PVT ANOMALIES IN He^3 NEAR ITS MELTING CURVE*

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Recent measurements[1] on the *PVT* melting properties of high-purity He^3 have been extended down to 0.3°K. A different apparatus and technique were required, however, because of the predicted[2] minimum, $P_{\min}$, in the melting curve, at a temperature, $T_{\min}$, recently reported[3,4] to be 0.32°K. A consequence of this phenomenon first pointed out[5] by one of us (SGS) is that all parts of a pressure sensing tube at temperatures below $T_{\min}$ will be plugged with solid. The present apparatus avoided this difficulty by use of a spring-loaded bellows, cell *A*, filled with high-pressure liquid He^3 and located inside a heavy-walled copper cell, *B*. The assembly was soldered to a He^3 bath equipped with a He^3 vapor pressure thermometer. The He^3 in cell *A* was raised above the melting pressure, P_m, by raising the *B*-cell pressure, P_B, from a low initial value. Subsequent variations in P_A were detected by observing P_B.

The difference in molar volume of liquid and solid He^3 at the melting point, $\Delta V_m \equiv V_l - V_s$, was determined as follows: Starting with *A* full of solid, small amounts of He^3 were removed from *A* into a low-pressure tank of known volume and measured pressure, p_t. The ratio, $\Delta P_B/\Delta p_t$, was used to indicate onset and completion of melting, being zero only during melting. Thermal disturbances verified these conditions. The material withdrawn from *A* during melting gave a measure of ΔV_m.

Calibrations of cell volumes $(v_A + v_B)$ and of v_A versus $(P_A - P_B)$ were made using liquid nitrogen and liquid He^3. For He^3 experiments at $T > T_{\min}$, $(P_A - P_B)$ was a directly observable constant during the entire melting process. At $T < T_{\min}$ the value of P_A at melting was determined from an extrapolation of pressure measurements made after P_A had dropped below $P_{\min}$.

Values of V_l were determined in a separate series of experiments with *A* and *B* cells interconnected, using a stepwise bleed-out method which also yielded the compressibility coefficient $\beta_l = -V^{-1}(\partial V_l/\partial P)_T$, and the thermal expansion coefficient $\alpha_l = V^{-1}(\partial V_l/\partial T)_P$ near the melting point.

Figure 1 shows the V_l data and the values of V_s obtained by subtracting experimental values of ΔV_m from the V_l curve. Since ΔV_m does not become negative at $T_{\min}$, the entropy change on melting must reverse sign at $T_{\min}$. This was, in fact, observed in the present experiments: melting produced heating and cooling at $T_m = 0.308$°K and $T_m > 0.382$°K, respectively, which

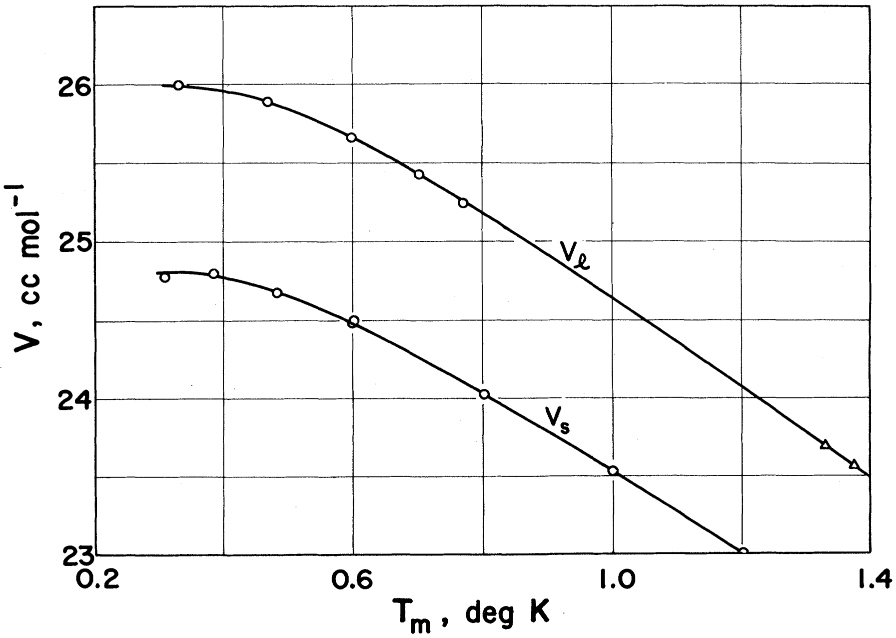

FIG. 1. Molar volume of liquid and solid He^3 along the melting curve. △ Grilly and Mills (reference 1), ○ present measurements.

