where λ is a normalization factor, so chosen that the experimental value is coincident with the theoretical one at 900 Mev. The data for 6 mrad confirm that the collimation operates an angular selection of the photons. For $\theta \approx 1$ mrad we do not draw the theoretical curve because at these angles the dependence on θ is very strong, and for the experimental data we have $\theta = 1 \pm 0.5$ mrad.

The preceding experiments[7,8] have shown a dependence on θ of the bremsstrahlung intensity without showing any central minimum. We think that this is due to insufficient angular resolution with respect to the low-energy detected photons.

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MUON ABSORPTION IN LIQUID HYDROGEN*

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The long-awaited measurement of the rate of μ^- absorption on protons could conceivably use hydrogen in any form, but there are technical advantages in working with liquids, as in bubble chambers. However, many physicists have expressed pessimism about the utility of such an experiment, because in liquid H_2 muons form $p-\mu-p$ molecular ions in addition to $p-\mu$ atoms. We shall show here how it is nevertheless possible to interpret the observable rate of muon absorption in liquid hydrogen in terms of the basic $pn\mu\nu$ interaction. The main problem is to calculate the rate for the process

$$(p-\mu-p) \to n+\nu+p \tag{1}$$

in terms of the muon absorption in atoms; because of the extreme spin dependence of the V-A interaction,[1] this requires knowledge of the orientation of proton and muon spins as well as the overlap of their wave functions at the instant of absorption. Almost all of our remarks will apply to solid as well as to liquid hydrogen, and we will confine ourselves to the case of isotopically pure $(H^1)_2$.

The only two bound orbital states of the $p-\mu-p$ system[2] are a para $1s\sigma g$ ground state with rotational angular momentum $L=0$ and binding energy 2771 ev, and a $1s\sigma g$ ortho state with $L=1$ which is $\Delta E = 148$ ev higher. The formation of these states by electron ejection in a collision of a $p-\mu$ atom with an H_2 molecule is respectively an $E0$ or $E1$ process, and hence the ratio of the formation rates is of order[3]

$$\text{Para/Ortho} \sim k_e{}^2 a_\mu{}^2 \sim 2\times 10^{-4}, \tag{2}$$

where k_e is the wave number of the ejected electron. Actually, detailed calculations[2] show this ratio to be less than 3×10^{-5}, so virtually all $p-\mu-p$'s are formed in the ortho state.

It seems furthermore that the $p-\mu-p$'s formed in the ortho state stay there during the few microseconds of muon lifetime. If we define the ortho→para electric dipole transition matrix element to be

$$|\langle P|\int\rho\vec{R}|O\rangle| \equiv e a_\mu D, \tag{3}$$

then the rate for de-excitation accompanied by ejection of an electron from a hydrogen molecule in a collision is

$$\omega_e(O\to P) = 16\pi n_e m_e a_\mu{}^2 e^4 D^2/3k_e$$
$$= 2.7\times 10^{10} D^2 \text{ sec}^{-1}. \tag{4}$$

(Here n_e is the number density of electrons in a bubble chamber, $n_e = 3.5\times 10^{22}$ cm^{-3}, and $k_e = [2m_e(\Delta E - E_H)]^{1/2}$, where $\Delta E = 148$ ev, and $E_H = 15.6$ ev is the electron separation energy in H_2.) It is also possible that the $p-\mu-p$ forms "ordinary" atoms or molecules with electrons and protons from the liquid.[4] The rate ω_A for Auger ortho→para conversion in such a system is again given by (4), with n_e replaced by the density n_e' of bound electrons at the $p-\mu-p$. If, for example, we take $n_e' = 1/\pi a_e^3$, then

$$\omega_A(O \to P) = 61\omega_e(O \to P) = 1.6 \times 10^{12} D^2 \text{ sec}^{-1}. \quad (5)$$

Of course, both ω_e and ω_A are much larger than the radiative conversion rate,

$$\omega_\gamma(O \to P) = 4(\Delta E)^3 a_\mu^2 e^2 D^2/3$$
$$= 8.1 \times 10^7 D^2 \text{ sec}^{-1}. \quad (6)$$

