lying charge transfer band (not shown in the figure). Consequently, we conclude that we were observing transitions between two excited optical states. The fact that the abruptly increased 3600A absorption also decayed with a 5-msec time constant is consistent with and strengthens the above conclusion.

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## NONADIABATIC THEORY OF THE SCATTERING OF ELECTRONS FROM HYDROGEN

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Rosenberg, Spruch, and O'Malley[1] (RSO) have recently calculated rigorous upper bounds, $a_S < 6.23$ and $a_t < 1.92$, for the singlet and triplet scattering lengths in the scattering of electrons from atomic hydrogen,[2] under the assumption that the H ion has only one bound state, with singlet spin. The fact that these upper bounds are 15% below the results of most of the elaborate calculations thus far performed[3] indicates that we are far from an adequate quantitative theory of this three-body problem.

Those calculations which yield scattering lengths close to these upper bounds make use of the adiabatic hypothesis, according to which the distortion of the atom follows the instantaneous position of the scattered electrons.[4,5] Objections to the validity of the adiabatic hypothesis have recently been raised.[6] We shall outline in this note a rigorous (and hence nonadiabatic) series for the phase shift. The proposed series serves two purposes: First, because it is rapidly convergent it provides a method for calculating phase shifts reliably. This is so because methods can be given for accurately approximating the higher order terms. Second, the successive terms of the projected series can be put in a one-to-one correspondence with an adiabatic series which has been derived for dealing with this problem.[4,7] It can be demonstrated that the main contribution to the nonadiabatic terms comes from the functions which enter the adiabatic development. This provides a justification for the qualitative validity of the adiabatic theory and an explanation of the fact that the adiabatic calculations give scattering lengths in better accord with the RSO upper bounds.

The method is an extension of a method used originally by Breit and more recently by Luke, Meyerott, and Clendenin[8] for calculating the energy of some excited states of 2-electron atoms and ions. One makes a decomposition of the zero angular momentum wave function $\Psi(r_1 r_2 \theta_{12})$,

$$\Psi(r_1 r_2 \theta_{12}) = \frac{1}{r_1 r_2} \sum_{l=0}^{\infty} (2l+1)^{1/2} \Phi_l(r_1 r_2) P_l(\cos\theta_{12}), \qquad (1)$$

and reduces the Schrödinger equation to the infinite set of coupled equations (for $r_1 > r_2$)

$$\left\{\left[\sum_{i=1}^{2} \frac{\partial^2}{\partial r_i^2} - \frac{l(l+1)}{r_i^2} + \frac{2}{r_i}\right] - 1 + k^2 - (2l+1)\sum_n \frac{r_2^n}{r_1^{n+1}} \int_0^{\pi} P_l^2 P_n \sin\theta_{12} d\theta_{12}\right\} \Phi_l$$

$$= \sum_{m(\neq l)} \left\{[(2l+1)(2m+1)]^{1/2} \sum_n \left( \int_0^{\pi} P_l P_m P_n \sin\theta_{12} d\theta_{12} \right) \frac{r_2^n}{r_1^{n+1}} \right\} \Phi_m. \qquad (2)$$

The asymptotic form of $\Psi$ for large $r_1$ in the scattering problem is such that all the $\Phi_l$ for $l>0$ vanish. For $\Phi_0$ we have

$$\lim_{r_1\to\infty}\Phi_0(r_1r_2)=\sin(kr_1+\delta)R_{1s}(r_2). \qquad (3)$$

$\delta$ is the required phase shift, and $R_{1S}(r_2)$ is $r_2$ time the radial ground-state wave function of the hydrogen atom.