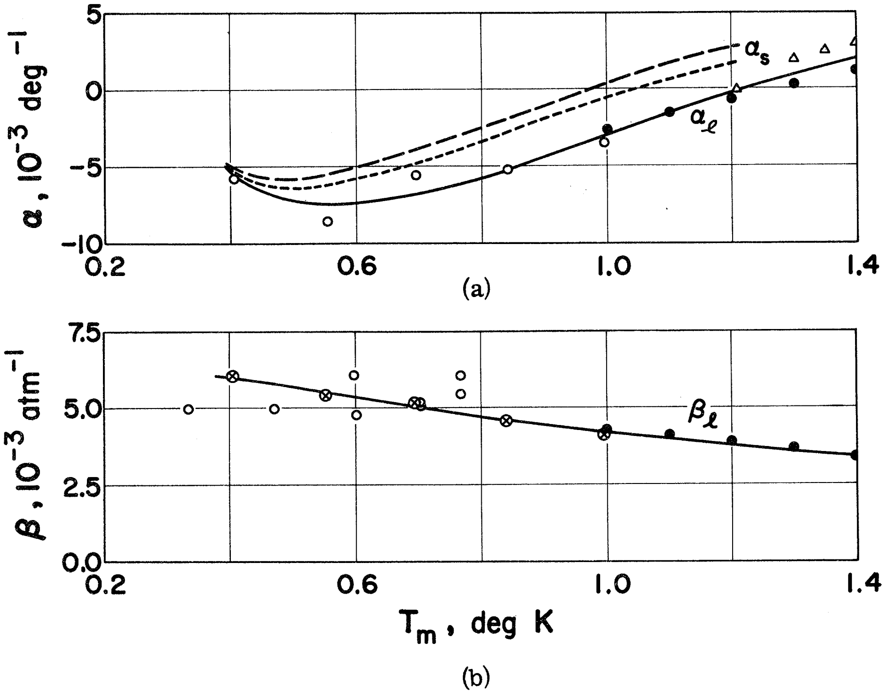

FIG. 2. (a) Thermal expansion coefficient (upper figure) and (b) compressibility coefficient (lower figure) of He^3 along the melting curve. △ Grilly and Mills (reference 1), ● Sherman and Edeskuty (reference 12), ○ present measurements, ⊗ computed from Eq. (1) using the present α_l points. Curves for α_S calculated from Eq. (2): ---- assuming $\beta_S = \beta_l$; ------ assuming $\beta_S = 0.99\beta_l$.

agrees qualitatively with the reversal temperature ~ 0.4°K of Walters and Fairbank.[6]

Values of α are shown in Fig. 2. The anomalous negative values of α_l found at lower pressures[7-9] are seen to persist up to the melting point. From the relation

$$\alpha_l = \left(\beta_l + \frac{1}{V_l}\frac{dV_l}{dP_m}\right)\frac{dP_m}{dT_m}, \tag{1}$$

and the observation that dV_l/dP_m is nearly constant as $P \to P_{\text{min}}$, it appears likely that $\alpha_l \to 0$ as $T \to T_{\text{min}}$. The data are inadequate, however, for a determination of the shape of the curve near T_{min}. Measured and calculated values of β_l near the melting curve are shown in Fig. 2.

