of 0.45×10^5 sec^{-1}, based on 1000 muons stopped in a propane bubble chamber.[5] Combining these two results with estimates from counter work[1] gives a weighted mean of

$$\Lambda_i = (0.40\pm0.03)\times10^5 \text{ sec}^{-1}. \qquad (3)$$

The theoretical value from Primakoff[6] is 0.44×10^5 sec^{-1}. This estimate has recently been modified by Flamand and Ford[7] to account for the effect of the finite nuclear size on the muon wave function. The experimental results are in good agreement with the modified prediction of 0.41×10^5 sec^{-1}.

Finally, it should be mentioned that several measurements recently have been made[8] of the partial rate for the muon-capture process leading to the ground state of B^{12}. These investigations provide a test of the Universal Fermi Interaction theory.[9] Although the value of the total rate, as found in this experiment, is somewhat lower than that previously accepted, the change does not significantly affect the measurements of the partial rate.

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MODIFIED ANALYSIS OF NUCLEON-NUCLEON SCATTERING. *p-p* PHASE SHIFTS AT 210 Mev*

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The modified phase-shift analysis program[1,2] has been extended to measurements at 210 Mev. We find only two acceptable solutions, which correspond to the two Stapp solutions[2,3] at 310 Mev. This is one of the few instances in which the data[4-7] have been complete enough to yield a definitive result. Since the phase-shift sets thus obtained are of great value in dispersion relation calculations of the energy variation of phase shifts, and since the present analysis indicates what further experiments should be done to eliminate one of the two remaining phase-shift solution sets at 210 Mev (and therefore also at 310 Mev), we feel that the present results are of considerable interest, even though based on some preliminary data.

In the notation of Wolfenstein,[8] we have analyzed the recent Rochester 210-Mev measurements[4-6] of P, R, and A combined with an earlier Rochester 240-Mev measurement[7] of σ. Search problems were run from 30 random sets of phase shifts, using one-pion exchange contribution (OPEC) with a coupling constant $g^2 = 14.4$ for G and higher waves,[1] and searching on the 9 parameters S, P, D, F, ϵ_2. Since 35 pieces of data were used in the search program (the smallest angle cross-section point was excluded), a 9-parameter solution has an expected least-squares sum χ^2 of 26. The 30 random searches yielded 7 solution sets with χ^2 less than 200,

Table I. Nine-parameter phase shift[a] fits to 210-Mev data ($g^2=14.4$).

Soln.	χ^2	χ^2 corr.[b]	$^1S_0$	$^1D_2$	$^3P_0$	$^3P_1$	$^3P_2$	ϵ_2	$^3F_2$	$^3F_3$	$^3F_4$
a	43.08	28	12.8	0.5	-32.0	23.0	4.9	1.5	-6.0	-4.3	0.8
b	45.61	28	5.9	7.5	2.1	-20.9	17.0	-1.6	-0.4	-2.0	0.3
c	78.42	56	-15.7	2.8	-28.4	-2.1	17.9	-5.8	1.2	-2.3	1.0
d	94.59	81	-2.0	5.3	-41.6	-13.0	10.4	-4.6	-0.2	-3.6	1.1

[a]Phase shifts are nuclear bar phase shifts (see reference 3) in degrees.
[b]χ^2 corr. is the result when the three largest angle differential cross-section points are adjusted to give a contribution of 1 each to χ^2, as they should.

which we label *a* through *g* in order of increasing χ^2. Sets *a*-*d* are listed in Table I. Sets *e*-*g*, with χ^2 values of 115, 135, and 181, respectively, all feature $^1S_0$ phase shifts larger than +30°, and hence are ruled out by comparison with the negative $^1S_0$ phase shift required at 310 Mev.[2] An examination of the remaining sets *a*-*d* shows that the three largest angles in the differential cross-section data contribute unreasonable values to χ^2. When a correction is applied to allow for this, solutions *a* and *b* have about the expected χ^2 value of 26, as shown in Table I. Solution *c* has a χ^2 that, accepted at face value, has only one chance in a thousand of being correct. However, in view of the uncertainty with regard to the differential cross-section data, this cannot be considered sufficient evidence to rule out *c*. Solution *d* may possibly be ruled out by the χ^2 argument, but we will also present other reasons for discarding it.

