NUCLEAR GIANT DIPOLE RESONANCE*

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The collective oscillation of neutrons and protons in nuclei which can be excited by an electromagnetic field leads to a resonant response of nuclei[1-3] which is called the giant dipole resonance. Several classical descriptions of this motion have been given and more recently attempts have been made to describe the resonance starting from an independent-particle description of the nucleus.[4,5] It is the purpose of this note to clarify the relationship between those approaches. We shall restrict ourselves to a discussion of the classical treatment and shall give the detailed quantum mechanical treatment in a separate paper.

The giant dipole resonance is characteristic of finite nuclei, but the same type of relative displacement of neutrons and protons can occur in nuclear matter.[6,7] The description of the collective oscillation is very much simplified in the absence of boundaries, so we shall restrict ourselves to the case of nuclear matter. In the interest of simplicity, we consider an unperturbed state with equal neutron and proton densities. In this case the type of collective motion of interest is one in which the sum of neutron and proton densities remains constant but the neutron and proton densities separately fluctuate 180° out of phase. The shift in neutron and proton densities is opposed by the change in zero-point kinetic energy and in potential energy which tends to restore the neutron and proton density to equality. This effect, together with the inertial reaction of the neutrons and protons, leads in the small-amplitude region to harmonic oscillation of the densities.

Since the oscillation for large wavelengths involves a large number of nucleons, a classical description is appropriate. Any description starting from the single-particle approximation must also lead to the classical result for long-wavelength oscillations.[8]

To carry out the classical calculations, we first determine the single-particle energies, allowing for departures from the unperturbed neutron-proton densities. The single-particle energies are in general given in the K-matrix approximation[9] by

$$E_i = \frac{p_i^2}{2m} + \sum_j n_j (K_{ij,ij} - K_{ij,ji}), \tag{1}$$

with n_j the occupation number of the states of the medium. We rewrite the interaction term as

$$V_i^{(0)} + V_\tau, \tag{2}$$

with

$$V_i^{(0)} = \sum_j n_j^{(0)} (K_{ij,ij} - K_{ij,ji}), \tag{3}$$

$$V_\tau = \sum_j [n_j - n_j^{(0)}] (K_{ij,ij} - K_{ij,ji}), \tag{4}$$

and $n_j^{(0)}$ as occupation number in the unperturbed medium. Near the Fermi surface, the unperturbed potential energy $V_i^{(0)}$ can be written

$$V_i^{(0)} = \text{constant} + \tfrac{1}{2} p_i^2 \left(\frac{1}{m^*} - \frac{1}{m} \right), \tag{5}$$

with m^* the effective mass at the Fermi surface.

The remaining term in the potential energy as given in Eq. (4) is most simply evaluated if the sum over j is separated into neutron and proton sums and the K matrix is resolved into its spin and isotopic spin substates. In this evaluation we set the momentum p_i equal to p_F, since for long wavelengths of the collective oscillation the excitations lie close to the Fermi surface. The result is

$$(V_\tau)_{\text{neutrons}} = \frac{N-Z}{N+Z} v_\tau, \qquad (V_\tau)_{\text{protons}} = \frac{Z-N}{N+Z} v_\tau, \tag{6}$$

with

$$v_\tau = \tfrac{1}{8}(N+Z) \Big[K_{\text{singlet, even}} + 3K_{\text{triplet, odd}} - K_{\text{singlet, odd}} - 3K_{\text{triplet, even}} \Big]_{\text{average}}. \tag{7}$$

The average indicated in Eq. (7) is

$$K_{\text{average}} = \int \frac{d\Omega_j}{4\pi} \left(K_{ij,ij} \right)_{p_i = p_j = p_F}. \tag{8}$$

A result of the form of Eq. (7) has previously been given by Brueckner and Gammel[9] in their evaluation of the nuclear symmetry energy.

Equations (1), (2), and (6) exhibit the important feature that the single-particle energies in general will depend on the fluctuation in neutron-proton densities typical of the collective density oscillations. This effect gives about half of the nuclear symmetry energy, and gives an appreciable shift in the frequency of the collective oscillation. This important contribution to the energy of the collective mode has not been included in previous discussions[4,5] starting from an independent-particle approximation.

