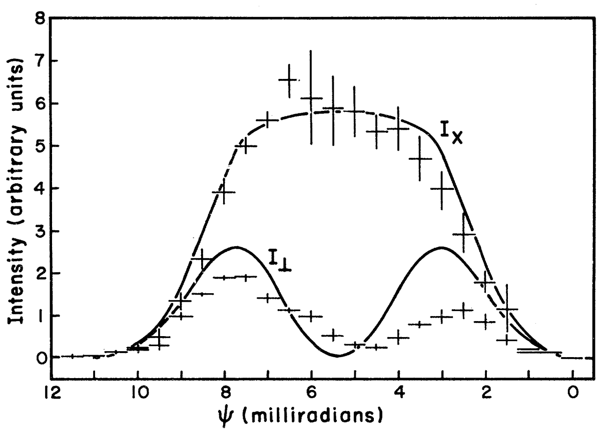

FIG. 2. Experimental and theoretical intensities of the two polarization components of synchrotron radiation plotted as a function of the angle.

value of 3.

To check the phase relation between J_x and $J_\perp$, a quarter-wavelength plate for the Hg 5461A line was inserted between window and analyzer. We placed the slit at 11 milliradians and opened it 3.8 milliradians. At this position the intensities were $I_x = 19.5$ units, $I_\perp = 13$ units. Inserting the quarter-wave plate and adjusting the analyzer at 45° between the two directions, gave for the maximum position 27 units, for the minimum 3. Thus the nearly circularly polarized light was converted into nearly linearly polarized light, which shows that the derived phase relation is correct.

The measurement of the magnitude and the angular dependence of the polarization show good agreement with the theory in its general shape. Since our measurement integrates J_x^2 and $J_\perp^2$ for one cycle and over all electrons, which might have an asymmetrical angular distribution changing with time, there are deviations in the details. The sensitivity to angular changes of the central orbit could enable one to make very fine beam studies.

I am most grateful to Professor Robert R. Wilson and the Staff of the Synchrotron for the hospitality extended to me.

*Supported in part by the joint program of the Office of Naval Research and the U. S. Atomic Energy Commission.

†Visiting Fellow on leave from Physikalisches Staatsinstitut, Universität Hamburg, Germany, supported by the Bundesministerium für Atomkernenergie und Wasserwirtschaft.

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PLASMA STABILITY AND BOUNDARY CONDITION

F. C. Hoh
Department of Electronics, The Royal Institute of Technology, Stockholm, Sweden
(Received May 10, 1960)

Recently, the validity of the classical theory of diffusion of charged particles across a magnetic field has been extensively discussed.[1-10] In the experiment made by Lehnert[1,7] agreement with the classical theory was found for magnetic fields weaker than a certain critical value B_c, beyond which this theory was observed not to be valid. The present note shows that this phenomenon can be explained by an instability of the wall sheath. The experiments[1,5-7,10] indicate that this instability causes the plasma to diffuse across the magnetic field much more rapidly than predicted by the classical theory.

Bohm's[3] criterion for a stable wall sheath formation is

$$v_i' \geq (kT_e/m_i)^{1/2}, \tag{1}$$

where v_i' is the ion velocity normal to the wall on entering the sheath, k is Boltzmann's constant, T_e is the electron temperature, and m_i is the ion mass. In ordinary gas discharges[1,5-7,11] it can be shown that this criterion remains approximately valid when the effect of a magnetic field (of the order of B_c) is taken into account.

Here it will be postulated that the plasma becomes unstable when the boundary condition Eq. (1) is not satisfied because v_i' can be reduced by a magnetic field.

