akoff, Phys. Rev. 111, 313 (1958). Also reference 7.

[2]S. Cohen, D. L. Judd, and R. J. Riddell, Phys. Rev. (to be published); and University of California Radiation Laboratory Reports UCRL-8390, 8391, 8802. This work will be referred to below as CJR. See also reference 12.

[3]In our notation $\hbar = c = 1$, $a_i \equiv 1/e^2 m_i$, $e^2 = \alpha = 1/137$.

[4]This possibility was suggested to the author by J. Steinberger.

[5]R. H. Dalitz (private communication).

[6]See, e.g., J. M. Blatt and V. F. Weisskopf, Theoretical Nuclear Physics (John Wiley & Sons, Inc., New York, 1952), Chap. XII, Eq. (3.34).

[7]H. Primakoff, Revs. Modern Phys. 31, 802 (1959). Also references 9 and 12.

[8]I would like to thank Mr. V. Brady for performing the numerical integrations needed on the UCRL IBM-650.

[9]T. Y. Wu (to be published) has stated that for protons fixed at a distance r_{12} of 2 mesonic units, these corrections raise the value of 2γ from 1.317 to 1.422. However, Wu does not seem to perform the averaging over the nuclear separation needed to obtain $2\gamma_O$ and $2\gamma_P$. This averaging places the nucleons at an effective distance of 2.90 and 2.45, respectively. Apart from corrections due to higher mesonic orbitals, the CJR wave functions are essentially exact, agreeing at $r_{12} = 2$ and 4 to within 1 % with those of D. R. Bates, K. Ledsham, and A. L. Stewart, Phil. Trans. Roy. Soc. (London) A246, 215 (1953).

[10]S. S. Gershtein, J. Exptl. Theoret. Phys. (U.S.S.R.) 34, 463, 993 (1958) [translation: Soviet Phys.-JETP 34(7), 318, 685 (1958)].

[11]For neutral p-μ atoms it is rearrangement collisions that give rapid $F=1 \rightarrow F=0$ conversion, as shown in reference 10.

[12]Ia. B. Zel'dovich and S. S. Gershtein, J. Exptl. Theoret. Phys. (U.S.S.R.) 35, 649 (1958) [translation: Soviet Phys.-JETP 35(8), 451 (1959)].

P-WAVE RESONANCE IN PION-PION SCATTERING*

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(Received April 25, 1960)

The possible p-wave resonance in pion-pion scattering has been a subject of much discussion lately.[1-4] The work of Chew and Mandelstam based on the double dispersion representation[5] for the transition amplitude suggests that there is more than one solution to the set of integral equations for the pion-pion partial wave scattering amplitudes.[3,4] In the present note, we wish to point out the possible source of such a multiplicity of solutions from the viewpoint of conventional Lagrangian field theory, and to examine the significance of the fact that the p-wave phase shift remains small throughout the physically acceptable range of the pion-pion coupling constant in the "s-wave dominant solution" of the Chew-Mandelstam equations,[3,4] although a solution containing a p-wave resonance may be obtainable from a nonadiabatic approach, the starting point of which we suggest here.

It is crucial for the following discussion to realize that the form of the double dispersion representation[5] is uniquely determined by the masses of the participating particles and the selection rules, aside from the subtle question of possible subtractions necessary to give the representation a valid meaning. If one assumes the existence of a vector boson of isotopic spin one, which interacts with the isotopic vector part of the pion current in the Lagrangian,[6] and insists that the renormalized mass (which we will define later) of the postulated particle be larger than twice the pion mass, then the particle will become unstable and one has not changed the selection rules of the theory from those of the conventional pseudoscalar meson theory. Such a theory, on the other hand, will necessarily predict a p-wave resonance in pion-pion scattering. The situation here is similar to the well-known Castillejo-Dalitz-Dyson ambiguity[7] in the static meson theory: the double dispersion relation does not imply one particular Lagrangian, but a class of Lagrangians which are selection-rule equivalent.[8]

It follows then that there exists a solution to the Chew-Mandelstam equations corresponding to a Lagrangian theory in which there exists, in addition to the usual couplings, a coupling term of the form

$$L_{\rm int} = \frac{1}{\sqrt{2}} f \sum_{ijk} \epsilon_{ijk} (\phi^i \partial_\mu \phi^j - \partial_\mu \phi^i \phi^j) B_\mu^{\ k}, \qquad (1)$$

where $B_\mu^{\ k}$ is the field operator for the vector boson of isotopic spin one, k being the isotopic spin index, and ϕ is the pion field operator.