The phase-shift analysis was extended for solution types *a*-*d* by treating the coupling constant g^2 as a parameter. A 9-parameter search gave the $\chi^2(g^2)$ values shown in Table II. The values of the coupling constant deduced from plots of these results are $g^2=14.3\pm2.5$, 15.8 ± 2.4, 21.0 ± 3.8, and ~28 for cases *a*, *b*, *c*, and *d*, respectively. Since the expected value of about 14 was obtained in a similar analysis at 310 Mev,[1] this casts some doubt on the validity of *c* and is another argument for ruling out *d*. Seven-parameter searches were also made in which first the $^3F_3$ and $^3F_4$ (high *J*) and then the $^3F_2$ and ϵ_2 (low *J*) phase shifts were calculated by OPEC, in addition to *G* and higher waves. The high-*J* case (Table II) gives parabolas for *a*, *b*, and *c* with minimum

Table II. Goodness-of-fit parameter χ^2 versus coupling constant g^2 for solutions *a*-*d*.

g^2	3	9.4	11.9	14.4	18	24	28.8
			9-parameter search S, P, D, F, ϵ_2				
a	121	49	44	43	45	52	55
b	65	50	47	46	46	53	64
c	105	87	82	78	75	75	84
d	150		104	95	85	78	84
			7-parameter search S, P, D, F_2, ϵ_2 (high *J*)				
a	227	105	73	54	48[a]	73	134
b	95	54	63	78	109[a]	102	178
c	106	103	113	129	90[a]	102	178
d	7723	105	73	54	90[a]	73	178
			7-parameter search S, P, D, F_3, F_4 (low *J*)				
a	390	253	235	326	263[a]	569	760
b	87	77[b]	278	419	151[a]	569	1561
c	524	278	169	97	151[a]	569	1561
d	306	449	386	458	263[a]	355	760

[a]$g^2=19.4$.
[b]$g^2=6$.

