111, 354 (1958).

[5]For a complete list of references on weak interactions see the review articles of E. J. Konopinski, Annual Review of Nuclear Science (Annual Reviews, Inc., Palo Alto, California, 1959), Vol. 9, p. 99; and J. Sakurai, Progress in Nuclear Physics (Butterworths-Springer, London, 1959), Vol. 7, p. 244.

[6]Starting from a somewhat different point of view, L. B. Okun and V. M. Shekhter, J. Exptl. Theoret. Phys. (U.S.S.R.) 34, 1250 (1958)[translation: Soviet Phys.-JETP 34(7), 864 (1958)], obtain the same result as in this paragraph. The author is indebted to Professor R. G. Sachs for this reference.

[7]If the intermediate boson does not exist, (11) is the only possible mechanism for the decay $\mu + p \rightarrow n + e$: see S. P. Rosen, Nuovo cimento 15, 7 (1960).

EFFECT OF THE PION-PION RESONANCE ON K^--p SCATTERING*

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Considerable theoretical interest has been shown in the interpretation of the low-energy K^--p scattering data. The major effort has been directed toward a convenient parameterization that takes into account the kinematic features of the many competing channels ($K^- + p \rightarrow \bar{K}^0 + n, \pi Y$) and the "standard" Coulomb and mass-difference corrections.[1-4] In attempts to draw conclusions about the nature of the basic K-meson nucleon interaction, the assumption has been made that $k \cot\delta$ is essentially constant and equal to the reciprocal (complex) scattering length. We have examined the validity of this approximation on the basis of the Chew-Mandelstam program.[5] We found that the two-pion exchange, which determines the long-range tail of the K-N interaction, gives a substantial energy dependence to $k \cot\delta$.

A qualitative determination that remains to be made is the sign of the real part of each ($I = 0, 1$) scattering length.[6] Two attempts to do this have been based on properties of the Coulomb-nuclear interference. The angular distribution at 172 Mev/c (laboratory-system momentum) favors a constructive interference[7] and implies a positive sign. The other attempt is based on the apparent leveling off and decrease of the elastic-scattering cross section with decreasing momentum ($P_L < 150$ Mev/c). Jackson and Wyld have suggested that this behavior is due to destructive interference and consequently concluded that the sign is negative.[3] The energy dependence arising from the two-pion exchange provides an alternative interpretation of the leveling off. This can be seen qualitatively from the following argument: by virtue of the small pion mass (compared with the K-meson mass) the two-pion exchange determines the longest range part of the K^--p potential. This suggests that we regard the nuclear interaction as made up of the two-pion contribution plus another part of shorter range, representing the net effect of everything else. The peculiar energy dependence is interpreted as the destructive interference between the two parts. Knowledge of the sign of the two-pion contribution then leads to a determination of the sign of the scattering length. Recent advances in the theory of the pion-pion interaction[8] and of the electromagnetic structure of the nucleon[9] make it possible to calculate the sign and estimate the magnitude of the two-pion contribution.

From their theoretical study of the nucleon electromagnetic structure, Frazer and Fulco[9] have inferred a resonance in the $I = 1$, $J = 1$ state of pion-pion scattering. Quantitative conclusions are still uncertain,[10] but it seems likely that the two-pion contribution accounts for a large fraction of the isotopic vector charge and magnetic moment. On the basis of their results, we suggest that the charge structure of the K meson also receives a sizable contribution from the two-pion state. The consistency of this hypothesis and its consequences for πK scattering have been studied.[11] This hypothesis provides an estimate of the order of magnitude of the matrix element for emission of two pions by a K meson $(\pi\pi | K\bar{K})$. Using this estimate, we have calculated the contribution of the two-pion exchange to the $\bar{K}$-nucleon interaction. (The details of this calculation will be presented in a separate report.)