According to (4), (5), and (6), we would need $D^2 \gtrsim 10^{-7}$ for any significant ortho→para conversion to occur during the muon lifetime, even if electrons were bound to all $p-\mu-p$'s. It is unlikely that the $E1$ transition moment is this large. The ortho and para states have total proton spins $S_p = 1$ and $S_p = 0$, and the only effects that can give rise to an $E1$ matrix element between such states are impurities in the states,[5] or the electric moments generated by moving magnetic dipoles.[6] The first effect would give at most a D of the order of the amplitude of the $1s\sigma u$ ($L=0$, $S_p=1$) admixture in the para state, which can be estimated roughly as a typical magnetic interaction energy, $\alpha^4 m_\mu \sim 0.3$ ev, divided by the minimum energy difference between $1s\sigma g$ and $1s\sigma u$ states whose wave functions overlap appreciably, roughly 3 kev, giving an amplitude of about 10^{-4}. Of course D is much smaller than this, since the $1s\sigma u$ para impurity will not have perfect overlap with the $1s\sigma g$ ortho wave function. The second effect mentioned would contribute to D an amount of order $2.79\Delta E/m_p \sim 4\times 10^{-7}$. If D were this small, $E1$ conversion would take as long as would be expected for conversion by $M2$ transitions.

De-excitation by rearrangement collisions of the $p-\mu-p$ with H_2 molecules is not retarded by the necessity to flip spins, but is hopelessly slow because of the high Coulomb barrier. The cross section for ortho→para conversion by this process contains the familiar factor e^{-G} where $G = 2\pi/137\beta$ and βc is the initial $p-\mu-p$ velocity. In order that $G < 100$, the $p-\mu-p$ would have to have kinetic energy greater than 200 ev, and this is impossible as it is formed with 0.02 ev, and $kT < 0.003$ ev.

The fact that process (1) occurs from the ortho state would not complicate matters badly if it were correct (as seems to have been previously assumed) that ortho $p-\mu-p$'s had definite total spin $(\vec{S}_\mu + \vec{S}_p)^2 = S(S+1)$. It has been shown[7] that if $\omega(S)$ is the rate for (1) in such states, and ω_F is the rate for muon absorption in $p-\mu$ atoms with total spin F, then

$$\omega(1/2) = 2\gamma_O(\tfrac{3}{4}\omega_0 + \tfrac{1}{4}\omega_1), \quad (7)$$

$$\omega(3/2) = 2\gamma_O\omega_1, \quad (8)$$

which may be compared with the smaller absorption rate in the para state,

$$\omega(\text{para}) = 2\gamma_P(\tfrac{1}{4}\omega_0 + \tfrac{3}{4}\omega_1). \quad (9)$$

Here γ_O, γ_P are the ratios of the muon density at one of the protons in an ortho or para $p-\mu-p$ molecular ion (averaged over the separation between protons) to the muon density at the proton in a $p-\mu$ atom. Using the CJR wave functions, we have found that[8]

$$2\gamma_O = 1.165, \quad 2\gamma_P = 1.308. \quad (10)$$

These values are subject to corrections[9] of order m_μ/m_p due to admixture of higher orbitals than $1s\sigma g$.

Of course, S is a good quantum number only if the forces coupling $\vec{S}_p$ and $\vec{S}_\mu$ to $\vec{L}$ are ignored. In this lowest approximation, the ortho state forms a degenerate quintuplet of states $|J, S\rangle$ which we can take with definite values of S and $(\vec{S}_\mu + \vec{S}_p + \vec{L})^2 = J(J+1)$; the states are: $|\tfrac{1}{2}, \tfrac{1}{2}\rangle$, $|\tfrac{1}{2}, \tfrac{3}{2}\rangle$, $|\tfrac{3}{2}, \tfrac{1}{2}\rangle$, $|\tfrac{3}{2}, \tfrac{3}{2}\rangle$, $|\tfrac{5}{2}, \tfrac{3}{2}\rangle$. The degeneracy among them is actually lifted by magnetic and relativistic effects which split the quintuplet into five states $|\psi_n\rangle$ separated by energies of order $\alpha^4 m_\mu = 0.3$ ev, with total angular momentum $J_1 = J_2 = \tfrac{1}{2}$, $J_3 = J_4 = \tfrac{3}{2}$, $J_5 = \tfrac{5}{2}$. [Each of these five states still retains a $(2J_n+1)$-fold degeneracy, which plays no role here. The field needed for a Paschen-Back effect is of order 10^9 gauss.]