The p-wave resonance in pion-pion scattering is then due to the resonant structure of the vector boson propagator, the relevant part of which is

$$D^{\mu\nu}(k) = -D(k^2)\, g^{\mu\nu} \tag{2}$$

(the terms proportional to $k^\mu k^\nu$ do not contribute to pion-pion scattering in the present approximation), where

$$D(k^2) = [k^2 - m^2 - \Sigma(k^2)]^{-1}, \tag{3}$$

where $m > 2\mu$ is the renormalized vector boson mass, and $\Sigma(k^2)$ is the vector boson self-energy operator; in the "resonance approximation," in which we consider only the sum of the iterated bubble diagrams (see Fig. 1), $\Sigma(k^2)$ is given simply by

$$\Sigma(k^2) = \frac{4i}{3(2\pi)^4} f^2 \int d^4q \frac{q_\mu q_\nu}{[q^2-\mu^2][(q+k)^2-\mu^2]}\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right). \tag{4}$$

The imaginary part of $\Sigma(k^2)$ can be calculated directly; it is given by

$$\operatorname{Im}\Sigma(s+i\epsilon) = -\frac{f^2}{48\pi}\frac{(s-4\mu^2)^{3/2}}{s^{1/2}}\,\theta(s-4\mu^2). \tag{5}$$

The real part can be calculated, for example by noting that $\Sigma(s)$ satisfies the dispersion relation,

$$\Sigma(s) = \Sigma(m^2) + (s-m^2)\Sigma'(m^2) - \lim_{\eta\to m^2}\frac{(s-m^2)}{\pi}\int\frac{\operatorname{Im}\Sigma(s')}{s'-m^2-i\epsilon} \times \left\{\frac{1}{s'-s-i\epsilon} - \frac{1}{s'-\eta-i\epsilon}\right\} ds', \tag{6}$$

where we have made two subractions at $s = m^2$, reflecting the fact that the integral in Eq. (4) is quadratically divergent. Note that while $\Sigma(m^2)$, $\Sigma'(m^2)$ are now complex, their imaginary parts explicitly appearing on the right-hand side of (6) are cancelled by the contributions from the $i\epsilon$'s in the denominators $(s'-m^2-i\epsilon)^{-1}$ and $(s'-\eta-i\epsilon)^{-1}$; the real parts of these constants are absorbed in the definition of the renormalized mass and coupling constant. Carrying out the indicated integrations then leads to an expression for the renormalized self-energy operator:

$$\Sigma(s) = J(s) - J(m^2) + i\operatorname{Im}\Sigma(s), \tag{7}$$

where

$$J(s) = \frac{f^2}{48\pi}\left\{\frac{(s-4\mu^2)^{3/2}}{s^{1/2}}\ln\left[\frac{s^{1/2}+(s-4\mu^2)^{1/2}}{s^{1/2}-(s-4\mu^2)^{1/2}}\right] - \xi s\right\} \tag{8}$$

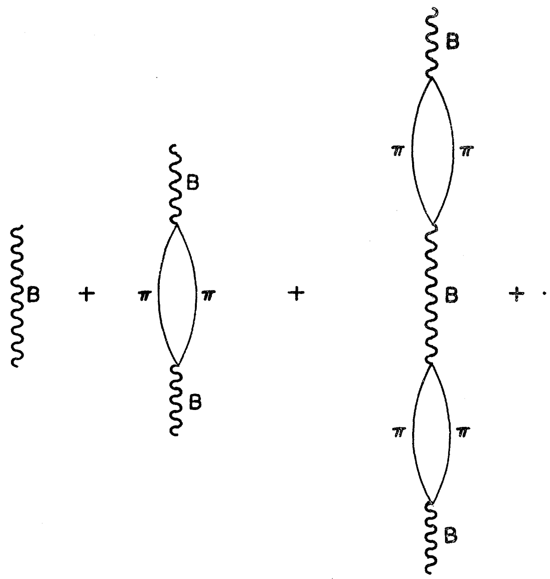

FIG. 1. Diagrams included in resonance approximation to boson propagator.

and the constant ξ is given by

$$\xi = \frac{m^2-4\mu^2}{m^2} + \left(\frac{m^2+2\mu^2}{m}\right)\frac{(m^2-4\mu^2)^{1/2}}{m}\ln\left[\frac{m+(m^2-4\mu^2)^{1/2}}{m-(m^2-4\mu^2)^{1/2}}\right]. \tag{9}$$

The $I=1$, $J=1$ pion-pion scattering phase shift δ is then given in this approximation by

$$\left(\frac{\nu+1}{\nu^3}\right)^{1/2} e^{i\delta}\sin\delta = \frac{1}{\nu_r K(\nu_r) - \nu K(\nu) - i\Gamma[\nu^3/(\nu+1)]^{1/2}}, \tag{10}$$

where $\nu = (s-4\mu^2)/4\mu^2$, $\nu_r = (m^2-4\mu^2)/4\mu^2$, $\Gamma = f^2/48\pi$, and

$$K(\nu) = 1 - \frac{\Gamma}{\pi}\left\{\left(\frac{\nu}{\nu+1}\right)^{1/2}\ln\frac{(\nu+1)^{1/2}+\nu^{1/2}}{(\nu+1)^{1/2}-\nu^{1/2}} - \xi\right\}.$$