The thermal expansion coefficient of the solid was calculated from

$$\alpha_s = \alpha_l + \frac{1}{V_s}\frac{dV_s}{dT_m} - \frac{1}{V_l}\frac{dV_l}{dT_m} - (\beta_l - \beta_s)\frac{dP_m}{dT_m}. \tag{2}$$

With the reasonable assumption that $\beta_l \geqslant \beta_s$, it is found (Fig. 2) that α_S becomes negative below $1.2 > T_m > 1.0$°K. This anomalous behavior is in accord with a suggestion of Goldstein[10] and in qualitative agreement with the theory of Bernardes and Primakoff.[11]

Each ΔV_m determination yielded a single P_m point. A modification of the above technique was used to increase substantially the number of points near T_{min}. The P_m data indicate $P_{\text{min}} = 28.91 \pm 0.02$ atm and $T_{\text{min}} = 0.330 \pm 0.005$°K. The 39 points obtained between 0.3°K and 0.5°K fit the equation

$$P_m = 28.91 + 32.20(T_m - 0.330)^2 \text{ atm}$$

with an rms deviation of 0.02 atm, which is one-seventh of the least count of the *A* gauge, a 1000-psi Heise gauge calibrated in situ against a free piston gauge accurate to 0.001 atm. At higher temperatures the P_m data join smoothly to the melting points of Grilly and Mills[1] and Sherman and Edeskuty.[12]

The data of Baum et al.[4] gave $T_{\text{min}} = 0.32$°K, in good agreement with the present determination. However, their $P_{\text{min}} = 29.3 \pm 0.1$ atm exceeds the value reported here by 0.27 atm more than the combined estimated experimental error of the two determinations. Lee et al.[3] also gave $T_{\text{min}} = 0.32$°K and reported a value of $P_{\text{min}} = 29.1$ atm with no mention of their experimental error.

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HYDROMAGNETIC STABILITY OF FLOW BETWEEN ROTATING CYLINDERS*

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The flow of an electrically conducting fluid between rotating cylinders in the presence of an axial magnetic field is one of the fundamental problems in magnetohydrodynamics. The stability of this flow was first discussed by Chandrasekhar[1] who gave theoretical estimates of the limit of stable flow for a number of magnetic field strengths. Chandrasekhar's calculations were for the case of perfectly conducting cylinder walls. Since then the corresponding problem for insulating cylinders has been considered by Niblett[2] and Chandrasekhar.[3] All these discussions are confined to the case where the gap between cylinders is small.

We have begun an experimental investigation of this problem using the rotating-cylinder viscometer technique developed by Donnelly.[4,5] In this experiment the cylinders are made of No. 302 stainless steel and the fluid is mercury. The inner cylinder is driven by means of a Graham variable-speed transmission. The outer cylinder is suspended freely on a tungsten wire 0.003 in. in diameter. A second wire holds the outer cylinder from below to balance the buoyant force of the mercury. The torque transmitted to the outer cylinder is measured by turning the upper torsion fiber until the torque just balances that due to the motion of the mercury. Thus the viscometer is a null-reading instrument. Fixed guard cylinders are provided at the top and bottom of the outer cylinder to shield the suspended part from end effects. The length of the suspended cylinder is 10 cm and the guard cylinders are each 0.95 cm long. These proportions are different from those of the earlier instrument because the wavelength of the motions at the critical speed in this experiment are quite large and it is desirable to make the suspended cylinder longer than a wavelength. The cylinders have radii $R_1 = 1.8$ cm and $R_2 = 0.2$ cm, leaving a gap $d = 0.2$ cm. The field was furnished by the magnet of the University of Chicago small cyclotron. The field was measured by a search coil calibrated against a nuclear resonance device.

The experimental procedure is to measure the effective viscosity of the fluid (which is proportional to the product of the torque and the period of rotation of the inner cylinder) as a function of speed of rotation. It was found that this curve shows a distinct change in slope at the critical rotation rate. Furthermore the time of response of the suspended cylinder to a small change in speed becomes longer and characteristic fluctuations in deflection appear as the critical speed is reached. All these features have been noticed with the earlier instrument.[4]

Since the properties of fluids are temperature dependent the results are best represented in dimensionless form. In Fig. 1 the critical Taylor number T_c is plotted as a function of Q, where

$$T = \frac{4R_1^2\Omega_0\Omega_1}{R_2^2 - R_1^2}\frac{d^4}{\nu^2}, \tag{1}$$

and

$$Q = \mu^2 H^2 \sigma d^2/\rho\nu. \tag{2}$$

In Eqs. (1) and (2), Ω_1 is the angular velocity of the inner cylinder, Ω_0 is a mean angular velocity