to ∞. When $M'^2 > B^2 + \mu^2$, therefore, we have to integrate along the solid curve shown. Note that the integrand in (8) is an analytic function of t in the relevant region, so that we are permitted to deform the contour—we have already pointed out that $N(t)$ has no cut along the positive real axis.

We may now let the curves in Fig. 1 approach the real axis. The difference between the value of (15) on the two sides of the cut for $t_0 < t < 4\mu^2$ is $-2\pi i/p'q$, so that (8) is replaced by

$$A^{(2)}(M', t) = -\frac{1}{4\pi D(t)}\int_{t_0}^{4\mu^2} dt' \frac{N(t')}{|p'|w'(t'-t)} + \frac{1}{8\pi^2 D(t)}\int_{4\mu^2}^{\infty} dt' \frac{q'N(t')A^{(1)}(M', t')}{w'(t'-t)}. \quad (19)$$

$A^{(2)}(M', t)$, given by Eq. (8) if $M'^2 < B^2 + \mu^2$ and by Eq. (19) if $M'^2 > B^2 + \mu^2$, is thus an analytic function of M'^2 if $t > 4\mu^2$. It might be thought that there is a branch point when $M'^2 = B^2 + \mu^2$, but this is not the case, as we would have obtained exactly the same result had we given M'^2 a negative instead of a positive imaginary part. The curves in Fig. 1 would then be reflected across the real axis, but the sign change in the difference of the logarithm on the two sides of the cut would be cancelled by the change of sign of q' in the integrand of Eq. (8).

When $M'^2 > B^2 + \mu^2$, we have seen that $A^{(1)}(M', t)$ behaves like $-2\pi i/p'q$ at $q = 0$, so that $\mathrm{Im}A^{(2)}(M', t')$ is equal to $-g^2(M')B(4\mu^2)/16\pi\mu(M'^2 - \mu^2)^{1/2}$ instead of zero at $t = 4\mu^2$. The cut in $A^{(2)}$ will extend to the anomalous threshold at $t = t_0$. By putting $M' = M$ in (19) we obtain the result of physical interest, with an anomalous threshold at $t = t_1$. The method of applying the unitarity condition only above the physical threshold, but for varying masses, and then continuing analytically in the masses, can thus handle both the normal and anomalous cases. The extra cut in the anomalous case is a mathematical consequence of the analytic continuation, and does not appear to have any precise physical significance.

*Supported by the U. S. Air Force Office of Scientific Research.

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NEW TEST FOR $\Delta I = 1/2$ IN K^+ DECAY*

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(Received December 28, 1959)

We wish to suggest a practicable test of the $\Delta I = 1/2$ rule, based on a comparison of the pion-energy distributions in τ^+ and $\tau^{+\prime}$ decay. At present, the only check of $\Delta I = 1/2$ in these processes is the successful prediction of the τ'/τ branching ratio.[1] However, it is well known that the branching ratio tells us that $\Delta I = 5/2$ and $\Delta I = 7/2$ are absent but tells us almost nothing about the possible presence of $\Delta I = 3/2$ terms. The only symmetric three-pion states have $I = 1$ or $I = 3$, and the other, nonsymmetric and hence inhibited states with $I = 1$ or $I = 2$ (which could be produced by a $\Delta I = 3/2$ term) cannot interfere with the symmetric states in a measurement of decay rates. It is of course very important to learn whether the nonleptonic weak interactions involve a mixture of $\Delta I = 1/2$ and $\Delta I = 3/2$. In particular, it has been noted that such a mixture would result if these interactions arose from a folding of a $\Delta I = 1/2$ strangeness-nonconserving current with the usual $\Delta I = 1$ β-decay current.[2] From experience with the τ'/τ ratio we see that a test for $\Delta I = 3/2$ terms must depend on measurements of pion asymmetries of some sort.[3]