We note that this expression for the scattering amplitude has a pole for $\nu = -\nu_0$ on the negative real axis; indeed, Eq. (10) was first written down, in a slightly altered form,[9] by Frazer and Fulco,[1] who derived it from the partial wave dispersion relations[3] with the left-hand branch cut replaced by a phenomenological pole. From the present point of view, this pole merely represents the failure of the resonance approximation for large $|s|$; in the resonance region, the effect of the pole is negligible if ν_0 is sufficiently large and Γ is not too large. The distinction between the two viewpoints may be seen by noting that if the pole is interpreted as a phenomenological representation of the left-hand branch cut, then $\nu_0 \sim m^2$, where m is some average mass of the intermediate states which contribute to the left-hand branch cut. Since $\nu_0 \sim 100 - 1000$ for values of the width Γ which fit the data on nucleon electromagnetic structure,[2] one is led to suspect that virtual baryon-antibaryon pairs play a prominent role in producing the p-wave resonance,[10] unless an unstable vector boson such as Sakurai and others have suggested[6] is present in the original field-theoretical Lagrangian.

If Eq. (10), with the pole removed, is inserted into the iterative procedure of Chew et al.,[4] then the resonance should persist, and, in addition, nonresonant terms will appear in other angular momentum states; in particular, one should be able to calculate the s-wave scattering lengths in terms of the resonance parameters. It also seems clear that a purely dispersion theoretic approach will require at least two parameters to obtain a p-wave resonance, since the one-parameter theory of Chew et al., which corresponds to a $\lambda\phi^4$ pion-pion interaction,[11] seems incapable of producing a resonance. Whether these two parameters are to be related to the pion-pion and pion-nucleon coupling constants, or to the mass and coupling constant of the conjectured vector boson, cannot be decided on the basis of present knowledge.

*Supported in part by the U. S. Atomic Energy Commission.

†Harrison Special Fellow.

[1]W. Frazer and J. Fulco, Phys. Rev. Letters 2, 365 (1959).

[2]W. Frazer and J. Fulco, Phys. Rev. 117, 1603, 1609 (1960).

[3]G. Chew and S. Mandelstam, Lawrence Radiation Laboratory Report UCRL-8728; Phys. Rev. (to be published).

[4]G. Chew, S. Mandelstam, and H. P. Noyes, Lawrence Radiation Laboratory Report UCRL-9001; Phys. Rev. (to be published).

[5]S. Mandelstam, Phys. Rev. 112, 1344 (1958).

[6]As has recently been suggested by J. Sakurai, Ann. Phys. (to be published). See also C. Yang and R. Mills, Phys. Rev. 96, 191 (1954).

[7]L. Castillejo, R. Dalitz, and F. Dyson, Phys. Rev. 101, 453 (1956).

[8]This point is discussed for the static theory by R. Norton and A. Klein, Phys. Rev. 109, 991 (1958).

[9]More precisely, $\Gamma' = \pi\Gamma(\pi + \xi\Gamma)^{-1}$, $\Gamma'\nu_\gamma' = \Gamma'\nu_\gamma K(\nu_\gamma)$, where ν_γ', Γ' are the quantities designated as ν_γ, Γ_γ by Frazer and Fulco.[1,2]

[10]K. Igi and K. Kawarabayashi, Progr. Theoret. Phys. (Kyoto) 20, 576 (1958), have noted that the contribution to the p-wave pion-pion scattering amplitude from a single nucleon closed loop corresponds to an attractive potential.

[11]S. Okubo, Phys. Rev. 118, 357 (1960).

NEW THEORETICAL VALUE FOR THE LAMB SHIFT*

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(Received May 9, 1960)

Although the good agreement between theory and experiment for the Lamb shift is one of the notable successes of quantum electrodynamics, the most recent tabulations[1] still have shown small but significant discrepancies for H, D, and He^+ which have made a further increase in the accuracy of the theoretical value desirable. Since the listed discrepancy for He^+ is roughly 60 times that of H or D, it was natural to try to reduce this disagreement by calculating the second order radiative correction of order $\alpha(\alpha Z)^6$, which is the next order in (αZ), the Coulomb interaction parameter, beyond the previously calculated[2,3] order of $\alpha(\alpha Z)^5$.

A closer mathematical analysis[4,5] shows that there exist the leading orders of $\alpha(\alpha Z)^6\ln^2(\alpha Z)$ and $\alpha(\alpha Z)^6\ln(\alpha Z)$. We have completed the calculation of these two orders. The analytic result is

$$\Delta E(2S - 2P_{1/2}) = -Lw[\tfrac{3}{4}\ln^2 w + \ln w(4\ln 2 + 1 + 7/48)], \qquad (1)$$

where $\Delta E(2S - 2P_{1/2})$ is the difference of the shifts of the $2S$ and $2P_{1/2}$ levels due to the two new orders, $w \equiv (\alpha Z)^2$, and L is Z^4 times the "Lamb constant": $L \equiv Z^4\alpha^3/(3\pi)$ ry $= Z^4(135.6)$ Mc/sec. The $\alpha w^3\ln^2 w$