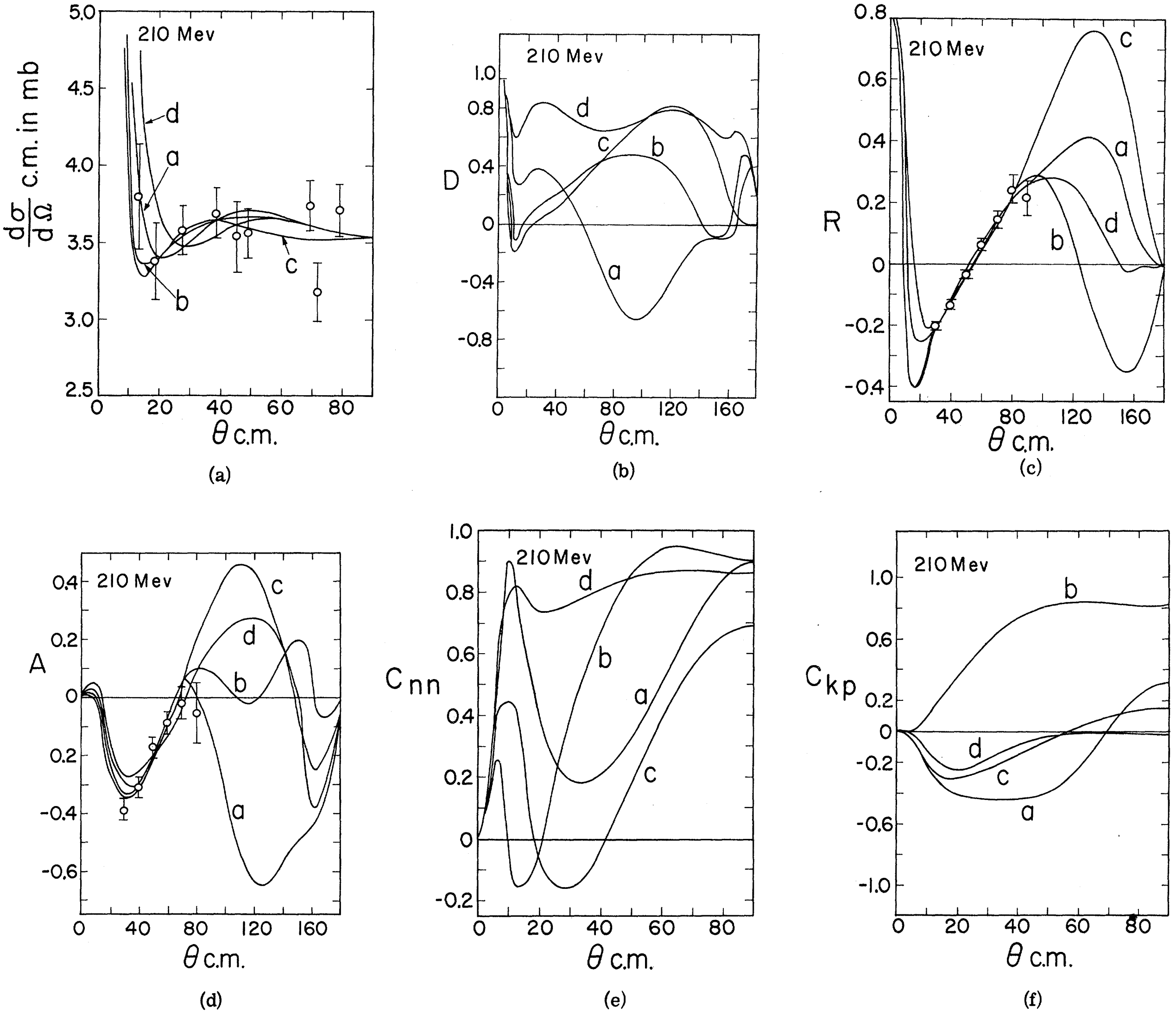

FIG. 1. Plots of the p-p scattering parameters σ, D, R, A, C_{nn}, C_{kp} predicted by solutions a-d, together with data used in the analysis.

χ^2 values of 48, 54, and 99 at $g^2=19$, 9, and 7, respectively. The shift to lower values for g^2 in the bases of b and c as the number of parameters is reduced from 9 to 7 is in agreement with a similar study at 310 Mev.[2] Solution d in the high-J case has lost its independent existence. The good χ^2 values for a and b show that they have $^3F_3$ and $^3F_4$ phase shifts similar to those predicted by OPEC. Solution c in the high-J approximation is not as good as a or b. When the low-J approximation was made, solutions a and d gave no χ^2 values less than 200 for any value of g^2. Solutions b and c had χ^2 minima of 77 and 97 at $g^2=5$ and 14, respectively. Hence the $^3F_2$ and ϵ_2 waves are not approximated by OPEC for a and d, and are only poorly approximated for b and c.

Comparison of the phase shift sets a-d with the Stapp solution sets[1-3] reveals that b is similar to Stapp solution 1, c is similar to 2, d is similar to 6, and a has no Stapp counterpart. Plots of the p-p scattering parameters σ, D, R, A, C_{nn}, and C_{kp} predicted by solutions a-d are given in Fig. 1. The polarization, not shown in the figure, is almost identical for the four solutions and is in good agreement with the experimental data. Solution d can be ruled out on the basis of χ^2 (Table I) and of erratic behavior in the $\chi^2(g^2)$ plots, as mentioned above, and also because the corresponding Stapp solution 6 was shown to be spurious.[2] Solution a can probably be ruled out on three counts: (1) The $^1D_2$ value of 0.5° is not compatible with a required $^1D_2$ of either 4.8° or 11.9° at 310 Mev,[2] and the $^3P_1$ value of +23° is not compatible with a required $^3P_1$ value of -8.1° or -27.5° at 310 Mev.[2] (2) The predicted value for D [Fig. 1(b)] given by a disagrees radically with the actual shape of D as measured at 310 Mev.[3] (3) A large positive $^3P_1$ phase shift is in contradiction with analysis of the photodisintegration of the deuteron.[9] Solution a seems to be a spurious solution arising because of the lack of a D measurement at 210 Mev. On the basis of the present analysis, we rule out all phase-shift solution sets except b and c, and b is slightly favored over c. An examination of Fig. 1 shows that a single measurement of R', A', C_{nn}, or C_{kp} at an appropriate angle would choose among a, b, and c.

The question always arises in this kind of analysis as to whether 30 random searches are sufficient to span the space. Only four of the search problems did not end up with one or another of the sets a-g. In addition, search problems started from Gammel and Thaler[10] and from Bryan, Signell, and Marshak[11] phase shifts both ended up at solution b. Early phase-shift sets of Garren[12] and Klein,[13] using only S and P waves and only σ and P data, gave surprisingly good χ^2 values when applied to the complete set of data. Search problems started from four Garren and four Klein phase-shift sets ended up with our solution sets a, c, d, e, and b, d, e (twice), respectively, for the two cases. At 310 Mev, Stapp[3] used 400 random starts in the search for solution sets. The analogy between the results obtained at 210 Mev and at 310 Mev[1-3] makes us feel that additional 210-Mev random search problems would have a very small probability of producing additional solution sets.

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