ROLE OF THE π-π INTERACTION IN HIGH-ENERGY π-NUCLEON INTERACTIONS

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In this note we describe in a simple qualitative way the salient features of pion-nucleon interactions in the energy range 0.5 to 1.5 Bev from the point of view that every interaction is initiated by means of the scattering of the incident pion by a virtual pion of the nucleon.[1]

From the behavior[2] of the very high-energy (5 Bev) elastic and low-multiplicity inelastic events it is clear that there must be a large π-π cross section. One may therefore reasonably hope to explain the "intermediate" energy region ($0.5<E<1.5$ Bev) in terms of the two principal facts of pion physics: the 3-3 resonance and the large π-π cross section. Peierls[3] already used this general idea to explain the 600- and 900-Mev resonances in π^--p scattering. He, as well as Wong and Ross,[4] consider these and other higher resonances as occurring primarily in the system N-π-π; while the total angular momentum of the system may be high (up to $L=3$), each of the two individual pions in this model has a small angular momentum, $l=1$ or 0, relative to the nucleon.

The question then arises as to how the incident single pion, which has the full angular momentum L, gets "caught" by the nucleon and makes a "transition" to a small l corresponding to the N-π-π resonant state. We wish to suggest that this transition is effected by the π-π force acting between the incident π and a virtual π in the nucleon. No matter how the primary N-π-π resonance comes about, the transition will be strongly enhanced by resonances in the π-π system. From the point of simplicity alone this model deserves attention. Aside from this mechanism of initiating the reaction we wish to point out (a) that despite first appearances, the over-all behavior of the π^+-p data from 0.8 to 1.2 Bev is similar to that of the π^--p data from 0.6 to 1.0 Bev; (b) that present information on charge state ratios in single pion production events provides evidence for the dominance of the π-π isotopic spin state $t=1$ at 900-Mev pion lab energy, where the "third" resonance is located, in accord with the work of Frazer and Fulco[5] on the problem of nucleon form factors; (c) that the over-all behavior can be explained by assuming that the $t=0$ interaction in the pion system is dominant in the vicinity of the "second" resonance, from 400 up to about 700 Mev; above this energy the $t=1$ amplitude develops rapidly.

Evidence for point (a) comes from a comparison of the (π^--p) and (π^+-p) total cross sections.[6] Drawing $\lambda\!\!\!^{-2}$ ($\lambda\!\!\!^-$ = reduced c.m. wavelength) curves through the two π^--p peaks, one notes that, although the 1.3-Bev π^+-p peak is perhaps too broad to be considered as a single resonance, $\sigma(\pi^+$-$p)$ is qualitatively similar to $\sigma(\pi^-$-$p)$ but shifted upwards somewhat in energy. (Given the same angular momentum states, this increase in energy naturally suppresses the magnitude of the total cross section.) The similarity lies deeper than this, however. 1. It has been suggested[7] that the sudden rise of $\sigma(\pi^+$-$p)$ above 700 Mev is associated with the advent of a D_{33} resonance. (The notation is $L_{2T,\,2J}$ in extension of Fermi's notation for low-energy pion states.) For $\sigma(\pi^-$-$p)$ it has been reasonably well established that the second resonance (600 Mev) is D_{13}. 2. The π^+-p angular distribution for elastic scattering at 990 Mev displays a backwards bump similar to (but smaller than) that in π^--p scattering between 0.8 and 1 Bev.[8-12] 3. The work of Crittenden et al.[12] on π^--p interactions shows that the inelastic cross section rises abruptly to a maximum near the third resonance (900 Mev) although it is already large before. A similar behavior, but at higher energies, is shown by the inelastic π^+-p cross section although data are more scarce: $\sigma(\pi^+$-$p)$ increases slowly from about 3 mb at 500 Mev[13] to about 10.5 mb at 990 Mev[8] and then rises quickly to 16.5 mb at 1.1 Bev.[14] According to Peierls, the third resonance in π^--p is F_{15}. Hence it seems likely that an F state becomes important in $T=3/2$ (π^+-p) at about a Bev.