Before we carry out the classical calculation,[10] we must determine the change of the zero-point kinetic energy as a function of the neutron and proton densities. For isotropic variation of the densities, the sum of the zero-point kinetic energy for neutrons and protons is

$$T=\sum_{i,N}\frac{p_i^2}{2m^*}+\sum_{i,Z}\frac{p_i^2}{2m^*}=\frac{1}{n_D}\frac{p_F^2}{2m^*}\frac{(N-Z)^2}{N+Z}+\frac{3}{5}\frac{p_F^2}{2m^*}(N+Z), \quad (9)$$

with $n_D=3$ for the three dimensions excited in the medium. The mean change per nucleon then is[3] (for $N+Z$ constant)

$$\left(\frac{p_i^2}{2m^*}\right)_{\text{average}}=\frac{\partial T}{\partial N}=\frac{1}{n_D}\frac{p_F^2}{m^*}\frac{N-Z}{N+Z},\ \text{neutrons}$$

$$=\frac{\partial T}{\partial Z}=\frac{1}{n_D}\frac{p_F^2}{m^*}\frac{Z-N}{N+Z},\ \text{protons.} \quad (10)$$

If the change in density is due to a one-dimensional motion, then only one degree of freedom is excited and n_D in Eqs. (9) and (10) is equal to one. The excitation of all three degrees of freedom takes place in ordinary sound propagation, for example, because of collisions. In a plasma oscillation, however, collisions are missing (except for weak damping effects) and the oscillation is one-dimensional only. The three-dimensional excitation can also occur because of cooperative pairing effects[11] analogous to those in superconductors; such effects are, however, expected to be absent in nuclear matter except for very long wavelengths. Thus we shall set $n_D=1$ in Eq. (10). In this we differ from the phenomenological treatment of Steinwedel and Jensen[3] who used the empirical nuclear symmetry energy which is obtained, of course, for isotropic density changes in nuclei.

To obtain the classical equations of motion, we now replace in Eqs. (6) and (10)

$$(N-Z)/(N+Z)=(\rho_N-\rho_Z)/(\rho_N+\rho_Z), \quad (11)$$

with ρ_N and ρ_Z the neutron and proton densities. This replacement is valid as long as the collective oscillation has wavelength large enough to include many nucleons. The classical equations then are

$$m_t\frac{d\vec{v}_Z}{dt}=-\vec{\nabla}\left[2u_\tau\left(\frac{\rho_Z-\rho_N}{\rho_N+\rho_Z}\right)\right],$$

$$m_t\frac{d\vec{v}_N}{dt}=-\vec{\nabla}\left[2u_\tau\left(\frac{\rho_N-\rho_Z}{\rho_N+\rho_Z}\right)\right], \quad (12)$$

with v_Z and v_N the average velocities of displacement of the protons and neutrons and

$$u_\tau=\frac{p_F^2}{2m^*}+\frac{1}{2}v_\tau. \quad (13)$$

If the factor of 1/3 in Eqs. (9) and (10) were included in the kinetic energy term in Eq. (12), v_τ would be the usual nuclear symmetry energy K. Since the calculation of Brueckner and Gammel gives $v_\tau\cong\frac{1}{3}p_F^2/m^*$, we have the approximate relation

$$u_\tau\cong 2K. \quad (14)$$

The effect of this change on the oscillation frequency is apparent in Eq. (20), which shows that the frequency varies at $(u_\tau)^{1/2}$.

In Eq. (12) we mean by m_t the mass appropriate to the collective displacement of the neutrons relative to the protons. This will not in general be equal to m^* since the excitation is collective, the neutrons for example not changing their motion relative to the other neutrons as they do when singly excited. However, since most of the potential energy of a neutron is due to its interaction with the protons, it is probable that m_t is nearly equal to m^* and we shall take it equal in the following.