It is easy to show that the $S = \tfrac{1}{2}$ and $S = \tfrac{3}{2}$ components of the true energy eigenstates $|\psi_n\rangle$ don't interfere in reaction (1) if no attempt is made to measure the momentum or polarization of the left-over proton. Furthermore there is no interference among the $|\psi_n\rangle$ states, since their energy separation is much larger than their natural width of 3×10^{-10} ev. Hence the rate for (1) is given as the sum

$$\omega_{p\mu p} = \xi\omega(1/2) + (1-\xi)\omega(3/2), \quad (11)$$

where

$$\xi \equiv \sum_n P_n \xi_n, \quad (12)$$

$$\xi_n \equiv |\langle J_n, S=\tfrac{1}{2}|\psi_n\rangle|^2, \quad (\xi_5 \equiv 0) \tag{13}$$

and P_n is the probability of the ortho $p-\mu-p$ occupying $|\psi_n\rangle$ at the instant of absorption.

It is easy to calculate the relative probabilities P_n^0 for formation of the five ortho states. Since the $p-\mu-p$ is formed from a proton and an $F=0$ $p-\mu$ atom,[10] it has $S=\frac{1}{2}$ initially, and hence

$$P_n^{\,0} = \tfrac{1}{6}(2J_n+1)\xi_n. \tag{14}$$

Furthermore, it seems reasonable to suppose that no transitions among the $|\psi_n\rangle$ occur during the muon lifetime. The Coulomb barrier prevents rearrangement collisions[11]; $M1$ radiative transitions would take a day; and probably $M1$ conversion in collisions with H_2 molecules is very slow because the energy available is insufficient for electron ejection. Assuming then that $P_n = P_n^0$, we obtain

$$\xi = \sum_n \tfrac{1}{6}(2J_n+1)\xi_n^{\,2}, \tag{15}$$

and our problem reduces to that of computing the ξ_n. However, there is one important inequality that can be derived without any calculation. Since $\xi_1+\xi_2=\xi_3+\xi_4=1$, it follows directly from (15) that

$$\tfrac{1}{2} \leq \xi \leq 1, \tag{16}$$

whereas in general we would only know that $0 \leq \xi \leq 1$. If we accept that $\omega_0 \sim 50\omega_1$,[7] then because we cannot have ξ near zero it is safe to approximate

$$x \equiv \omega_{p\mu p}/\omega_0 \cong (2\gamma_O)\tfrac{3}{4}\xi, \tag{17}$$

and furthermore the neutrons emitted should have almost 100% longitudinal polarization.

The Hamiltonian H' that couples $\vec{S}_p$, $\vec{S}_\mu$, and $\vec{L}$ takes the general form

$$\begin{aligned} H' = {} & E_1\vec{S}_p\cdot\vec{L} + E_2\vec{S}_\mu\cdot\vec{L} + E_3\vec{S}_\mu\cdot\vec{S}_p \\ & + E_4[\tfrac{1}{2}(\vec{S}_\mu\cdot\vec{L})(\vec{S}_p\cdot\vec{L}) + \tfrac{1}{2}(\vec{S}_p\cdot\vec{L})(\vec{S}_\mu\cdot\vec{L}) - \tfrac{2}{3}(\vec{S}_\mu\cdot\vec{S}_p)] \\ & + E_5[(\vec{S}_p\cdot\vec{L})^2 + \tfrac{1}{2}(\vec{S}_p\cdot\vec{L}) - \tfrac{4}{3}]. \end{aligned} \tag{18}$$

One term in E_1 represents the proton spin-orbit force; E_2 and another term in E_1 come from the magnetic interaction between the proton momenta and the muon and proton spins; E_3 and E_4 come from the interaction between proton and muon magnetic moments; and E_5 arises from the interaction between the proton magnetic moments. We have derived formulas (too lengthy for this note) which give the ξ_n as simple algebraic functions of the E_j coefficients, and which express the E_j in terms of averages over the molecular ion orbital wave function. All the E_j seem roughly of the same magnitude, containing factors $e^2/m_\mu m_p$ or e^2/m_p^2; e.g.,

$$E_3 = 8e^2(2.79)\gamma_O/3m_\mu m_p a_\mu^{\,3} = 0.292 \text{ ev}.$$

If E_1-E_2 and E_4-2E_5 turn out to be small compared to E_3, then ξ will be near unity. We have not performed the necessary integrations to obtain the E_j and hence ξ numerically, as it seems most important at this stage to recognize that the task is straightforward.

