ORDER OF LEVELS IN THE SHELL MODEL AND SPIN OF Be^{11}*

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It is often assumed that the order of single-nucleon levels is the same for neighboring odd nuclei. This, however, is not always the case if the residual (effective) two-body interactions are taken into account. The detailed consideration of these interactions leads sometimes to interesting "competition" between orbits. For example, the order of filling of neutron shells may depend on the proton configuration. In this note, we discuss from this point of view the ground-state spin of Be^{11}. The experimental data recently obtained[1] indicate a spin and parity 1/2+. This assignment seems to be very unexpected since the standard order of levels in the shell model suggests a 1/2- assignment. We shall show that a 1/2+ spin of Be^{11} due to a last $2s_{1/2}$ neutron is plausible and even preferred on the basis of the detailed quantitative scheme of the shell model.

In C^{13} the $s_{1/2}$ level is 3.09 Mev above the $p_{1/2}$ ground state. However, there are nuclei in which the $s_{1/2}$ orbit is below the $p_{1/2}$ orbit (e.g., Be^{9} and also Be^{10}). It is, therefore, difficult to decide what is the order of these orbits in Be^{11} without a quantitative calculation. Such a calculation is demonstrated in Fig. 1. The first two levels in B^{12} are due to the coupling of one $p_{3/2}$ proton (hole) to the $p_{1/2}$ neutron. The next two levels are similarly obtained by coupling to an $s_{1/2}$ neutron. The spin and parity assignments are based on the B^{12} data as well as on the data for the first $T=1$ levels in C^{12}.[2] The center of mass of the two $p_{1/2}$ levels lies at 0.59 Mev while that of the two $s_{1/2}$ levels lies at 2.03 Mev above the ground state. A linear extrapolation of the $p_{1/2}$ - $s_{1/2}$ difference in C^{13} (3.09 Mev) and the corresponding difference between centers of mass in B^{12} (1.44 Mev) gives the predicted difference in Be^{11}. The $s_{1/2}$ state is predicted to be the ground state, 0.21 Mev below the $p_{1/2}$ level.

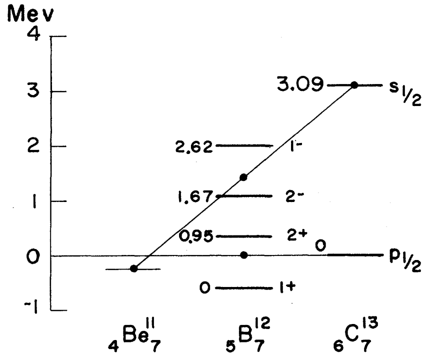

FIG. 1. Competition between $s_{1/2}$ and $p_{1/2}$ levels.

The linear extrapolation in Fig. 1 is based on the fact that the interaction of a j' neutron with two j protons coupled to $J=0$ is given by

$$\langle j^2(J=0)j'|V_{1n}+V_{2n}|j^2(J=0)j'\rangle = 2\sum_{J=|j-j'|}^{J=j+j'}(2J+1)\langle jj'J|V|jj'J\rangle\Big/\sum_{J=|j-j'|}^{J=j+j'}(2J+1). \quad (1)$$

Thus, the change in interaction energy of a $p_{1/2}$ or $s_{1/2}$ neutron when two $p_{3/2}$ protons are removed is twice the change due to the removal of one $p_{3/2}$ proton. This latter change is given by the position of the center of mass as expressed in (1). An analogous case which demonstrates the usefulness of the linear extrapolation is given in Fig. 2. The competition between the $d_{5/2}$ and $s_{1/2}$ orbits is clearly seen. The linear extrapolation from O^{17} and N^{16} gives for C^{15} an $s_{1/2}$ ground state with a $d_{5/2}$ level at 0.58 Mev above it. This agrees very well with the measured spin of C^{15} and the position of its first excited state (0.66 Mev).[2]

Our result about the C^{15} spin was actually based not only on O^{17} and N^{16} but on many more data.[3] In the present case there are not enough data and the result obtained above on Be^{11} is not that certain. The error involved in our estimate can be roughly obtained by considering the Be^{11} binding energy. The separation energy of the $s_{1/2}$ neu-

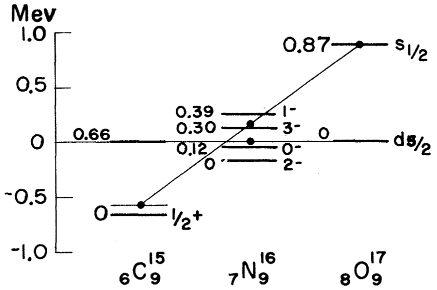

FIG. 2. Competition between $d_{5/2}$ and $s_{1/2}$ levels.

tron in C^{13} is 1.86 Mev. That of the center of mass of the 2- and 1- levels in B^{12} is 1.34 Mev. The linear extrapolation to Be^{11} yields 0.82 Mev for the $s_{1/2}$ neutron separation energy. This value (i.e., the binding energy difference between Be^{11} and Be^{10}) leads to an energy difference between Be^{11} and B^{11} of 11.20 Mev as compared to the experimental value[1] of 11.48 ± 0.15 Mev. The only definite prediction is thus that the 1/2+ and 1/2- levels should be very close in Be^{11}. It would not be surprising if the Be^{11} spin is 1/2-. However, the expectation that the spin and parity of the ground state of Be^{11} are indeed 1/2+, as suggested by the experiment, is at least as firm.

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PANOFSKY RATIO[*]

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The Panofsky ratio, P, which is defined as the relative probability of mesonic to radiative capture of π^- mesons from the K shell in hydrogen,

$$P = \frac{\text{Rate } (\pi^- + p \to n + \pi^0)}{\text{Rate } (\pi^- + p \to n + \gamma)},$$

has been measured many times by various people as is shown in Table I. In those experiments, the two reactions were detected either by the use of a pair spectrometer magnet, a large glass Čerenkov counter, or a cloud chamber. The values obtained for P vary quite widely and are statistically incompatible. The Panofsky ratio can be connected through detailed balancing arguments to low-energy charge exchange scattering and photomeson production as has been pointed out by Marshak[1] and Fermi and Anderson.[2] In making the comparison it is necessary to know the cross section at threshold for charge exchange scattering, the cross section at threshold for π^+ photoproduction on hydrogen, and the ratio of π^-/π^+ photoproduction from a single nucleon which is related to photoproduction of π^-/π^+ in deuterium. Agreement or disagreement can be attained between these phenomena and P depending upon extrapolation pro-

Table I. List of previous measurements of the Panofsky ratio.

P	Reference
0.94 ± 0.30	W. K. H. Panofsky, R. L. Aamodt, and J. Hadley, Phys. Rev. 81, 565 (1951).
1.10 ± 0.50	C. P. Sargent, R. Cornelius, M. Rinehart, L. M. Lederman, and K. Rogers, Phys. Rev. 98, 1349 (1955).
1.50 ± 0.15	J. M. Cassels, G. Fidecaro, A. Wetherell, and J. R. Wormald, Proc. Phys. Soc. (London) A70, 405 (1957).
1.60 ± 0.17	J. Kuehner, A. W. Merrison, and S. Tornabene, Proc. Phys. Soc. (London) 73, 545 (1959).
1.87 ± 0.10	J. Fischer, R. March, and L. Marshall, Phys. Rev. 109, 533 (1958).
1.46 ± 0.10	L. Koller and A. M. Sachs, Phys. Rev. 116, 760 (1959).