For a cylindrical positive column in an axial magnetic field the radial ion velocity is [1,5]

$$v_i = \frac{k(T_e + T_i)}{e}\left(\frac{\beta_e\beta_i}{\beta_e+\beta_i}\right)\left(\frac{\lambda}{R}\right)\frac{J_1(\lambda r/R)}{J_0(\lambda r/R)}, \qquad (2)$$

where

$$\beta_i = (\beta_{i1}/p_0)[1+(B\beta_{i1}/p_0)^2]^{-1}, \qquad (3)$$

and the corresponding equation when i is replaced by e. Here $\lambda \approx 2.4$, R is the tube radius, r is the distance from the axis, J_1 and J_0 are the Bessel functions of orders 1 and 0, e is the electron charge, p_0 is the gas temperature at 0°C, T_i is the ion temperature in the plasma, β_{e1} and β_{i1} are the electron and ion mobilities at unit pressure and at the corresponding temperatures, and B is the magnetic field strength. Combining Eqs. (1), (2), and (3) and putting $r=r'$, corresponding to the position of the sheath edge, we obtain the stability criterion:

$$B \leq B_c = (\beta_{e1}\beta_{i1})^{-1/2}(g'p_0/R - p_0^2)^{1/2}, \qquad (4)$$

where

$$g' = \left(\frac{km_i}{T_e}\right)^{1/2}\frac{T_e+T_i}{e}\frac{\beta_{e1}\beta_{i1}}{\beta_{e1}+\beta_{i1}}\frac{J_1(\lambda r'/R)}{J_0(\lambda r'/R)}. \qquad (5)$$

Near the wall $J_1(\lambda r'/R) \approx 0.52$. $J_0(\lambda r'/R) = n_e'/n_0$ is the ratio of the electron density at $r=r'$ to that at the axis, and can be written as[1]

$$J_0^{\,3}(\lambda r'/R) = \frac{\epsilon_0 k}{n_0 e^2}\,\frac{T_e\beta_e - T_i\beta_i}{\beta_e+\beta_i}\left(\frac{\lambda J_1(\lambda r'/R)}{R}\right)^2\frac{n_e'}{n_i'-n_e'}. \qquad (6)$$

Here ϵ_0 is the dielectric constant of vacuum, and n_i' is the ion density at r'. The position r', where the plasma merges into the sheath, can be quite well defined by requiring[12] $(n_i' - n_e')/n_e' \approx 0.01 - 0.05$, as B_c is rather insensitive to this value [see Eqs. (4), (5), and (6)].

Earlier measurements[7,11] of B_c are plotted against $(p_0/R)^{1/2}$ in Fig. 1. Under these discharge conditions we have on the average $T_e/V_i \approx 1200$ °K/volt (V_i is the ionization potential), $T_i \approx 400$°K, and $J_0(\lambda r'/R) \approx 0.013$ [Eq. (6)]. With the average mobility data,[13] B_c is calculated from Eqs. (4) and (5) for helium (full curves in Fig. 1). Similar results are also obtained for discharges with krypton,[7,11] argon,[6,7,11] and neon.[6] The present theory may also explain the critical magnetic field observed by Bostick and Levine[5] in the afterglow of a microwave discharge (Fig. 2). Their experiment indicates that the onset of instability is due to wall conditions and is not associated with an axial electric field.

The above discussions are valid for partially ionized gases. The corresponding situation for a fully ionized plasma remains to be investigated. However, the initial stage of the discharge in the B-3 Stellarator[4,8] is not very far from that of the present experiments,[7,11] but with magnetic fields probably too strong to sustain a stable wall sheath. If the plasma is kept from the wall,

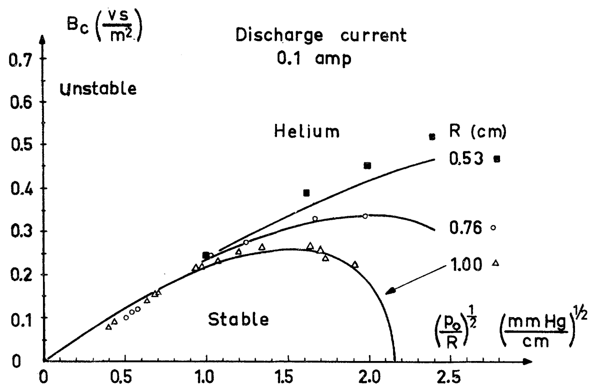

FIG. 1. Earlier measurements (references 7 and 11) of B_c as a function of $(p_0/R)^{1/2}$. The curves are calculated from Eqs. (4) and (5).

