Core-Polarization and Exchange-Current Effects on the Magnetic Form Factor of ^{17}O

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(Received 2 February 1978)

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The magnetic form factor of ^{17}O is studied by taking account of the effects of core polarization and mesonic exchange currents. It is found that the core polarization reduces strongly the single-particle value of the form factor at the diffraction minimum. In other words, the $M3$ part of the single-particle value is strongly canceled out by the core polarization which, however, does not contribute so much to other multipoles, i.e., $M1$ and $M5$. The pion-exchange current, on the other hand, tends always to enhance the single-particle form factor. The enhancement due to the exchange current is much weaker than the hindrance caused by the core polarization for the $M3$ but stronger than the effect of the core polarization for the $M5$.

The effect of core polarization on electron-scattering Coulomb form factors has been studied in nuclei with either a single particle or hole outside an LS closed shell.[1] The effect on the magnetic form factors of ^{51}V and ^{209}Bi has also been calculated.[2]

In this Letter, we will report our theoretical results on the magnetic form factor of ^{17}O. We take the core polarization into account and estimate also the contribution of exchange currents, because the exchange magnetic moment appears to be important in certain nuclear magnetic moments.

A magnetic form factor $F_M(q)$ only contributes to the transverse electromagnetic form factor $F_T(q)$:

$$|F_T(q)|^2 = |F_M(q)|^2 = \frac{4\pi}{Z^2}\frac{1}{2J+1}\sum|\langle J\|\hat{T}_M{}^{(\lambda)}(q)\|J\rangle|^2, \tag{1}$$

where $\hat{T}_{M\mu}{}^{(\lambda)}(q)$ is the magnetic multipole operator defined as

$$\hat{T}_{M\mu}{}^{(\lambda)}(q) = \int d^3r\, j_\lambda(qr)\vec{Y}_\mu{}^{(\lambda\lambda 1)}(\Omega)\vec{J}(r). \tag{2}$$

In the following, one-body currents together with two-body meson exchange currents are taken into account.

We begin with the one-body current whose contributions are calculated in the impulse approximations. The core polarization is treated by using first-order perturbation theory:

$$\langle\langle j\|\hat{T}_M{}^{(\lambda)}\|j\rangle\rangle = \langle j\|\hat{T}_M{}^{(\lambda)}\|j\rangle - \sum_{phJ}\frac{2}{\Delta E_{ph}}(2J+1)W(phjj;\lambda J)\langle pjJ|V|jhJ\rangle\langle h\|\hat{T}_M{}^{(\lambda)}\|p\rangle. \tag{3}$$

The first term on the right-hand side is the contribution of the single particle which for a neutron with quantum numbers n, l, and j is

$$\langle lj\|\hat{T}_M^{(\lambda)}\|lj\rangle = g_s\frac{eh}{MC}\frac{iq}{8\sqrt{\pi}}\frac{(2j+1)^{1/2}}{[\lambda(\lambda+1)(2\lambda+)]^{1/2}}(j\tfrac{1}{2}\lambda 0|j\tfrac{1}{2})\tfrac{1}{2}\{1-(-1)^{\lambda}\}$$
$$\times\{\lambda(\lambda+1-2\epsilon)(j|j_{\lambda+1}(qr)|j)+(\lambda+1)(\lambda+2\epsilon)(j|j_{\lambda-1}(qr)|j)\}, \qquad (4)$$

where $j_\kappa(x)$ are spherical Bessel functions and $\epsilon=(-1)^{l+1/2-j}(j+\frac{1}{2})$.

A harmonic-oscillator radial dependence is assumed for single-particle wave functions. The length parameter b of the oscillator is taken to be 1.8 fm, which is very close to the value found by elastic electron scattering[3] on ^{17}O. We shall use also $b=1.7$ fm to see how sensitive the calculated form factor depends on the b parameter. The center-of-mass correction and proton finite-size effects are taken into account by using the harmonic-oscillator model; the calculated results are multiplied by a factor $\exp[-\frac{1}{4}(a_p^2-b^2/17)q^2]$ with $a_p=0.657$ fm.

