Determination of the Quark and Gluon Moments of the Nucleon

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The $n = 2, 4, 6$ moments of the nucleon structure function F_2 were evaluated from muon and electron deep-inelastic scattering measurements on hydrogen and deuterium done at Stanford Linear Accelerator Center and Fermilab. From these moments the quark and gluon content of the nucleon was determined by application of the theory of asymptotic freedom. With the values $0.25^{-0.11}_{+0.10}$ for $R = \sigma_L/\sigma_T$, the scale parameter of the theory was determined to be $\Lambda = 600 \pm 80$ MeV.

Recent measurements of the deep-inelastic scattering of high-energy muons by hydrogen and deuterium at Fermilab[1] have extended considerably the range in the variable[2] $x = Q^2/2M\nu$ over which the structure function $F_2(x, Q^2) = \nu W_2(x, Q^2)$ is known. In this Letter we conbine these with the extensive set of measurements of deep-inelastic electron scattering done at Stanford Linear Accelerator Center[3] (SLAC), to evaluate the moments,

$$\tilde{M}(n, Q^2) = \int_0^1 x^{n-2} F_2(x, Q^2)\, dx. \tag{1}$$

Using a modification of these moments in various bands of Q^2, we were able to fit directly the formulas of asymptotically free gauge theory[4] and thus determine the coupling strength of the theory as well as the moments of the quark and gluon momentum distributions in the nucleon. By working directly with the moments, we avoid the uncertainties involved in attempting to deal with the formulas via Mellin transforms.[5,6]

In asymptotically free gauge theory, the Q^2 behavior of the nucleon structure function is given in terms of its moments by the formula[6]

$$M^{(N)}(n, Q^2) = M_{\rm NS}{}^{(N)}(n, Q_0{}^2) \exp[-\lambda_{\rm NS}(n)s] + M_+(n, Q_0{}^2) \exp[-\lambda_+(n)s] + M_-(n, Q_0{}^2) \exp[-\lambda_-(n)s], \tag{2}$$

where N is either p for the proton or n for the neutron. The Q^2 dependence resides in the variable $s = \ln[\ln(Q^2/\Lambda^2)/\ln(Q_0{}^2/\Lambda^2)]$, with the scale set by the parameter Λ, which determines the coupling strength. The coefficients $\lambda_{\rm NS}(n)$, $\lambda_+(n)$, and $\lambda_-(n)$ are given explicitly by the theory. For the popular quark-gluon theory with three colors and four flavors, $\lambda_{\rm NS}(n) = 0.426\,67$, $0.837\,33$, and $1.080\,38$ for $n = 2$, 4, and 6, respectively; similarly, $\lambda_+(n) = 0.746\,67$, $1.852\,34$, and $2.460\,39$, and $\lambda_-(n) = 0.0$, $0.816\,99$, and $1.074\,27$. The matrix elements $M_{\rm NS}{}^{(p)}$, $M_{\rm NS}{}^{(n)}$, M_+, M_- are not calculated by the theory but can be expressed in terms of the quark and gluon momentum distributions of parton theory.[7] The subscripts refer to the nonsinglet (NS) and singlet (+, −) terms, respectively. The quantity $Q_0{}^2$ is a reference value of Q^2 which may be chosen somewhat arbitrarily, provided that it is large compared with Λ^2. The analysis in this paper was carried out for four flavors. We obtained similar results with three flavors but found a somewhat larger value of Λ.

The quark and gluon moments are defined for $Q^2 = Q_0{}^2$. For up quarks,

$$\langle u\rangle_n = \int_0^1 x^{n-1} u(x, Q_0{}^2)\, dx, \tag{3}$$

where $u(x, Q_0{}^2) = N_u(x, Q_0{}^2) + N_{\bar{u}}(x, Q_0{}^2)$, and similarly for the d, s, and c quarks; for gluons, $G(x, Q_0{}^2) = N_G(x, Q_0{}^2)$. Here, N_i is the number of quarks, antiquarks, or gluons per unit momentum interval, in units of the nucleon momentum.