Since some muon absorptions will occur with muons still in $F=0$ $p-\mu$ atoms, the observed total rate of all absorptions in liquid hydrogen is $\omega_{abs} = \omega_0(1+xy)/(1+y)$, where y is the product of the muon mean life times the rate per $p-\mu$ atom of $p-\mu-p$ formation. According to CJR, $y=14.4$ at a proton number density $n_p = 3.5\times10^{22}$ cm^{-3}. Since x [as given by (17), (10), and (16)] must be between 0.44 and 0.88, the value of ω_0 determined by a measurement of ω_{abs} (and calculation of x) will be quite insensitive to the precise value of y, providing y isn't too small.

It is possible to test many of our assumptions by measuring ω_{abs} over a range of hydrogen densities. If CJR are correct, varying n_p from 1.8 to 4.5×10^{22} cm^{-3} (possible in a counter experiment) would change y from 7.4 to 18.9, giving according to (17) a decrease in ω_{abs} of between 1% and 8%. If an earlier calculation,[12] giving a y five times smaller than that of CJR, were correct, then the decrease in ω_{abs} would be between 2.5% and 16.5%. If our estimates are wrong, and ortho → para conversion becomes important at the higher densities, x and ω_{abs} can drop by almost a factor of 3, while if ortho → ortho transitions became important ω_{abs} could rise or fall.

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[8]I would like to thank Mr. V. Brady for performing the numerical integrations needed on the UCRL IBM-650.

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P-WAVE RESONANCE IN PION-PION SCATTERING*

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The possible p-wave resonance in pion-pion scattering has been a subject of much discussion lately.[1-4] The work of Chew and Mandelstam based on the double dispersion representation[5] for the transition amplitude suggests that there is more than one solution to the set of integral equations for the pion-pion partial wave scattering amplitudes.[3,4] In the present note, we wish to point out the possible source of such a multiplicity of solutions from the viewpoint of conventional Lagrangian field theory, and to examine the significance of the fact that the p-wave phase shift remains small throughout the physically acceptable range of the pion-pion coupling constant in the "s-wave dominant solution" of the Chew-Mandelstam equations,[3,4] although a solution containing a p-wave resonance may be obtainable from a nonadiabatic approach, the starting point of which we suggest here.

It is crucial for the following discussion to realize that the form of the double dispersion representation[5] is uniquely determined by the masses of the participating particles and the selection rules, aside from the subtle question of possible subtractions necessary to give the representation a valid meaning. If one assumes the existence of a vector boson of isotopic spin one, which interacts with the isotopic vector part of the pion current in the Lagrangian,[6] and insists that the renormalized mass (which we will define later) of the postulated particle be larger than twice the pion mass, then the particle will become unstable and one has not changed the selection rules of the theory from those of the conventional pseudoscalar meson theory. Such a theory, on the other hand, will necessarily predict a p-wave resonance in pion-pion scattering. The situation here is similar to the well-known Castillejo-Dalitz-Dyson ambiguity[7] in the static meson theory: the double dispersion relation does not imply one particular Lagrangian, but a class of Lagrangians which are selection-rule equivalent.[8]

It follows then that there exists a solution to the Chew-Mandelstam equations corresponding to a Lagrangian theory in which there exists, in addition to the usual couplings, a coupling term of the form

$$L_{\text{int}} = \frac{1}{\sqrt{2}} f \sum_{ijk} \epsilon_{ijk} (\phi^i \partial_\mu \phi^j - \partial_\mu \phi^i \phi^j) B_\mu{}^k, \quad (1)$$

where $B_\mu{}^k$ is the field operator for the vector boson of isotopic spin one, k being the isotopic spin index, and ϕ is the pion field operator.

The p-wave resonance in pion-pion scattering