The core-polarization effects are calculated up to $(\lambda_{max}+1)\hbar\omega$ excitations, where λ_{max} is the highest possible multipole in the transition; in the present case $\lambda_{max}=5$. For the states contributing to the core polarization, the single-particle energies are assumed to be degenerate and the value of $\hbar\omega$ is fixed to be 14.4 MeV. As the effective interaction V in Eq. (3), two phenomenological interactions are employed: one with the Rosenfeld mixture (RM) and the other which has the same exchange character as the RM except for a sign change in the attractive triplet odd state (ATO). The same Gaussian radial dependence is assumed for both interactions. The range parameter $r_0(\frac{1}{2}\nu)^{1/2}$ is assumed to be 0.67, where r_0 is the range of the interactions and $\nu=m\omega/\hbar$. The strength of the potentials is assumed to be -72 MeV in the triplet even state. The numerical results are shown in Figs. 1(a), 1(b), and 2.

For the two-body exchange currents, we take account only of those arising from one-pion–exchange processes, among which pionic and pair diagrams are considered.[4] Making use of the nonrelativistic reduction of the Feynman diagrams, one can write the two-body current densities in momentum space

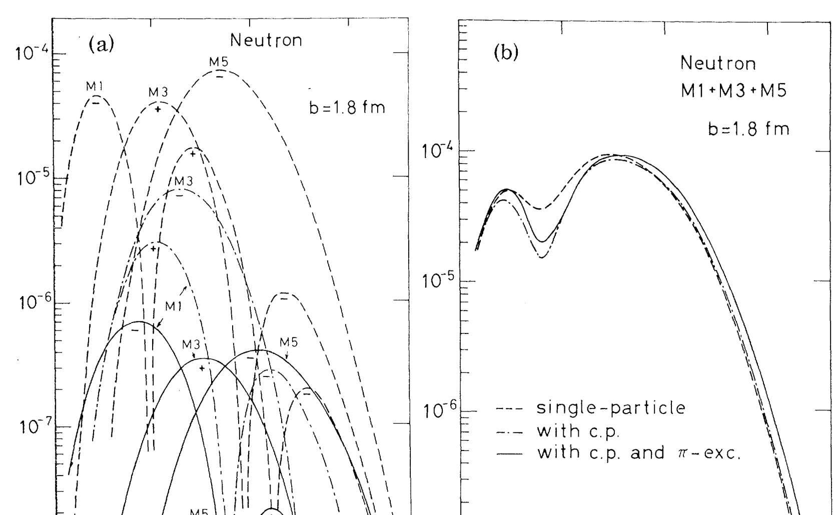

Fig. 1. (a) The square of $M1$, $M3$, and $M5$ form factors for the $0d_{5/2}\to 0d_{5/2}$ transition. The dashed, dash-dotted, and solid curves represent the individual contribution of the single-particle value, the core-polarization effects, and the one-pion–exchange current contributions, respectively. The sign + or − denotes the relative sign of the form factors. (b) The square of the magnetic form factors for the $0d_{5/2}\to 0d_{5/2}$ transition. The dashed curve is calculated with the single-particle harmonic-oscillator wave function. The dash-dotted curve includes the core polarization and the solid curve includes further the one-pion–exchange current effects.

as

$$\vec{J}_{\text{pionic}}(\vec{k},\vec{q}) = -2\sqrt{2}\,[\vec{\tau}_1\otimes\vec{\tau}_2]_0^{(1)} e(g/2M)^2[(\vec{\sigma}_1\cdot\vec{k}_1)(\vec{\sigma}_2\cdot\vec{k}_2)\vec{k}/\omega_1^2\omega_2^2], \tag{5a}$$

$$\vec{J}_{\text{pair}}(\vec{k},\vec{q}) = -\sqrt{2}\,[\vec{\tau}_1\otimes\vec{\tau}_2]_0^{(1)} e(g/2M)^2[\vec{\sigma}_1(\vec{\sigma}_2\cdot\vec{k}_2)/\omega_2^2 - \vec{\sigma}_2(\vec{\sigma}_1\cdot\vec{k}_1)/\omega_1^2], \tag{5b}$$

where $\vec{k}_1 = \vec{p}_1{}' - \vec{p}_1$, $\vec{k}_2 = \vec{p}_2{}' - \vec{p}_2$, $\omega_i^2 = \mu^2 + \vec{k}_i^2$, $\vec{k} = \frac{1}{2}(\vec{k}_2 - \vec{k}_1)$, and $\vec{q} = \vec{k}_1 + \vec{k}_2$.

