Substituting Eq. (5) into the right-hand side of Eq. (3) and using Eq. (4) and Eq. (7), one verifies that the solution Eq. (5) satisfies the unitarity condition Eq. (3a) provided r_{ij} is analytic in R.

If the scattering solutions [i.e., $r_{ij}(\omega)$] are known, the solutions to the problem of coupled form factors Eq. (3b) are

$$F_j(\omega) = \sum_k \frac{f_k D^{(jk)}}{D}, \tag{8}$$

with the f_k analytic in R. If F_j has no other singularities, the f_k are constants.

A practical procedure of handling these equations is to expand r_{ij} and D in the coupling parameter. Alternatively one may approximate the singularities of r_{ij}—and thus the corresponding singularities of T_{ij}—by poles, and fitting the residues of r_{ij} to the singularities of T_{ij}. If the singularities of T_{ij} are known, the procedure reduces to a system of coupled algebraic equations.

Applications of these methods to double pion production, associated production, and K^--N scattering are in progress.

---

*Supported in part by the U. S. Air Force through the Air Force Office of Scientific Research.

[1]R. Omnes, Nuovo cimento 8, 316 (1958).

[2]G. F. Chew and S. Mandelstam, University of California Radiation Laboratory Report UCRL-8728, April, 1959 (unpublished).

[3]We may equally well use these arguments for other variables such as ω^2 or the square of the center-of-mass momentum.

[4]By this we mean that cases involving "anomalous thresholds" are excluded. See, however, reference 6.

[5]M. Baker, Ann. Phys. 4, 271 (1958).

[6]S. Mandelstam, Phys. Rev. Letters 4, 84 (1960).

EFFECT OF PION-PION RESONANCES ON $\pi^- p$ INTERACTIONS*

L. Landovitz and L. Marshall
Brookhaven National Laboratory, Upton, New York
(Received April 8, 1960)

Some recent experiments[1,2] on the production of π mesons by π mesons at about a Bev have led to contradictory results with respect to the spectra of emitted pions: in one,[1] the π^- from the process $\pi^- + p \to \pi^- + p + \pi^0$ emerges predominantly fast whereas in the other,[2] the π^- is predominantly slow. The former result is quite adequately explained by the isobar model[3] (even if the possibility of excitation of the $T = 1/2$ isobar is included) whereas the latter results are directly contradictory to this model. It may be that the pion spectrum is extremely energy sensitive, and that the two sets of data differ because the incident pion energies are sufficiently different, 960 Mev and 1 Bev, respectively. It is useful, therefore, to find a model which could leave the interpretation of the first pion spectrum unaltered but which could fit the second spectrum owing to the rapid onset of another process.

We would like to indicate here a possible explanation based on a model which is very closely tied to the isobar model and which retains much of its conceptual simplicity: Namely, the inclusion of a π-π interaction predominantly in a $T = 1$ state[4] along with the production of an isobar is considered to be the mechanism for the production of pions.

The picture we use is that the incoming pion collides with a pion in the cloud, scattering, causing one of the pions to be emitted, and leaving the nucleon in an excited state which then decays to the ground state with the emission of the second pion. We will consider the possibility that both the 3/2 and 1/2 isobars can be excited. We can easily calculate the energy of the emerging pions in either case and using the mean of the energies from the two isobars as that of the peak in the spectrum, we have a momentum of 340 Mev/c for 950-Mev incident pions.

On the basis of the above picture, we can readily calculate the ratio of fast to slow π^-: we then obtain, for the case of π-π interaction in $T = 1$ state,

$$\frac{N(\pi^- \text{ fast})}{N(\pi^- \text{ slow})} = \frac{\frac{4}{3}(\frac{1}{6})^2 |a_{1/2}|^2}{\frac{1}{3}(\frac{1}{2})^2 |a_{1/2}|^2 + \frac{1}{3}(\frac{1}{6})^2 |a_{3/2}|^2}$$

$$= \frac{4}{9}\left(\frac{|a_{1/2}|^2}{|a_{1/2}|^2 + \frac{1}{9}|a_{3/2}|^2}\right),$$

$$\frac{4}{9} \geq N(\pi^- \text{ fast})/N(\pi^- \text{ slow}) \geq 0. \tag{1}$$

Of course, there is some production by mechanisms other than that proposed here, and thus these limits may be changed somewhat.