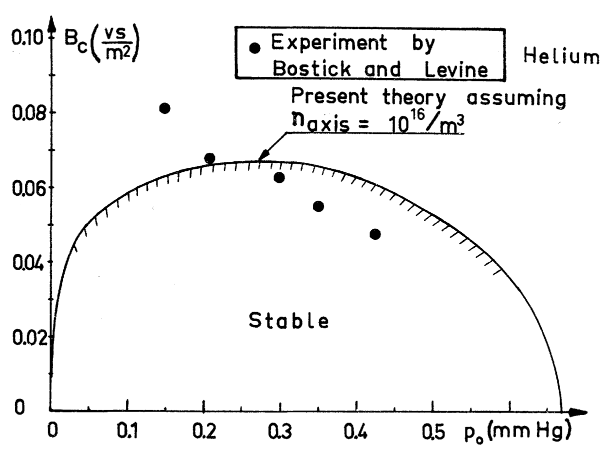

FIG. 2. Critical magnetic field observed by Bostick and Levine (reference 5) in the afterglow of a microwave discharge, compared with the present theory.

e.g., by means of apertures inserted in the discharge tube, the wall conditions are modified. This also seems to be supported by experiments.[8] Further, the present theory seems to connect the classical diffusion theory[1,5] and the ion wave instability theory[4] in a natural way.

An important consequence of the present discussions is that a reversal of the radial electric field in a plasma by means of a magnetic field seems to be impossible if the boundary condition Eq. (1) has to be satisfied. Finally, for large values of Rp_0, Eq. (4) shows that the sheath is unstable even in the absence of a magnetic field. This may be of some interest in the study of high-pressure glow discharges.[14]

The author wishes to express his sincere thanks to Professor H. Alfvén, Dr. B. Lehnert, Dr. S. Lundquist, and Dr. E. Åström for many valuable discussions.

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ZEEMAN EFFECT IN THE RECOILLESS γ-RAY RESONANCE OF Zn^{67}†

P. P. Craig, D. E. Nagle, and D. R. F. Cochran
Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico
(Received May 4, 1960)

Recoil-free resonance emission and absorption (Mössbauer effect)[1] of the 93-kev γ ray in Zn^{67} gives rise to the most precise energy definition thus far reported.[2] Despite the numerous difficulties which beset the experimenter searching for the resonance,[3] a small but definite Mössbauer effect has been found.[2] This Letter reports measurements on the influence of the nuclear Zeeman effect and other perturbing factors upon the Mössbauer effect in Zn^{67} embedded in an enriched ZnO absorber lattice.

The relatively high energy of this γ ray makes it necessary to embed the source and the absorber atoms in rigid crystalline lattices, and to perform the experiments at low temperatures. The first of these requirements was met by using ZnO for both the source and the absorber lattices. The second requirement was more than satisfied by using temperatures below the helium lambda transition (2.175°K). In addition to these basic requirements, several experimental difficulties are consequences of the extreme narrowness of the line (4.84×10^{-11} ev). It is accordingly necessary to take into account the effect of various perturbing influences. Here we list the more important of these perturbations, and in the following paragraphs indicate how they enter into the design of the experiment.

One class of shifts arises from the change of nuclear mass upon γ-ray emission or absorption, with a resultant change in the phonon spectrum of the lattice. The change in energy of the emitted γ ray is given by[4]

$$\Delta E = -(E/Mc^2)\langle T\rangle, \qquad (1)$$

where E is the γ-ray energy, M is the mass of the emitting nucleus, and $\langle T\rangle$ is the expectation value for the kinetic energy per atom of the lattice. If any parameter x should differ between source and absorber lattice, the recoil-free peaks will occur at different energies in the emission spectrum and in the absorption spec-