Using the Fourier transformation of the current densities

$$J(q,\vec{r}_1,\vec{r}_2) = \int d^3k\,(2\pi)^{-3}\exp[i(\vec{q}\cdot\vec{R}) - i(\vec{k}\cdot\vec{r})]\,J(k,q), \tag{6}$$

where $\vec{R} = \frac{1}{2}(\vec{r}_1 + \vec{r}_2)$ and $\vec{r} = \vec{r}_1 - \vec{r}_2$, one can define a two-body multipole operator as

$$\hat{T}_{M\mu}{}^{(\lambda)}(q,\vec{r}_1,\vec{r}_2) = [(-i)^\lambda/4\pi]\int d\Omega_q [Y^{(\lambda)}(\Omega_q)\otimes J(q,\vec{r}_1,\vec{r}_2)]_\mu{}^{(\lambda)}. \tag{7}$$

Making use of Eqs. (5a) and (6), one can further rewrite $\hat{T}_{M\mu}{}^{(\lambda)}$ as

$$\hat{T}_{M\mu}{}^{(\lambda)}(q,\vec{r}_1,\vec{r}_2) = \sum_{lLAJ\gamma S}(-i)^{\lambda+L+l}(-1)^{L+J+l+S}[3(2J+1)]^{1/2}\langle A\|Y^{(L)}\|\lambda\rangle\begin{Bmatrix}J&L&l\\A&\gamma&\lambda\end{Bmatrix}\begin{Bmatrix}J&\lambda&\gamma\\1&S&\lambda\end{Bmatrix}\int dk\,k^2 F(k,q,\vec{r}_1,\vec{r}_2), \tag{8a}$$

$$F(k,q,\vec{r}_1,\vec{r}_2) = G_{AlS}{}^{(\gamma)}(k,q)[\vec{\tau}_1\otimes\vec{\tau}_2]_0^{(1)} j_L(qR)j_l(kr)\{[Y^{(L)}(\hat{R})\otimes Y^{(l)}(\hat{r})]^{(J)}\otimes[\vec{\sigma}_1\otimes\vec{\sigma}_2]^{(S)}\}^{(\lambda)};$$

$$G_{AlS}{}^{(\gamma)}(k,q) = (2\gamma+1)^{1/2}\pi^{-1}\sum_{\alpha,\beta,\delta}(-1)^{l+\alpha+\gamma}\langle\delta\|Y^{(\alpha)}\|A\rangle\langle\delta\|Y^{(\beta)}\|l\rangle\begin{Bmatrix}A&l&\gamma\\\beta&\alpha&\delta\end{Bmatrix}\int_{-1}^{1}P_\delta(x)T_{\alpha\beta S}{}^{(\gamma)}(k,q,x)\,dx. \tag{8b}$$

Here $T_{\alpha\beta S}{}^{(\gamma)}(k,q,x)$ are simple mermorphic functions of k, q, and x. The integration over x in Eq. (8b) can be carried out analytically; the integration over k in Eq. (8a) is done numerically. The expectation value of the operator

$$\sum_{i>j} T_{M\mu}{}^{(\lambda)}(q,\vec{r}_i,\vec{r}_j)$$

for the ground state of ^{17}O provides the contribution of the two-body exchange currents to the magnetic form factor. The numerical results are shown in the figures.