Suppose we let $A_\tau(T_1T_2T_3)$ and $A_{\tau'}(T_1T_2T_3)$ be the Lorentz-invariant amplitudes for K^+ decay into $\pi^+\pi^+\pi^-$ or $\pi^0\pi^0\pi^+$ with kinetic energies T_1, T_2, and T_3, respectively. The Bose statistics of pions implies that

$$A_j(T_1T_2T_3) = A_j(T_2T_1T_3) \quad (1)$$

for $j = \tau$ or τ'. We shall break up A_j into symmetric and nonsymmetric parts:

$$A_j(T_1T_2T_3) = A_j^S(T_1T_2T_3) + A_j^N(T_1T_2T_3), \quad (2)$$

$$A_j^S(T_1T_2T_3) \equiv \tfrac{1}{3}[A_j(T_1T_2T_3) + A_j(T_3T_1T_2) + A_j(T_2T_3T_1)], \quad (3)$$

$$A_j^N(T_1T_2T_3) \equiv \tfrac{1}{3}[2A_j(T_1T_2T_3) - A_j(T_3T_1T_2) - A_j(T_2T_3T_1)], \quad (4)$$

Now, the six amplitudes corresponding to 3π states with definite total I can be expressed as linear combinations of the six amplitudes obtained by permuting the arguments of A_τ and $A_{\tau'}$. When we set the $I=2$ amplitudes equal to zero, we obtain

$$A_\tau^N = A_{\tau'}^N. \quad (5)$$

Also, by setting the $I=3$ amplitude equal to zero, we have

$$A_\tau^S = -2A_{\tau'}^S. \quad (6)$$

Hence we see from Eqs. (2), (5), and (6) that, under the $\Delta I = 1/2$ rule, the energy dependence of the τ' matrix element is entirely determined by the behavior of the matrix element for τ decay. The presence of $\Delta I = 3/2$ terms would invalidate Eq. (5), though not Eq. (6).

To make this result more concrete, let us use the coordinates of the Dalitz-Fabri plot,[4]

$$x \equiv \rho \sin\phi \equiv \sqrt{3}(T_1 - T_2)/Q,$$
$$y \equiv \rho \cos\phi \equiv (3T_3 - Q)/Q, \quad (7)$$

where we have $Q \equiv T_1 + T_2 + T_3$. We shall expand A in the series

$$A_j(T_1T_2T_3) = \sum_{n,l} a_j(n,l)\left(\frac{Q_j}{m_K}\right)^n \rho^n \cos l\phi, \quad (8)$$

where $a(n,l)=0$ unless $n-l$ is even and non-negative. [Equation (8) may be justified by performing a multipole expansion of A, which will yield a power series in $|\vec{P}_1|^2$, $|\vec{P}_2|^2$, $|\vec{P}_3|^2$. Since we have $|\vec{P}|^2 = 2mT + T^2$, this can be rewritten as a power series in T_1, T_2, and T_3, and hence in Qx and Qy. Since, according to Eq. (1), we cannot have terms odd in x, this power series can be written as the cosine series of Eq. (8).]

It may easily be seen that the terms in Eq. (8) with $l = 0, 3, 6, 9, \cdots$ are completely symmetric and belong to A^S, while the others belong to A^N. Thus, according to Eqs. (5) and (6), we obtain

$$a_\tau(n,l) = a_{\tau'}(n,l) \quad (9)$$

for $l = 1, 2, 4, 5, 7, 8, \cdots$, and

$$a_\tau(n,l) = -2a_{\tau'}(n,l) \quad (10)$$

for $l = 0, 3, 6, 9, \cdots$. These equations are expected to be correct up to Coulomb and mass-shift corrections of a few percent.[5] Since the $a_j(n,l)$ are only slowly varying functions of Q_j, the 12% difference between Q_τ and $Q_{\tau'}$ will also not alter Eqs. (9) and (10) by more than a few percent at most, though it will have a very large effect on the total available phase space.

In principle, for each (n,l), Eq. (9) or (10) can serve as a test of the presence of $\Delta I = 3/2$, $5/2$ terms or of $\Delta I = 5/2$, $7/2$ terms. There is one case, however, for which Eqs. (9) and (10) can be tested by merely observing T_3 in a reasonable number of τ and τ' events, without having to measure any neutral pion energies. We expect theoretically that all of the $a_j(n,l)$ should be roghly comparable in magnitude. [Very strong final-state interactions might give rise to some factors of $(m_\pi/m_K)^n$, but this seems unlikely.] Since Q_j/m_K is small, we can try neglecting all terms in $|A_j|^2$ except for $|a_j(0,0)|^2$ and the interference between $a_j(0,0)$ and $a_j(1,1)$. We then have

$$|A_j(T_1T_2T_3)|^2 \simeq \mathcal{a}_j\left[1 + \beta_j(Q_j/m_K)y + O\left(\frac{Q_j^2}{m_K^2}\right)\right], \quad (11)$$