This evidence that the dynamical behavior of the π^+-p system is similar to that of the π^--p system, but develops at a higher energy, may be understood simply if we assume that the high-energy dynamics of the π-nucleon system, however complicated they may be, must be initiated by π-π interactions. The small π^+-p ($T=3/2$) cross section below 800 Mev is considered to be a consequence of the lack of any appreciable interaction in the $t=1$ and $t=2$ π-π states in this energy region. The rapid increase in $\sigma(\pi^+$-$p)$

above 800 Mev is interpreted as due to a strong $t=1$ interaction which reaches a maximum at about 900 Mev, according to the evidence considered in the next paragraph. On the other hand, the $T=1/2$ state of the π-nucleon system will, in our interpretation, be dominated by a $t=0$ resonance in the π-π system [point (c) above]. The simplest assumption for this $t=0$ state is that it is a two-pion S wave in the π-π center-of-mass system (P states are forbidden for two pions with $t=0$ by the Pauli principle). Another possibility would be the $t=0$, $J=1$ three-pion resonance, or bound state, suggested by Chew,[15] but this assumption would make it much more difficult to interpret the transition from a single incident pion to the resonant N-π-π state. Rodberg[16] has reported the results of a calculation of pion production using a π-π interaction, in which the s-wave $t=0$ amplitude dominates for energies less than 400 Mev. A reasonable fit to the data of Perkins et al.[17] is thereby obtained.

In order to get evidence for point (b) above, we consider the ratio r of the cross sections for the reactions $\pi^- + p \to \pi^- + \pi^+ + n$ and $\pi^- + p \to \pi^- + \pi^0 + p$.

$$r = \sigma(n + -)/\sigma(p - 0). \qquad (1)$$

Note that the final pion state π^--π^0 cannot have $t=0$, while π^+-π^- is a mixture of $t=0$, 1, and 2. It is clear from the experiments that near the third resonance (900 Mev) the inelastic scattering is much greater in the $T=1/2$ state than in the $T=3/2$ state.[8,12,13] If now we combine first the isotopic spins of the pions in the final state and then couple these states of well-defined t to the nucleon to form a state with total isotopic spin $T=1/2$ (this eliminates $t=2$), then a simple use of Clebsch-Gordan coefficients gives the π-π amplitudes in terms of the charge state ratio:

$$|a_1|^2/|a_0|^2 = (r - \tfrac{1}{2})^{-1}. \qquad (2)$$

a_t is the amplitude for the state in which the π-π system has isotopic spin t. Figure 1 shows the value of the ratio (2) in the vicinity of the third resonance as measured by a number of authors.[10,11,18-20] Equation (2) says that $|a_0|$ decreases from about $1.4|a_1|$ at 800 Mev to only $\frac{1}{2}|a_1|$ at 900 Mev, rising again to about $1.2|a_1|$ at a Bev. The precise ratio of $|a_1|$ to $|a_0|$ depends on the experimental accuracy, and also on the $T=3/2$ admixture. However, if the data of Fig. 1 are reasonably correct, then it is easily seen that for any sensible amount of $T=3/2$ amplitude the same feature persists: $|a_1|^2/|a_0|^2$ goes through a pronounced maximum at 900 Mev.[21]