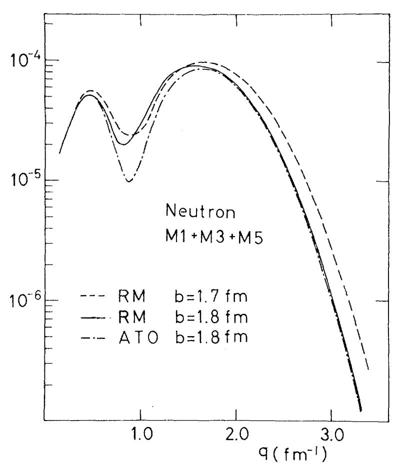

FIG. 2. The square of the magnetic form factors for the $0d_{5/2}\to 0d_{5/2}$ transition. All curves include both the core-polarization and exchange-current contributions. The solid and dashed curves are obtained by using the Rosenfeld interaction with different values of the b parameter, while the dash-dotted curve is given by an attractive triplet odd interaction (ATO).

For magnetic multipole moments, it has long been known that the core polarization reduces the single-particle values of low multipole operators[5,6] such as dipole $M1$ and octupole $M3$, though its effect vanishes for the magnetic dipole moments of nuclei with a particle or hole outside an LS closed shell.[5] The core polarization, however, may or may not reduce the single-particle values for higher multipole moments such as $M5$ in ^{17}O, depending on the exchange nature of the residual interaction.

A similar behavior is expected for the magnetic form factor particularly when q is small. The core polarization is thus expected to reduce the $M3$ form factor very much. On the other hand, the core polarization affects the $M5$ form factor very little. These statements are confirmed by the present calculation as shown in Fig. 1(a).

As for the $M1$ form factor, the q dependence of the form factor due to the core polarization is very different from that of the single-particle form factor, since the main contributions given by the core polarization come from particle-hole excitations with a node change such as $0s\to 1s$ and $0p\to 1p$ [see Fig. 1(a)]. This fact is related to the vanishing correction of the magnetic moment in ^{17}O.[5] Hence, the core-polarization effect on the $M1$ form factor of ^{17}O is small near its first peak.

The results of the present calculation are shown in Figs. 1(a), 1(b) and 2. As was already discussed, the core polarization makes the diffraction minimum around $q \approx 1\ \mathrm{fm}^{-1}$ much deeper than that of the single-particle value, which is shown in Fig. 1(b). This effect comes mainly from the large reduction of the $M3$ form factor in its first peak. On the other hand, the pion-exchange current always enhances the single-particle value, and this effect is noticeable particularly in a region of large q. However, since the form factor for such a large q is sensitive to the radial wave functions, unfortunately the present result does not necessarily provide a good test of exchange currents. Figure 2 shows the dependence of the total form factor on the b parameter. As just mentioned, the form factor at the large-q region is sensitive to the choice of the b parameter.

The dependence of the core polarization on the interaction assumed is also shown in Fig. 2, from which one sees that the effect is more comspicuous for the ATO interaction than for the case of the Rosenfeld interaction (RM). This feature is the same as observed in the Coulomb form factors.[1]

A recent observation showed that the minimum of the form factor is much deeper than that given by the single-particle value.[7] In other words, the experiment does not see the $M3$ contribution at all. The data seem to confirm at least qualitatively the present theoretical picture and in particular the importance of the core polarization. In heavier nuclei ^{51}V and ^{209}Bi, the $M3$ form factors have been known to be hindered very much both theoretically[2] and experimentally.[8]

In the present work, we have not considered the following possibly important contributions: the impulse term due to the core polarization induced by a residual tensor force, and the interference terms between this core polarization and the exchange currents. If these contributions are included, the q dependence and the magnitude of the magnetic form factors may be altered significantly. We expect that this occurs at the high-q region in particular for $M5$ but not at the lower-q region. Therefore our conclusion regarding the $M3$ form factor should remain the same. These points shall be discussed in detail in a separate paper.

The authors would like to thank Professor W. Bertozzi and other members of the Massachusetts Institute of Technology group for showing us their experimental results before publication. Our thanks are also due to Professor D. Strottman for his critical reading of the manuscript. This work was supported in part by the U. S. Department of Energy under Contract No. EY-76-S-02-3001.

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