Following Nachtmann[7] we use the convenient combinations $\langle u+c\rangle$, $\langle d+c\rangle$, and $\langle s-c\rangle$. In terms of these moments, the matrix elements of Eq. (2) may be written

$$M_{\rm NS}{}^{(p,n)}(n, Q_0{}^2) = \tfrac{1}{6}(\pm\langle u+c\rangle_n \mp \langle d+c\rangle_n - \langle s-c\rangle_n) \tag{4}$$

and

$$M_\pm(n, Q_0{}^2) = [\tfrac{5}{18} a_n{}^\mp(\langle u+c\rangle_n + \langle d+c\rangle_n + \langle s-c\rangle_n) - \langle G\rangle_n]/(a_n{}^\mp - a_n{}^\pm). \tag{5}$$

The constants $a_n{}^\pm$ are given by the theory. For the four-flavor model, they have the values[7] $a_2{}^+ = -\frac{18}{5}$, $a_2{}^- = \frac{24}{5}$, $a_4{}^+ = -20.761\,608$, $a_4{}^- = 0.416\,153$, $a_6{}^+ = -39.518\,542$, and $a_6{}^- = 0.174\,905$.

For values of Q^2 accessible at present, the Nachtmann[8] moments,

$$M(n, Q^2) = \int_0^{\xi_{\max}} \xi^{n-2}[1-(M^4/Q^4)\xi^4](1+Q^2/\nu^2)(1+3\eta_n)F_2(\xi, Q^2)d\xi, \tag{6}$$

where $\eta_n = [(n+1)M\nu\xi - (n+2)Q^2]/[(n+2)(n+3)(\nu^2+Q^2)]$, $\xi = [(\nu^2+Q^2)^{1/2} - \nu]/M$, and $\xi_{\max} = [\frac{1}{2}+(\frac{1}{4}+M^2/Q^2)^{1/2}]^{-1}$, should be used in Eq. (2), since the moments defined in Eq. (1) are correct only in the large-Q^2 limit.

We evaluated $M(n, Q^2)$ by simple numerical integration. In a selected band of Q^2, the data were ordered in increasing ξ. At each point, the integrand of Eq. (6) was multiplied by the half-interval in ξ between nearest neighbors. The integral was obtained by summing these products. The contribution from zero to the point with smallest ξ was estimated by assuming zero derivative for F_2 at $\xi = 0$. The measurements extended to values of ξ close enough to $\xi_{\max}$ to make negligible the correction in this region. This method gives greater weight to the more numerous electron data. The muon data receive emphasis at values of x and Q^2 not accessible to the electron experiments. Some of the uncertainties in normalization were taken into account by adding a 2.5% systematic error in quadrature with the experimental statistical error. To avoid ambiguities in both theory and experiment, at least to some extent, we limited our use of the data to $Q^2 \geq 3.0$ GeV^2.

A major uncertainty arises from a lack of knowledge of $R = \sigma_L/\sigma_T$, needed in deducing the value of F_2 from the measured cross section. Present experiments[9] give $R = 0.25 \pm 0.10$, with no dependence on either x or Q^2. Accordingly, the moments were evaluated with the central and extreme values for R.

The contribution of the elastic peak was included in the moments and calculated using the simple scaling law and the empirical dipole formula for elastic scattering.[10] The moments for deuterium were evaluated as the simple sum of proton and neutron contributions, i.e., the impulse approximation. To compensate for the effect of the Fermi motion,[11] we reduced the elastic form factor of the neutron to 42% of its normal value. This reduces the $n = 6$ moment for $Q^2 = 3.25$ GeV^2 by 6%; less for larger Q^2 and for smaller n.

Fitting was done by least squares with use of an improved version of the program MINUIT, written by F. James at CERN. There were twelve parameters, the coupling-strength parameter Λ, and the four quark and gluon moments for $n = 2$, 4, and 6, except that the moment $\langle G\rangle_2$ was not varied independently but was taken from the energy-momentum sum rule as given by $\langle G\rangle_2 = 1 - \langle u+c\rangle_2 - \langle d+c\rangle_2 - \langle s-c\rangle_2$. In the fitting, only those solutions were retained that satisfied the positivity constraints[7,8]: $\langle u+c\rangle_2 \geq \langle u+c\rangle_4 \geq \langle u+c\rangle_6 \geq 0$ and $\langle u+c\rangle_2\langle u+c\rangle_6 - (\langle u+c\rangle_4)^2 \geq 0$, and similarly for $\langle d+c\rangle$, $\langle s-c\rangle$, and $\langle G\rangle$. The fit to the experimental moments, $n = 2$, 4, and 6 obtained for $R = 0.25$ and $Q_0^2 = 30.0$ GeV^2 is shown in Fig. 1. The χ^2 of the fit is 20.4 for 49 data points and twelve free parameters. Although the points are not independent, being different moments of the same data set, the fit for each separate moment is good. Thus, the data can be fitted by the theory to within 2.5% rms, well within the estimates of error that have been given.