It is of some interest to see the effect of other states in the π-π interaction on this ratio and whether, if the above mechanism is accepted, a $T=1$ "resonance" is needed. Thus, if the $T=2$ state is dominant,

$$\frac{N(\pi^- \text{ fast})}{N(\pi^- \text{ slow})} = \frac{\frac{4}{3}(\frac{1}{9})^2|a_{3/2}|^2 + \frac{4}{3}(\frac{5}{18})^2|a_{1/2}|^2}{\frac{1}{3}(\frac{1}{18})^2|a_{3/2}|^2 + \frac{1}{3}(\frac{5}{18})^2|a_{1/2}|^2},$$

$$16 \geq N(\pi^- \text{ fast})/N(\pi^- \text{ slow}) \geq 4, \qquad (2)$$

or if the $T=0$ is dominant,

$$\frac{N(\pi^- \text{ fast})}{N(\pi^- \text{ slow})} = \frac{\frac{4}{3}(\frac{1}{9})^2|a_{3/2}|^2 + \frac{4}{3}(\frac{1}{9})^2|a_{1/2}|^2}{\frac{4}{3}(\frac{1}{9})^2|a_{3/2}|^2 + \frac{4}{3}(\frac{1}{9})^2|a_{1/2}|^2} = 1. \qquad (3)$$

In neither case could one obtain a predominance of slow π^- mesons.

It is interesting to note that for the $T=1$ case, the π^+ meson is <u>always</u> fast by the above mechanism in the reaction $\pi^- + p \rightarrow n + \pi^+ + \pi^-$.

We can also consider the total number of $\pi^+ + \pi^-$ reactions to $\pi^- + \pi^0$ reactions: we then have, for $T=1$,

$$\frac{N(\pi^+ + \pi^-)}{N(\pi^- + \pi^0)} = \frac{18}{1}\frac{[|a_{3/2}|^2 + |a_{1/2}|^2]}{[|a_{3/2}|^2 + 13|a_{1/2}|^2]},$$

$$18 \geq N(\pi^+ + \pi^-)/N(\pi^- + \pi^0) \geq \tfrac{18}{13}. \qquad (4)$$

Experimentally, this number is found to be greater than one.

However, it should be mentioned that this model, using pion-nucleon isobars with pion-pion interaction only in $T=1$ state, allows no fast π^- to be produced in the reaction $\pi^- + p \rightarrow \pi^+ + \pi^- + n$. Correspondingly the data of reference 2 suggest that the $T=1$ pion-pion resonance is active, but that other processes are important as well.

We thank Maurice Goldhaber and R. M. Sternheimer for valuable comment and criticism.

Note: After completion of this work, two preprints have come to our attention which contain similar ideas.[5,6]

*Work performed under the auspices of the U. S. Atomic Energy Commission.

[1]V. Alles Borelli, S. Bergin, E. Perez-Mendez, and P. Waloschek, Nuovo cimento <u>14</u>, 211 (1959).

[2]I. Derado, G. Lütjens, and N. Schmitz, Ann. Physik <u>4</u>, 103 (1959); I. Derado and N. Schmitz, Phys. Rev. (to be published).

[3]R. M. Sternheimer and S. J. Lindenbaum, Phys. Rev. <u>109</u>, 1723 (1958).

[4]See L. R. Rodberg, Phys. Rev. Letters <u>3</u>, 58 (1959).

[5]Franco Selleri, CERN Report No. 8511/TH 90 (to be published).

[6]I. Derado, CERN Report No. 8113 (to be published).

ERRATUM

ANALYSIS OF THE EXPERIMENTAL τ'^+ DECAY SPECTRUM AS A TEST OF THE $\Delta T = 1/2$ RULE. S. Björklund, E. L. Koller, and S. Taylor [Phys. Rev. Letters <u>4</u>, 424 (1960)].

Due to an error, the predicted value of the parameter $a_{\tau'}$ was given as -9.8. Actually, Weinberg's predicted value is $a_{\tau'} \approx -5.4$. Using the latter value, the question of p becoming negative does not arise. At $a_{\tau'} = -5.4$, the likelihood function for the experimental data is down by a factor of ~1.5 from its maximum at $a_{\tau'} = -7.1$. A six-division χ^2 test with $a_{\tau'} = -5.4$ gives $\chi^2 = 4.0$ with a probability of 0.53. Hence the previous conclusion remains unchanged: The present data, as tested by Weinberg's analysis, are consistent with a $\Delta T = 1/2$ rule.