where

$$\mathcal{a}_j = |a_j(0,0)|^2, \quad (12a)$$

and

$$\beta_j = 2\mathrm{Re}[a_j(1,1)/a_j(0,0)]. \quad (12b)$$

If we apply Eqs. (9) and (10) to Eqs. (12) and (13), we obtain the familiar result $\mathcal{a}_{\tau'} = \mathcal{a}_\tau/4$, and the new prediction that

$$\beta_{\tau'} = -2\beta_\tau. \quad (13)$$

(It is of course helpful experimentally that $|\beta_{\tau'}|$ is twice $|\beta_\tau|$.) To calculate rates from Eq. (11), we must multiply by the relativistic phase-space density, which is conveniently a constant for both τ and τ' decays, and integrate over unobserved energies. We then obtain the differential decay probability

$$\omega_j(y)dy \sim [1 + \beta_j(Q_j/m_K)y]x_j(y)dy, \quad (14)$$

where $x_j(y)$, the maximum value of x for a given

y, is given exactly by

$$x_j(y) = \left[\frac{(1+y)(C_j+\eta_j y)(A_j - B_j y)}{1+3\eta_j(A_j - B_j y)}\right]^{1/2}. \qquad (15)$$

Here we have $A_j = 1 - \delta_j + \eta_j$, $B_j = 1 + \delta_j + 2\eta_j$, and $C_j = 1 + 3\delta_j + \eta_j$, where $\delta_\tau = 0$, $\eta_\tau = Q_\tau/6m_\pi$, $\delta_{\tau'} = (m_\pi - m_{\pi^0})/3m_{\pi^0}$, $\eta_{\tau'} = Q_{\tau'}/6m_{\pi^0}$.

It follows from Eq. (14) that the distribution in unlike and like pion energies will be $1 + a_j T_3/m_K$ and $1 + \gamma_j T_1/m_K$, respectively [except for a phase-space factor similar to Eq. (15)], and that we have

$$a_j = \frac{3\beta_j}{1 - \beta_j(Q_j/m_K)}, \qquad (16)$$

and

$$\gamma_j = -\tfrac{1}{2}a_j. \qquad (17)$$

In a recent study of 959 τ events,[6] it was observed that the deviations from "phase space" of the distributions in T_3 and in T_1 were roughly linear, with $a_\tau = 6.8 \pm 1.2$, $\gamma_\tau = -2.2 \pm 0.3$, in fair agreement with Eq. (17) and hence with our neglect of quadratic terms in Eq. (11). We then have $\beta_\tau \sim 1.3$, in agreement with our expectation that the $a_j(n,l)$ should be of comparable magnitude. We predict $\beta_{\tau'} \sim -2.6$. The intrinsic uncertainty in this prediction, due to neglect of quadratic terms in Eq. (11), is probably about 20%. By careful study of existing τ data, it should be possible to estimate $a_\tau(2,0)$ and $a_\tau(2,2)$ [for example by checking (17) more accurately] and hence refine the accuracy of our prediction of the π^+ energy distribution in τ' decay.[7]

It is a pleasure to thank Professors R. H. Dalitz and M. A. Ruderman for their comments, and Professor D. Glaser, Professor G. Goldhaber, and Professor S. Goldhaber for their suggestions on experimental possibilities.

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*This work was done under the auspices of the U. S. Atomic Energy Commission.

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HIGH-ENERGY LIMIT OF SCATTERING CROSS SECTIONS

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Pomeranchuk[1] has shown that under rather plausible assumptions concerning the very high energy dependence of total cross section—the main one being that they behave almost as constants at infinity—the difference of the cross sections for a particle and its charge conjugate on the same target tends to vanish at infinity. More explicitly: if $\sigma^+(E)$ and $\sigma^-(E)$ are the total cross sections for a particle and its charge conjugate ($\pi^+ - \pi^-$, proton-antiproton, $K^+ - K^-$, $K^0 - \bar{K}^0$, etc.) at total energy E of the incoming particle in the laboratory system on a specific target, and if

$$\lim_{E\to\infty} \sigma^+(E) = \sigma^+(\infty),$$

$$\lim_{E\to\infty} \sigma^-(E) = \sigma^-(\infty), \qquad (1)$$

$\sigma^+(\infty)$ and $\sigma^-(\infty)$ being finite constants (or zero),