By carrying out the fit for various values of Q_0^2, we obtained the Q^2 behavior of the various quark and gluon moments shown in Fig. 2. The moments $\langle s-c\rangle_6$ and $\langle G\rangle_6$ were not well determined. A multiplicity of solutions could be found differing mainly in their $\langle s-c\rangle_6$ and $\langle G\rangle_6$ content. The solutions shown in Fig. 2 were selected to give these a smooth behavior. Solutions were also found for $R = 0.14$ and 0.35. These are indicated in Fig. 2 wherever they differ appreciably from the $R = 0.25$ curve. For the coupling-strength parameter Λ, we found the values 600 MeV for $R = 0.25$, 680 MeV for $R = 0.14$, and 520 MeV for $R = 0.35$. The χ^2 was 16.8 and 25.7 for $R = 0.14$ and 0.35, respectively. The value of Λ might turn out to be different

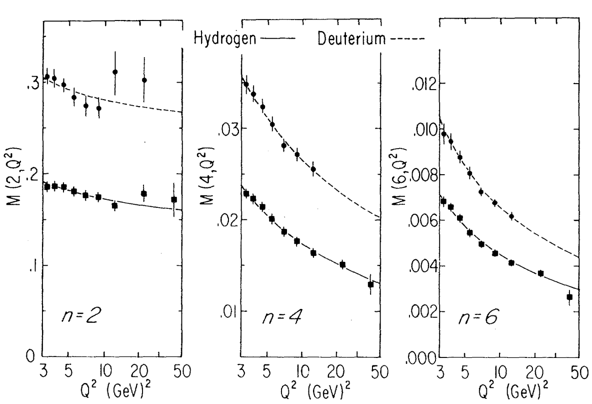

FIG. 1. Least-squares fit to the Nachtmann moments of the nucleon structure function F_2 for hydrogen and deuterium. The fit is for $R = 0.25$, $Q_0^2 = 30.0$ GeV^2 and has $\Lambda = 600$ MeV.

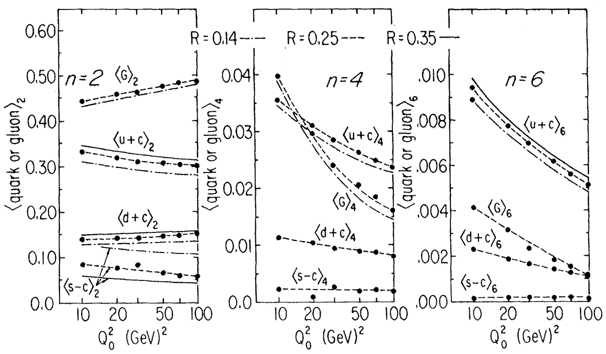

FIG. 2. The quark and gluon moments at $R = 0.25$ which give the best fit to the data. Results for $R = 0.14$ and $R = 0.35$ are indicated where these differ appreciably from the $R = 0.25$ curve.

if there were a significant Q^2 dependence of R. Further uncertainties in the determination of Λ, estimated to be about 10%, arise from systematic errors in the measurements, in the manner of extracting the moments, and in correcting for the elastic peak and the effect of the Fermi motion.

The second moments (Fig. 2), which remain substantially constant, show that the gluons carry about 46% of the total energy momentum, while up and down quarks (c is presumably small) carry about 31% and 14%, respectively. The moment $\langle s - c\rangle_2$ accounts for from 5–12%, depending on what R is. The striking decrease in $\langle G\rangle_4$ (and $\langle G\rangle_6$) with increasing Q^2 indicates that the average momentum carried by the gluons decreases with Q^2. Since the total momentum carried by the gluons remains fairly constant, it is their number seen in the scattering at high Q^2 that increases. The same effect, though less marked, occurs for the up and the down quarks. This is just the behavior described by Kogut and Susskind[12] as a consequence of asymptotic freedom. Our results make these features quantitative.

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[3]R. E. Taylor and W. B. Atwood kindly made available a complete set of the electron scattering results from the Massachusetts Institute of Technology–Stanford Linear Accelerator Center experiments. H. W. Kendall and J. I. Friedman made available a table of the final cross sections from the SLAC experiments (A. Bodek *et al.*, to be published).

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