Now consider the equation obtained by neglecting the right-hand side of the $l=0$ equation:

$$\left(\frac{\partial^2}{\partial r_1^2}+\frac{\partial^2}{\partial r_2^2}+\frac{2}{r_2}-1+k^2\right)\Phi_0^{(0)}(r_1r_2)=0, \quad r_1\geq r_2. \qquad (4)$$

We seek a solution of this equation in which $\Phi_0^{(0)}$ has the same asymptotic form as (3), but with a phase shift $\delta_0$ in general different from $\delta$. The main formula of this approach, which may readily be derived from (2) and (4), relates $\delta$ and $\delta_0$:

$$k\sin(\delta-\delta_0)=-\sum_{l=1}^{\infty}2(2l+1)^{-1/2}\int_0^{\infty}dr_1\int_0^{r_1}dr_2\Phi_0^{(0)}\frac{r_2^{\,l}}{r_1^{\,l+1}}\Phi_l. \qquad (5)$$

The demonstration that the successive terms on the right-hand side of (5) become rapidly small depends on showing that the major contribution of each term comes from the region $r_2\ll r_1$. That this is so follows from the asymptotic form of $\Phi_l$ in that region,

$$\lim_{r_1\to\infty}\Phi_l=\frac{-2\sin(kr_1+\delta)}{(2l+1)^{1/2}r_1^{\,l+1}}e^{-r_2}\left(\frac{r_2^{\,l+2}}{l+1}+\frac{r_2^{\,l+1}}{l}\right), \quad l>0. \qquad (6)$$

This satisfies (2) when one neglects all terms proportional to $r_1^{-n}$, where $n\geq l+2$. It also indicates that $\Phi_l$, for small $r_2$, vanishes as an inverse power of $r_1$, as opposed to its exponentially decreasing behavior for $r_1$ and $r_2$ both large. [The right-hand side of (6) is essentially the $l$th term of an adiabatic series which was proposed in references 3 and 4 for dealing with this problem.] In the region where $r_1$ and $r_2$ are both small, $\Phi_l$ is itself small by virtue of the centrifugal potential term in (2). The diminishing of the terms in (5) as a function of $l$ can be explicitly shown by using (3) and (6) for $\Phi_0^{(0)}$ and $\Phi_l$.

The physical meaning of (4) corresponds to electron 1 coming in from infinity and seeing no force at all until it is actually inside electron 2, at which point it sees the complete nuclear charge. Or to put it another way, when electron 1 gets inside electron 2, the latter becomes the scattered particle. Equation (4) is only valid for $r_1>r_2$, and the symmetry of $\Phi_0^{(0)}$ is taken into account by demanding that $\Phi_0^{(0)}$ or its normal derivative (corresponding to triplet or singlet scattering) be zero along the line $r_1=r_2$. The totality of separable solutions of (4) can readily be written down. In Table I we have given $\delta_0$ obtained from including two such terms so that their sum "best" satisfies the boundary condition along $r_1=r_2$.[9]

Table I. Approximate $\delta_0$ (in radians) based on two terms in the expansion of $\Phi_0^{(0)}$.

| $k$ | 0.05 | 0.1 | 0.3 | 0.5 | 0.75 |
|---|---|---|---|---|---|
| $\delta_0$ (singlet) | 2.76 | 2.41 | 1.48 | 0.92 | 0.41 |
| $\delta_0$ (triplet) | 3.03 | 2.93 | 2.49 | 2.09 | 1.71 |

The scattering lengths obtained from the $k=0.05$ entries in the table are $a_s=7.6$ and $a_t=2.2$. These are above the Spruch-Rosenberg limits, and they are similar to the results of calculations using correctly symmetrized wave functions that do not naturally describe a long-range polarization of the atom, the exchange approximation for example.[10] Our main point is that the inclusion of the first term of (5) will bring the scattering lengths near or within the Spruch-Rosenberg limits. This has already been indirectly tested by the method of polarized orbitals[4] in which the exchange wave function is modified by a polarization term, which is essentially the asymptotic form (6) of $\Phi_1$, and which reduces the scattering lengths below the RSO upper bounds.