We now linearize Eq. (12) for small fluctuations, writing

$$\vec{v}_i=\partial\vec{\xi}_i/\partial t,$$

$$\rho_i=\tfrac{1}{2}\rho_0+\rho_i', \quad (15)$$

and also making use of the fact that $\rho_N + \rho_Z = \rho_0$ is constant. The result is

$$\tfrac{1}{2}\rho_0 \operatorname{div}(\partial \vec{\xi}_i/\partial t) + (\partial \rho_i'/\partial t) = 0 \qquad (16)$$

or

$$\rho_i' = -\tfrac{1}{2}\rho_0 \operatorname{div}\vec{\xi}_i. \qquad (17)$$

Equation (12) then becomes (we drop terms quadratic in the velocities, and set $m_t = m^*$)

$$m^* \partial^2 \vec{\xi}_Z/\partial t^2 = u_\tau \vec{\nabla} \operatorname{div}(\vec{\xi}_Z - \vec{\xi}_N),$$

$$m^* \partial^2 \vec{\xi}_N/\partial t^2 = u_\tau \vec{\nabla} \operatorname{div}(\vec{\xi}_N - \vec{\xi}_Z). \qquad (18)$$

Now assuming one-dimensional motion and combining the neutron and proton equations, we find

$$\frac{\partial^2(\xi_Z - \xi_N)}{\partial t^2} = \frac{2u_\tau}{m^*}\frac{\partial^2(\xi_Z - \xi_N)}{\partial x^2}. \qquad (19)$$

This is a wave equation of usual form, giving the dispersion relation between frequency and momentum q,

$$\omega^2 = \frac{2u_\tau}{m^*}\left(\frac{q}{\hbar}\right)^2, \qquad (20)$$

or, from Eq. (13),

$$\hbar\omega = \frac{qp_F}{m^*}\left(1 + \frac{m^* v_\tau}{p_F^2}\right)^{1/2}. \qquad (21)$$

Since $v_t \cong \frac{1}{3}p_F^2/m^*$, the correction to $\hbar\omega$ is about 15%. It is not possible to extrapolate this result to finite nuclei, but it is clear that this collective effect can lead to appreciable elevation of the eigenvalue above the single-particle value.

The result previously obtained by Landau[6] and by Glassgold, Heckrotte, and Watson[7] is of the form

$$\hbar\omega = (qp_F/m^*)(1 + e^{-\Delta}), \qquad (22)$$

with

$$\Delta = 4\pi^2/[V(0)m^* p_F], \qquad (23)$$

and $V(0)$ the matrix element of the two-particle interaction for small momentum transfer. The correction term $e^{-\Delta}$ arises from the collective character of the motion, the derivation of this term following the usual treatment of the collective excitations in the electron gas. This treatment omits, however, the collective effect we have considered. The estimate by Glassgold et al., using reasonable nuclear matrix elements, gives a value for $e^{-\Delta}$ of about 1% so that a negligible shift in the frequency occurs from this effect. This is in contradiction with the suggestion by Brown and Bosterli[5] that the large shift in the giant dipole frequency from the single-particle value can be attributed to this term.

In summary, we wish to re-emphasize the large role played in the nuclear symmetry energy by the change in potential energy resulting from density changes. This well-known effect necessarily appears in a classical discussion of the giant dipole resonance. Finally, as we have pointed out, a correct discussion of the collective motion starting from a single-particle approximation will lead to the classical result.

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[7]A. E. Glassgold, W. Heckrotte, and K. M. Watson, Ann. Phys. 6, 1 (1959).

[8]This situation also arises in the electron gas. The relation of the classical result to the single-particle description has been discussed in detail by D. Bohm and D. Pines, Phys. Rev. 92, 609 (1953) and K. Sawada, K. A. Brueckner, N. Fukuda, and R. Brout, Phys. Rev. 108, 18 (1957).

[9]See, for example, K. A. Brueckner and J. L. Gammel, Phys. Rev. 109, 1023 (1958). This paper also gives references to the related work in this field.

[10]The classical treatment has been given in detail in reference 2, and we only sketch the derivation.

[11]This point is discussed in detail by N. Bogoliubov, V. Tolmacher, and D. Shirkov, A New Method in the Theory of Superconductivity (Academy of Sciences of U.S.S.R., Moscow, 1958; Consultant's Bureau, Inc., New York, 1959). N. Bogoliubov, Uspekhi Fiz. Nauk 67, 549 (1959) [translation: Soviet Phys. – Uspekhi 67(2), 236 (1959)]. P. W. Anderson, Phys. Rev. 112, 1900 (1958). T. Soda (unpublished paper, 1959).