We denote the S-wave $\bar{K}$-nucleon elastic-scattering amplitude by g. It is an analytic function in the cut s-plane (s is the square of the total energy in the $\bar{K}$-N center-of-mass system). The physical branch cut starts at $s = s_0 = (M_N + m_K)^2$ and extends to $+\infty$. There are other "right-hand" cuts starting from the thresholds of the dynamically coupled channels, $(m_\pi+M_\Lambda)^2$ and $(m_\pi+M_\Sigma)^2$.[12] The unitarity condition, which establishes certain nonlinear relations between the amplitudes g_{ij} of various coupled processes, is expressed by

$$\mathrm{Im}(g^{-1})_{ij} = -\frac{k_i\theta_i\delta_{ij}}{s^{1/2}(E_i+M_i)}, \qquad (1)$$

$$\theta_i = 1 \quad \text{if} \quad W_i < s^{1/2},$$

$$= 0 \quad \text{if} \quad W_i > s^{1/2},$$

where k_i, E_i, and W_i are the center-of-mass momentum, baryon energy, and threshold energy of the ith channel. The dynamical singularity arising from the two-pion exchange is a branch cut extending from the left up to the point $s=s_2 = [(M_N^2-m_\pi^2)^{1/2}+(m_K^2-m_\pi^2)^{1/2}]$. Relative to the other dynamical singularities, this cut is very close to the physical region $s \geq s_0$, and should produce the strongest energy dependence in $k\cot\delta$.

We denote the elastic $\bar{K}N$ amplitude in the state of isotopic spin I by g_I, where $I=0,1$. These two amplitudes are related by crossing symmetry to amplitudes $g^{(+)}$ and $g^{(-)}$ that depend on the isotopic spin $I'=0,1$ of the exchanged pion pair, namely

$$g^0 = g^{(+)} - 3g^{(-)},$$

$$g^1 = g^{(+)} + g^{(-)}. \qquad (2)$$

The magnitude of the discontinuity across the two-pion cut is expressed for our purpose as

$$\int_{s_1}^{s_2} ds\, \mathrm{Im}\, g^{(-)}(s) = R_1 \approx 4M_N^4 \text{ fermi}, \qquad (3)$$

where s_1 is approximately equal to $80\, m_\pi^2$. Assuming that only the resonant $g^{(-)}$ amplitude is important, we have three specific predictions: (a) the isotopic spin ratio R_0/R_1 is -3, (b) the sign of R_1 is positive, and (c) the expected magnitude of R_1 is rather large.

The normalization of our amplitude g is given by its relation to the $\bar{K}$-N elastic scattering phase shift,

$$g(s) = s^{1/2}(E+M)/(k\cot\delta - ik). \qquad (4)$$

In this note we use only a very simple model in which the effect of the two-pion cut is represented by a δ function at the position $s=a=93\, m_\pi^2$ in the amplitude g. The dynamical singularities in all other channels are represented by subtraction constants.

This model, combined with the many-channel unitarity condition, Eq. (1), leads to the following expression for $k\cot\delta$:

$$k\cot\delta_I = \left(l(s) + \frac{1}{f_I(s)+z_I}\right) s^{1/2}(E+M), \qquad (5)$$

where

$$l(s) = \frac{s-s_0}{\pi} P\int_{s_0}^{\infty} \frac{ds'\,\mathrm{Im}(g^{-1})_{11}}{(s'-s)(s'-s_0)}$$

$$-\frac{a-s_0}{\pi}\int_{s_0}^{\infty} \frac{ds'\,\mathrm{Im}(g^{-1})_{11}}{(s'-s_0)(s'-a)},$$

$$f_I(s) = \frac{1}{\pi}\left(\frac{1}{s-a} - \frac{1}{s_0-a}\right)\frac{R_I}{1+\beta R_I},$$

$$\beta = \frac{1}{\pi^2}\int_{s_0}^{\infty} \frac{ds'\,\mathrm{Im}(g^{-1})_{11}}{(s'-a)^2},$$

and z_I is a complex quantity which depends on the subtraction constants and on the slowly varying π-Y center-of-mass momenta. For each isotopic spin channel, we approximate z_I by a complex constant.[1, 12] The calculations were done by adjusting the parameters R_I and z_I to fit the at-rest branching ratio[7]

$$\lambda^{-1} = \sigma_{\mathrm{abs}}(I=1)/\sigma_{\mathrm{abs}}(I=0) = 0.18 \pm 0.09,$$

the total cross section at 172 Mev/c ($k\sigma_{\mathrm{tot}}/4\pi = 0.7$ fermi), and the elastic and charge-exchange cross sections at 100 Mev/c and 172 Mev/c.

