real axis:

$$\hat{S}_L^{\mathrm{IAR}}(E;I,\epsilon)\equiv\frac{\gamma_A^2}{2\pi I}\left(\tan^{-1}\frac{E+I-\hat{E}_A}{\hat{\Gamma}_S/2}-\tan^{-1}\frac{E-I-\hat{E}_A}{\hat{\Gamma}_S/2}\right). \tag{9}$$

The parameters $\hat{\Gamma}_S$ and $\hat{E}_A$ are Lorentzian averages related to ϵ in the same way as Γ_S and $\bar{E}_A$ are related to I, i.e., $\hat{\Gamma}_S=\Gamma_\epsilon^\downarrow+2\epsilon$ and $\hat{E}_A=E_A+\Delta_\epsilon$. Both quantities were taken to be constant in doing the contour integral, requiring that $\epsilon\gtrsim D/2$. Direct integration of the area under $\hat{S}_L^{\mathrm{IAR}}$ also gives γ_A^2. Comparing Eq. (5) with Eq. (9) we see that the energy dependence of the Lane SF is completely different from the Lorentzian form assumed by Lane and his collaborators[5-8] in their attempts to determine the IAR parameters by fitting the SF. In fact, $\hat{S}_L^{\mathrm{IAR}}$ can be expressed as the arctangent of a Lorentzian, with some care necessary in choosing the proper branch.

Numerical confirmation of each of these results is shown in Figs. 1 and 2 by application to the $J^\pi=\frac{3}{2}^-$ IAR state in ^{41}K at $E_p=1.875$ MeV. For this purpose all the above results were generalized to include intrinsic widths $\{\gamma_i^2\}$ of the uncoupled fine-structure states. These widths introduce an asymmetry described by two additional parameters, $\varphi=\tan^{-1}(-\langle M_i\gamma_i/D_i\rangle/\gamma_A)$ and $S_0=\langle\gamma_i^2/D_i\rangle$. For $I=2D=20$ keV, Fig. 2 shows the result of a least-squares fitting of $S^{\mathrm{IAR}}(E;I)$ to the microscopic Lorentz-weighted average

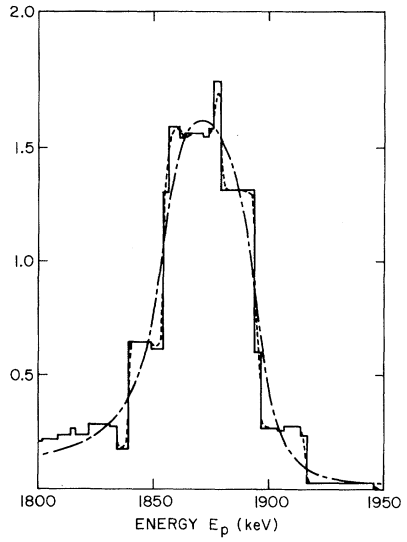

FIG. 1. Lane strength functions for $^{41}K\ \frac{3}{2}^-$ for $I = 20$ keV, $\epsilon = 0.1$ keV. Solid, $S_L(E;I)$; dashed, $\hat{S}_L(E;I,\epsilon)$; and dash-dotted, $S_L^{IAR}(E;I,\epsilon)$.

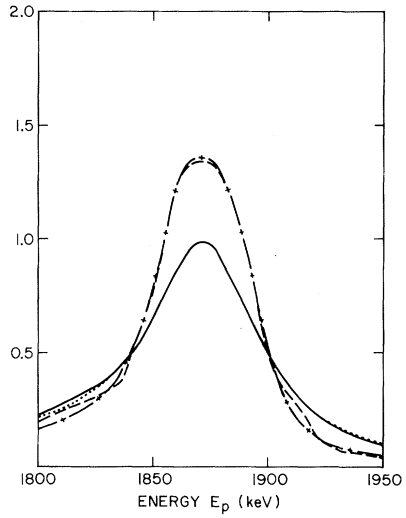

FIG. 2. Lorentzian and Lane strength function for $^{41}K\ \frac{3}{2}^-$ for $I = 20$ keV, $\epsilon = 10$ keV. Solid, Lorentzian $S(E;I)$; dotted, Lorentzian $S^{IAR}(E;I)$; dashed, Lane $\hat{S}_L(E;I,\epsilon)$; and dashed with crosses, Lane $\hat{S}_L^{IAR}(E;I,\epsilon)$.