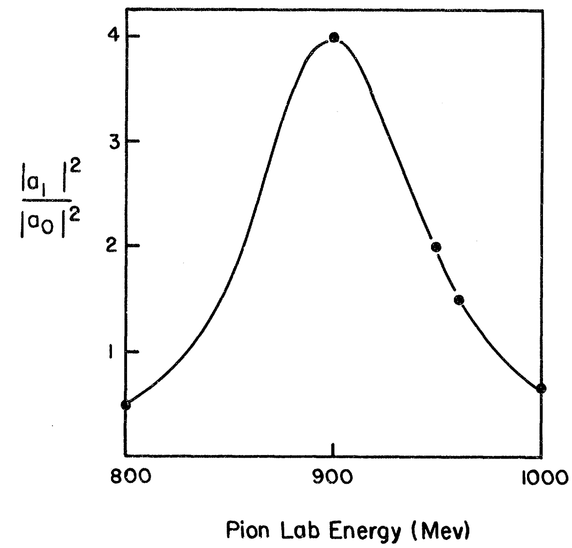

FIG. 1. The ratio of the $t=1$ π-π amplitude to the $t=0$ π-π amplitude, as deduced from Eq. (1), goes through a distinct maximum near 900 Mev. We consider this to be evidence for the $t=1$ π-π resonance suggested by Frazer and Fulco.

Thus Fig. 1 seems to indicate a resonance in the $t=1$ state of the π-π system at 900-Mev lab pion kinetic energy. Since f waves in the π-π center-of-mass system are surely negligible at this energy, it seems that we have evidence for the $t=1$, p-wave π-π resonance suggested by Frazer and Fulco.[5] If the pion in the cloud is assumed to be essentially at rest, this lab energy corresponds to a total π-π center-of-mass energy $W \approx 4\mu$, also compatible with their work. Additional experimental information on charge state ratios would be very desirable. One might check to see if the isotopic spin zero amplitude shows a resonant behavior at 600 Mev, by methods similar to those of Fig. 1.

However, the narrow widths of $|a_1|^2$ makes it extremely unlikely that the third resonance is due only to a π-π resonance, owing to the momentum distribution of the virtual pions.[22] Therefore, it is of interest that the $J=5/2$, even-parity configuration of the 2-pion–nucleon system is just that in which both pions can interact with the nucleon in the 3-3 state, as pointed out by Peierls.[3] Further, Wong and Ross,[4] examining the two-meson propagator by means of the static theory, predict the energy of the $J=5/2$, $T=1/2$ even state to be near a Bev. Hence the third resonance and also the sharpness of the π-π $t=1$ amplitude are very likely due to the effect of the resonant (3-3 state) interaction of both mesons with the nucleon. If this is true, then one cannot yet conclude that there is really a resonance in the π-π system, but only that the reaction goes through that channel in which the $t=1$ π-π interaction dominates.[23]

It is important to note that the "double isobar" picture of the F_{15} state by itself does not cause the behavior of $|a_1|^2/|a_0|^2$ shown in Fig. 1. This is seen by forming the $T=1/2$ state in three different ways: (a) first couple the nucleon and one meson together to form $T'=3/2$; (b) first couple the two mesons together to form a $t=1$ state; (c) first couple the two mesons in a $t=0$ state. Now the square of the projection of state (b) onto state (a), relative to the square of the projection of state (c) onto state (a), is just

$$3[W(\tfrac{1}{2}\,1\,\tfrac{1}{2}\,1;\ \tfrac{3}{2}\,1)]^2/[W(\tfrac{1}{2}\,1\,\tfrac{1}{2}\,1;\ \tfrac{3}{2}\,0)]^2=\tfrac{1}{2}, \quad (3)$$

where W is the Racah coefficient. Evidently $t=0$ is preferred, so far as the $T=1/2$ state is concerned.

Briefly, then, our picture of the high-energy resonances is as follows (see Fig. 2). The two-meson-plus-nucleon propagator is strongly enhanced by 3-3 resonance effects; for the channel with two p-wave mesons this amplitude probably displays a resonance (near 990 Mev). This two-meson channel is then considered to be coupled to the single (incident) meson channel only by the π-π interaction, which in turn may display resonances. With the over-all behavior controlled by the π-π channel, one may understand the difference between the interactions of positive and