Two sets of solutions were found, corresponding to constructive or destructive Coulomb-nuclear interference in the angular distribution at 172 Mev/c (Table I). The elastic and charge-exchange cross sections resulting from the "constructive" solution are shown in Fig. 1. The energy dependence of $(k\cot\delta_I)^{-1}$ is shown in Fig. 2, where the (a+) solution of Dalitz and Tuan is also reported for comparison. Note that only the values of R_I and the ratio R_0/R_1 given by the "constructive" solution agree with those calculated theoretically. From this consideration we reject the "destructive" solution. Our principal result is that $k\cot\delta_I$ has a rather substantial energy dependence due to the two-pion exchange.

Table I. Two sets of parameters that fit the available experimental data. The solutions are characterized by the constructive or destructive nature of the Coulomb-nuclear interference in the angular distribution at 172 Mev/c.

Solution	Constructive	Destructive
R_1	$1.0 M_N^4$ fermi	$2.0 M_N^4$ fermi
R_0/R_1	-3.8	+1.50
z_0	$(1.34+i0.74)M_N^2$ fermi	$(0.93+i1.46)M_N^2$ fermi
z_1	$(0.67+i0.12)M_N^2$ fermi	$(-0.22+i1.00)M_N^2$ fermi
λ^{-1}	0.13	0.29

Figure 2 shows that Re(k cotδ_I) is positive for each isotopic spin state in the momentum range above 100 Mev/c. However, the real parts of the scattering lengths appear to have opposite signs, that of the $I=0$ state being small and negative.

Finally, we would like to remark that, by crossing symmetry, the two-pion contribution to the K^+-proton amplitude can be determined from the parameters R_0 and R_1. In this case there is a destructive interference and flattening of the cross section for a repulsive K^+-proton short-range interaction.

We are deeply indebted to Professor Geoffrey F. Chew and Professor Robert Karplus for many illuminating discussions.

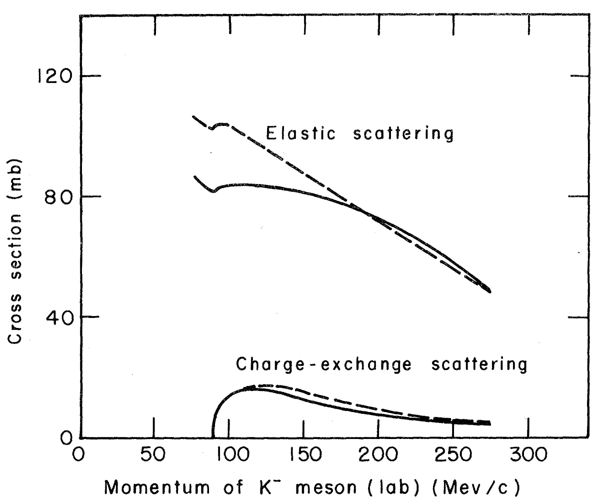

FIG. 1. Cross sections for elastic and charge-exchange K^--proton scattering. The (a+) solution of Dalitz and Tuan (dashed lines) is included for comparison.

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[1]R. H. Dalitz and S. F. Tuan, Ann. Phys. 8, 100 (1959).

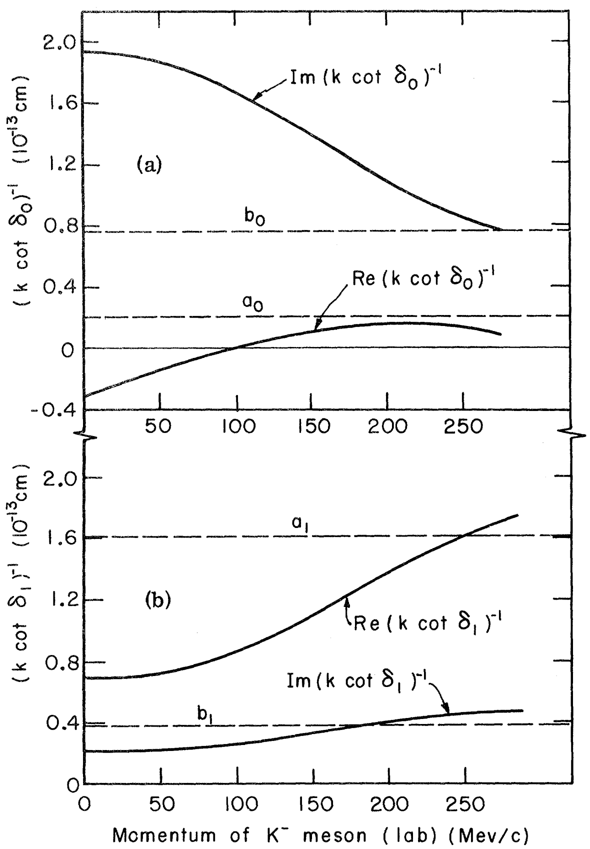

FIG. 2. Momentum dependence of the real and imaginary parts of (k cotδ)$^{-1}$, (a) for isotopic spin $I=0$, (b) for isotopic spin $I=1$. The dashed lines are the (a+) solution of Dalitz and Tuan.