One is, of course, interested in the phase shifts as well as the scattering lengths (and not only in upper bounds for these quantities). The relevance of the higher terms in (5) is that they allow one to calculate the phase shifts in a convergent manner. To all these calculations, the solution $\Phi_0^{(0)}$ of (4) is basic. For if that is known accurately, the contribution of the other terms can be reasonably approximated from the asymptotic form (6) of $\Phi_l$, modified for smaller values of $r_1$ and $r_2$ with parameters which can be determined by additional sum rules relating the $\Phi_l$ to $\Phi_0^{(0)}$.

*Most of this work was performed while the author was at the National Bureau of Standards.

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# METHOD FOR DETERMINING THE ORBITAL ANGULAR MOMENTUM IN $K^-$-$d$ CAPTURE*

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In a recent communication, Day, Snow, and Sucher predicted that $K^-$ mesons when stopped in liquid hydrogen or deuterium would be captured from high-lying $S$ atomic orbitals of the $K^-$-$p$ or $K^-$-$d$ system.[1] They found that the Stark-effect mixing of angular-momentum states induced by the strong electric fields experienced by the neutral $K^-$-$p$ or $K^-$-$d$ system as it passed within the Bohr orbitals of other atoms of the liquid allowed rapid capture through the $S$-wave $K$-nucleon interaction. Their prediction has been used to draw important conclusions about the properties of hyperons[1] and the nature of the low-energy, $S$-wave $K$-nucleon interaction.[2] In addition, estimates indicate that Stark-effect mixing determines the course of the antiproton annihilation reaction at rest.[3] Because of the wide applicability of the prediction, an experimental test is highly desirable. The obvious experiment of looking for $K$-shell x rays when $K^-$ mesons are stopped in liquid $H_2$ or $D_2$ is technically difficult and probably will not be done in the near future.

It may be noted that several of the $K^-$-$d$ capture reactions are sensitive to the orbital angular momentum of the $K^-$-$d$ system. This is implied for example in the calculations of Pais and Treiman[4] and Day and Snow[5] on the $\Sigma^- n$-hyperfragment production rate. Their work shows that if the bound $\Sigma^- n$ system existed and its characteristics were known, the production rate for stopped $K^-$ mesons would provide a clear determination of the orbital from which capture occurs. It is the purpose of this note to point out that the $K^-$-$d$ orbital angular momentum determines the rates for another class of reactions that is more accessible experimentally, the nonmesonic capture reactions in deuterium,

$$K^- + d \rightarrow \Sigma^- + p, \tag{1a}$$

$$K^- + d \rightarrow \Sigma^0 + n, \tag{1b}$$

$$K^- + d \rightarrow \Lambda + n, \tag{1c}$$

where the rate for reaction (1a) is twice that for (1b) by charge independence. We shall confine our attention to reaction (1a) since it is easily recognized in a deuterium-filled bubble chamber and is known to occur at a rate of $\sim 0.7\%$ when stopped $K^-$ mesons are captured.[6] For $K^-$ capture at rest, the center-of-mass momentum of the $\Sigma^- p$ system is 511 Mev/$c$, and production is inhibited unless the $K^-$ is absorbed while the two nucleons are within a distance $\sim 0.4 \times 10^{-13}$ cm. The small size of this effective interaction volume implies that the rate for (1a) will be proportional to $|\phi_{nS}(r=0)|^2$ or to $|\nabla\phi_{nP}(r=0)|^2$ depending upon the atomic orbital from which capture occurs. On the other hand, mesonic absorption can occur over the entire volume of the deuteron and will proceed through the $S$-wave $K^-$-nucleon interaction for capture from either $S$ or $P$ atomic orbitals.[5] Therefore, if capture occurs predominantly from $S$ orbitals as predicted by Day, Snow, and Sucher, the fraction of nonmesonic absorptions observed at rest and in flight must be a continuous and slowly varying function of $K^-$ momentum.

Making the explicit assumption that the behavior of the nonmesonic transition amplitude is