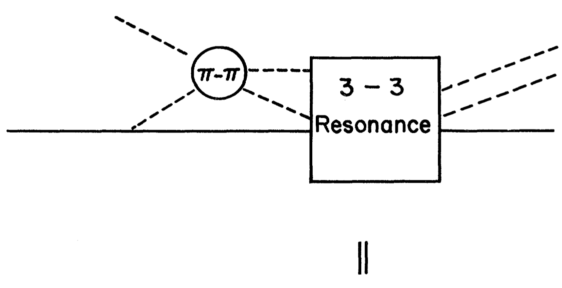

FIG. 2. A pion production event is pictured according to the model described in this Letter. The two-meson intermediate state, in which one or both mesons interact with the nucleon via the low-energy 3-3 resonance, is coupled to the incident meson through the π-π channel, which also may have resonances. (The mesons are represented by dashed lines, the physical nucleon by a solid line.)

negative mesons with protons as outlined above. This picture emerges in a natural way (except for S-wave mesons) when a π-π interaction is added to the static theory. Limitations of space prevent our showing how this happens.

The D_{13} and D_{33} resonances can presumably not be explained by the 3-3 interaction alone because it would be difficult to get a sharp peak in the S-P amplitude. Crudely speaking, increasing the energy of the system would merely increase the energy of the nonresonant S meson, leaving the p-wave at the resonant (3-3) energy. We therefore consider these resonances as due to cooperation between the 3-3 and a π-π resonance, of $t=0$ or $t=1$, respectively. The entrance into this state would also be through the π-π channel.

It was remarked earlier that the 1.3-Bev peak seems rather too broad to be made up from one state. Thus it is tempting to speculate that the $t=2$ π-π interaction becomes strong at this energy, since this would contribute to the $T=3/2$ (π^+-p) but not $T=1/2$ scattering. This interaction could possibly excite the P_{33} isobar found by Wong and Ross[4] in this energy range. It should be easy to check this experimentally; for example, one might expect the reaction $\pi^+ + p \to \pi^+ + \pi^+ + n$ to become important above a Bev, since two π^+ mesons have $t=2$.

Finally we wish to point out that the model here proposed explains why resonances exist in the $D_{3/2}$ (and $F_{5/2}$) state but not in $D_{5/2}$ (and $F_{7/2}$). We have assumed that the resonances arise from the two-meson configurations S-P (and P-P). Now from the vector model, the highest J obtainable from a state containing two mesons of orbital momenta l_1, l_2 and a nucleon of spin $s=1/2$ is

$$J_{\max}=l_1+l_2+1/2, \quad (4)$$

which gives $J_{\max}=3/2$ (5/2) for the S-P (P-P) configuration. This excludes $D_{5/2}$ and $F_{7/2}$.

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[1]References to most of the original work on π-π interactions may be found in a recent paper by C. J. Goebel, Phys. Rev. Letters 1, 337 (1958). See also G. F. Chew and S. Mandelstam, Lawrence Radiation Laboratory Report UCRL-8728, 1959 (unpublished).

[2]G. Maenchen et al., Phys. Rev. 108, 850 (1957); W. D. Walker, Phys. Rev. 108, 872 (1957).

[3]R. F. Peierls, Phys. Rev. 118, 325 (1960).

[4]W. N. Wong and M. Ross, Phys. Rev. Letters 3, 398 (1959).

[5]W. Frazer and J. Fulco, Phys. Rev. Letters 2,

365 (1959); Lawrence Radiation Laboratory Report UCRL-8880, 1959 (unpublished).

[6]Total $\pi^{\pm}-p$ cross sections in the energy range of interest in this paper are given by B. C. Barish, W. N. Hess, V. Perez-Mendez, and J. Solomon, Phys. Rev. Letters 4, 242 (1960).

[7]P. Carruthers, Phys. Rev. Letters 4, 303 (1960).

[8]A. Erwin, J. Kopp, and A. M. Shapiro (private communication from Dr. Erwin).

[9]A. Erwin and J. Kopp, Phys. Rev. 109, 1364 (1958).

[10]W. D. Walker, F. Hushfar, and W. D. Shephard, Phys. Rev. 104, 526 (1956).

[11]L. Bagget, Lawrence Radiation Laboratory Report UCRL-8302, 1958 (unpublished).

[12]R. R. Crittenden et al., Phys. Rev. Letters 2, 121 (1959).

[13]W. J. Willis, Phys. Rev. 116, 753 (1959).