[2]P. T. Matthews and A. Salam, Phys. Rev. Letters 2, 226 (1959).
[3]J. D. Jackson and H. W. Wyld, Phys. Rev. Letters 2, 355 (1959).
[4]M. Ross and C. Shaw, Phys. Rev. 115, 1773 (1959).
[5]G. F. Chew, Annual Review of Nuclear Science (Annual Reviews, Inc., Palo Alto, California, 1960), Vol. 9, p. 29; S. Mandelstam, Phys. Rev. 112, 1344 (1958).
[6]We would like to reiterate that the attractive nature of the K-nucleon interaction is already determined by the large magnitude of at least one of the scattering lengths; R. Karplus, L. Kerth, and T. Kycia, Phys. Rev. Letters 2, 510 (1959).
[7]Data of A. H. Rosenfeld, F. T. Solmitz, R. D. Tripp, and M. Ross, reported by L. W. Alvarez at Ninth Annual Conference on High-Energy Nuclear Physics, Kiev, 1959 (unpublished).
[8]G. F. Chew and S. Mandelstam, Lawrence Radiation Laboratory Report UCRL-8728 (to be published).
[9]W. R. Frazer and J. R. Fulco, Phys. Rev. 117, 1609 (1960).
[10]J. Ball and D. Wong, Lawrence Radiation Laboratory (private communication).
[11]Graham Frye (to be published).
[12]S. W. MacDowell, Phys. Rev. 116, 774 (1959); F. Ferrari, M. Nauenberg, and M. Pusterla, University of California Radiation Laboratory Report UCRL-8985, November, 1959 (unpublished); M. Nauenberg, Ph. D. dissertation, Cornell University (unpublished); F. Ferrari, M. Nauenberg, and M. Pusterla (to be published).

THEORY OF Σ-HYPERON DECAYS

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In this note we consider a model of the Σ hyperon as a bound state of a Λ^0 and a pion. By means of this model, we find it possible to explain some observed features of Σ decay simply in terms of the observed properties of Λ^0 decay.

For simplicity we shall consider the bound state to be an S state. This would lead to the following obvious features: (a) The spin of the Σ would be 1/2. (b) The isotopic spin of the Σ would be 1. (c) The relative parity of the Σ and Λ would be odd.[1]

Properties (a) and (b) are known to be satisfied.[2] Property (c) has not yet been investigated. However, a model of the Σ as a bound Λ and pion in a $P_{1/2}$ state will give similar predictions to those that follow.

We consider the following three decay modes of the Σ hyperons:

$$\Sigma^+ \to \pi^+ + n, \tag{1}$$

$$\Sigma^+ \to \pi^0 + p, \tag{2}$$

$$\Sigma^- \to \pi^- + n. \tag{3}$$

The mechanism for the decay is assumed to be the decay of the Λ hyperon within the Σ into a pion and a nucleon, with subsequent absorption by the nucleon, of either the pion within the Σ or the pion from the Λ decay, to form the appropriate final state of reactions (1), (2), or (3). The virtual decay of the Λ particle will be assumed to satisfy the $\Delta I = \frac{1}{2}$ rule and time-reversal invariance. This decay will be assumed to proceed with an amplitude A into an S-wave pion-nucleon system and with either an amplitude A or an amplitude $-A$ into a P-wave pion-nucleon system. For the present discussion, whose purpose is a qualitative picture of Σ decays, we will neglect multiple-scattering effects in the pion-nucleon systems of isotopic spin and total angular momentum $\frac{1}{2}$, and will consider the amplitude A as real. The approximate equality in magnitude of the S- and P-wave amplitudes is inferred from the assumption of a near maximal asymmetry parameter in the decay of a real Λ.[3] The positive and negative values of the P-wave amplitude correspond to the asymmetry parameter, α_Λ, equal to +1 and -1, respectively. The nucleon from the decay of the Λ will absorb a pion in a P wave relative to it. This may be the pion from P-wave decay of the Λ, in which case the absorption will be characterized, again approxi-