$S(E;I)$ to determine $\overline{E}_A = 1875$ keV, $\varphi = 0.084$, $\gamma_A{}^2 = 85$ keV, $\Gamma_I{}^{\downarrow} = 17.2$ keV, and $S_0 = 0.04$. Assuming $\Gamma_I{}^{\downarrow} = \Gamma_\epsilon{}^{\downarrow}$ and $\Delta_\epsilon = \Delta_I$ since these are the averages of uncorrelated quantities which are independent of the averaging parameter for ϵ, $I \gtrsim D/2$, I calculate the parameters for $\hat{S}_L{}^{IAR}(E;I,\epsilon)$ and have shown in Fig. 2 the excellent agreement between this function and the continuous microscopic $\hat{S}_L(E;I,\epsilon)$. I have also verified that the *area-preserving* smoothing operations which lead from Eq. (6) to Eq. (9) provide a function $\hat{S}_L{}^{IAR}$ which is a reasonable interpolation of the discontinuous function S_L by plotting $\hat{S}_L{}^{IAR}$ in Fig. 1 for $\epsilon = 0.01D = 0.1$ keV *using the values for the IAR parameters found above.* Clearly $\hat{S}_L{}^{IAR}$ does indeed provide a reasonable interpolating function.

The area under every curve in Figs. 1 and 2 integrates to $\gamma_A{}^2$; so their shapes are directly comparable. The qualitative difference between the Lane SF in Fig. 1 and that defined by Lorentz averaging in Fig. 2 is apparent. The parameter γ_A can be found from the area under any curve, but any attempt to fit the Lane SF in Fig. 2 to a Lorentzian must lead to an incorrect value for the spreading width $\Gamma^{\downarrow}$. In the central region the Lane SF is more strongly peaked than a Lorentzian. In the wings, i.e., for $\hat{\Gamma}_S I \ll (E - \overline{E}_A)^2 + (\hat{\Gamma}_S{}^2/2)^2 - I^2$, the $\hat{S}_L$ follows a Lorentzian, but of width $W = (\hat{\Gamma}_S{}^2 - 4I^2)^{1/2}$. For the parameters used in Fig. 1 this "width" in the wings has the imaginary value $W = 36.1i$ keV!

Lane, Lynn, and Moses[8] have also suggested using the (discontinuous) accumulating strength

$$\sum_{\epsilon_S < E_\lambda > E} \gamma_\lambda{}^2,$$

to determine the IAR parameters. Assuming this to be the integral of a Lorentzian, Bilpuch *et al.*[5] and Mitchell[7] find more stable values for the IAR parameters from fits to this function than from fits to the strength function. This result seems surprising in light of the above results. It can be shown, however, that the accumulating strength is *not* equal to the integral of the (non-Lorentzian) Lane strength function, S_L, as these authors assumed. Nevertheless, when the above area-preserving smoothing procedure is applied to the accumulating strength the resulting continuous function is equal to the indefinite integral of the Lorentzian strength function $S(E;I)$ for $I = \epsilon$. It follows and can be shown explicitly that the parametric dependence of the *smoothed* accumulating strength is therefore equal to the indefinite integral of the Lorentzian $S^{IAR}(E;I=\epsilon)$.[11] This surprising result explains the success of Bilpuch *et al.*, in using the accumulating strength to obtain IAR parameters[5] despite their assumption of an incorrect form for the Lane SF.

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Extended Time-Dependent Hartree-Fock Approximation with Particle Collisions

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We formulate an extended time-dependent Hartree-Fock approximation which includes particle collisions. As the configuration-space analog of the quantum Boltzmann equation, it can be utilized to study the dynamics of nuclear or other fermion systems when irreversible dissipation is present.

Recent renewed interest in the time-dependent Hartree-Fock approximation (TDHF) for the microscopic description of the dynamics of nuclear systems was pioneered by Bonche, Koonin, and Negele.[1] Since then, many TDHF calculations were carried out[2] and many different theoretical investigations were initiated.[2,3] However, in the TDHF approximation, the fermions are assumed to interact only through the mean field and the collisions between particles due to residual interactions are neglected. Because particle collisions are capable of altering the occupation probabilities and dissipating energies, the pattern of behavior of the quantum fluid can be that of hydrodynamics[4] or elastic response,[5,4] depending on the degree of particle collisions. A careful analysis of particle collision also helps our understanding of dissipative phenomena for which much progress has been made recently.[6] However, the incorporation of particle collisions into the TDHF approximation has, up to now, not been formulated.

Previous extension of the TDHF approximation was discussed in terms of a multideterminant representation and Pauli's master equation.[4] In this Letter, we present a different extended time-dependent Hartree-Fock approximation (ETDHF) in which collisions between particles are explicitly taken into account. The final set of equations turns out to be simple and physically transparent and may be of practical interest to those working in the field of the dynamics of nuclear or other fermion systems. Furthermore, concepts such as entropy, temperature, thermal equilibrium, and local equilibrium can be naturally and quantitatively introduced. The approach from nonequilibrium to thermal equilibrium can be quantitatively studied.

Being the configuration-space analog of the quantum Boltzmann equation, our set of equations should retain certain characteristics of the Boltzmann equation. It should be closed Markovian in the sense that all the quantities are to be specified at the same time coordinate.[7] This requirement necessitates an integration over the collision history analytically. Just as in the Boltzmann equation, we wish to keep terms only up to the second order in the residual interaction. Finally, as the Boltzmann equation violates time-reversal invariance, the latter concept needs to be properly introduced.

The starting point of our formulation is the