[14]L. O. Roellig and D. A. Glaser, Phys. Rev. 116, 1001 (1959).

[15]G. F. Chew, Phys. Rev. Letters 4, 142 (1960).

[16]L. S. Rodberg, Phys. Rev. Letters 3, 58 (1959).

[17]W. Perkins et al., Phys. Rev. Letters 3, 56 (1959).

[18]V. Alles-Borelli et al., Nuovo cimento 14, 211 (1959).

[19]1958 Annual International Conference on High-Energy Physics at CERN, edited by B. Ferretti (CERN, Geneva, 1958), p. 68.

[20]I. Derado and N. Schmitz, Phys. Rev. 118, 309 (1960).

[21]It should be noted that according to the standard isobar model [R. M. Sternheimer and S. J. Lindenbaum, Phys. Rev. 109, 1723 (1958)] the value of r should increase, essentially because of the presence of the combination $\pi^- - n$ in the final state of the reaction $\pi^- + p \rightarrow \pi^- + \pi^+ + n$.

[22]For a discussion of this point see R. Cool, O. Piccioni, and D. Clark, Phys. Rev. 103, 1082 (1956).

[23]Peierls has noted[3] that the angular distribution [K. Berkelman and J. Waggoner, Phys. Rev. (to be published)] of the π^0 in the reaction $\gamma + p \rightarrow p + \pi^0$ in the energy range 950-1050 Mev photon energy (corresponding to 800-900 Mev pion energy) can be interpreted as due to $D_{13}-F_{15}$ interference. The D state responsible for this behavior is probably not the 600-Mev resonance (D_{13}) but the 800-Mev resonance (D_{33}) because the latter is closer and has a larger Clebsch-Gordan coefficient. Since the angular distributions determine only the relative parity of the 800-900 Mev states, the F_{15} assignment for the 900-Mev resonance depends critically on the correctness of D_{33} for the 800-Mev state which in turn rests on the argument of reference 7 (i.e., if the 800-Mev state were P_{33}, then the 900-Mev state would be D_{15}). Earlier $\pi^+ - p$ scattering experiments seemed to indicate that the D_{33} resonance could not appear below 850 Mev. However, the results of reference 7 suggest that 800 Mev is possible, which is more comfortable if the amplitude of this state is to be appreciable between 600-800 Mev.

ERRATA

REGENERATION AND MASS DIFFERENCE OF NEUTRAL K MESONS. Francis Muller, Robert W. Birge, William B. Fowler, Robert H. Good, Warner Hirsch, Robert P. Matsen, Larry Oswald, Wilson M. Powell, Howard S. White, and Oreste Piccioni [Phys. Rev. Letters 4, 418 (1960)].

We neglected to mention the results of Boldt et al.[1] which indicated that the mass difference (divided by $\hbar/\tau_1$) is more probably a number of the order of one than larger than ten. These authors emphasized that the principal outcome of their experiment was to demonstrate the feasibility of the method that they used.

There are two typographical errors: In the third line above Eq. (1), c_1 should read τ_1. In the paragraph below Eq. (3), $K3$ should read $K2$.

[1]E. Boldt, D. O. Caldwell, and V. Pal, Phys. Rev. Letters 1, 150 (1958).

RECOILLESS RAYLEIGH SCATTERING IN SOLIDS. C. Tzara and R. Barloutaud [Phys. Rev. Letters 4, 405 (1960)].

Instead of

$$\varphi_T = \exp\left\{-\frac{3}{2}\frac{E_R}{k\theta}\left[\frac{1}{4}+\frac{1}{x}\int_0^x \frac{u\,du}{e^u-1}\right]\right\},$$

where $x = T/\theta$, Eq. (1) should read:

$$\varphi_T = \exp\left\{-\frac{3}{2}\frac{E_R}{k\theta}\left[\frac{1}{4}+\frac{1}{x^2}\int_0^x \frac{u\,du}{e^u-1}\right]\right\},$$

where $x = \theta/T$.