

single measurement of  $T_1$  on the same constant-volume curve as that of Fig. 3 was about 40 minutes at 0.1°K. This is shorter by about 50 orders of magnitude than it would have been if it had continued to follow the exponential curve  $T_1 = 3 \times 10^{-4} \exp(14/T)$  of Fig. 3. One complete set of  $T_1$  measurements has been obtained between 0.7 and 0.14°K at a relatively low density (starting temperature and pressure 0.8°K and 500 psi). In this case  $T_1$  was about 20% higher at 0.14°K and 0.7°K than at intermediate temperatures but seemed roughly constant at  $0.55 \pm 0.05$  second between these limits.

In summary, the following information about solid He<sup>3</sup> is obtained from the results: (1) In the cases where there is a volume change associated with the  $\alpha$ - $\beta$  transition there are also large changes in the relaxation times. Since the relaxation in the  $\alpha$  phase in these cases is primarily due to diffusion, there must also be changes in the diffusion coefficient across the phase boundary. (2) In the cases where the volume change approaches zero, there are no changes in the relaxation time across the boundary and, consequently, if there are changes in the diffusion coefficients, there must be compensating changes in other relaxation mechanisms. Even in these cases there remains a discontinuity in the rate of change with pressure of the relaxation times at the phase boundary. (3) For temperatures above 1.37°K the  $\alpha$  phase behaves as an ordinary solid with a large diffusion coefficient.

That is, the relaxation is determined by an activated diffusion of the atoms through the lattice. However, at temperatures somewhat below 1°K the relaxation is much too fast to be explained by the classical diffusive motion of the Bloembergen, Purcell, and Pound theory.

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### SPECIFIC HEAT OF INDIUM BELOW 1°K\*

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Heat capacity measurements on two indium specimens indicate that the  $T^3$  term in the specific heat is smaller in the superconducting than in the normal state, contrary to the usual assumption that it is the same in both states. The expected temperature dependence in the normal state is

$$C_n = C_q + \gamma T + \alpha_n T^3 + \beta_n T^5, \quad (1)$$

and in the superconducting state,

$$C_s = C_q + a\gamma T_c \exp(-bT_c/T) + \alpha_s T^3 + \beta_s T^5, \quad (2)$$

$$T < 0.7T_c$$

In each case, the first term,  $C_q$ , is the contribution from the nuclei due to an electric quadrupole interaction similar to that found in rhenium.<sup>1</sup>

From the microwave resonance measurements of Hewitt and Knight<sup>2</sup> it follows that  $C_q = 9.0 \times 10^{-4}$  millijoule/mole deg, which is 11% of  $C_s$  at 0.35°K, the lowest temperature reached in the measurement, and decreases to less than 1% of  $C_s$  at 0.6°K. The second term in  $C_s$  is the electronic specific heat,  $C_{es}$ , of a superconductor at temperatures below  $0.7T_c$ , and is less than 1% of  $C_s$  below half a degree. In this term,  $a$  and  $b$  are quasi-universal constants, and  $T_c$  is the transition temperature of indium (3.4°K). A

well-known expansion of the general expression for the electronic specific heat in the normal state yields  $\gamma T$  for the first term, which is generally a very good approximation.

For estimating  $\alpha_n$ ,  $\alpha_s$ , and  $\gamma$ , the specific heat data, now denoted by  $C_n$  and  $C_s$ , are plotted as  $(C_n - C_q)/T$  and  $(C_s - C_q)/T$  versus  $T^2$  in Fig. 1. It is evident that the upper points can be represented by Eq. (1), and since they lie on a nearly straight line,  $\beta_n T^5$  is very small in this region. A straight line through these points has a slope  $\alpha_n = 1.55$  millijoule/mole deg<sup>4</sup> and yields  $\gamma = 1.61$  millijoule/mole deg<sup>2</sup>. Although our data agree with those of Clement and Quinell<sup>3</sup> within the experimental error up to 4°K, they find  $\alpha_n = 1.50$  and  $\gamma = 1.81$ , using the 1948 temperature scale and extrapolating their data below 1.7°K.

Under the usual assumption that the lattice specific heat in both states is the same, one would expect the lower set of points to have the limiting slope  $\alpha_n$  as  $T \rightarrow 0$ . A line of slope  $\alpha_n$  has been drawn through the origin, showing that  $C_s < \alpha_n T^3$  for  $T^2 < 0.6$ . In fact, the lattice term in  $(C_s - C_q)$

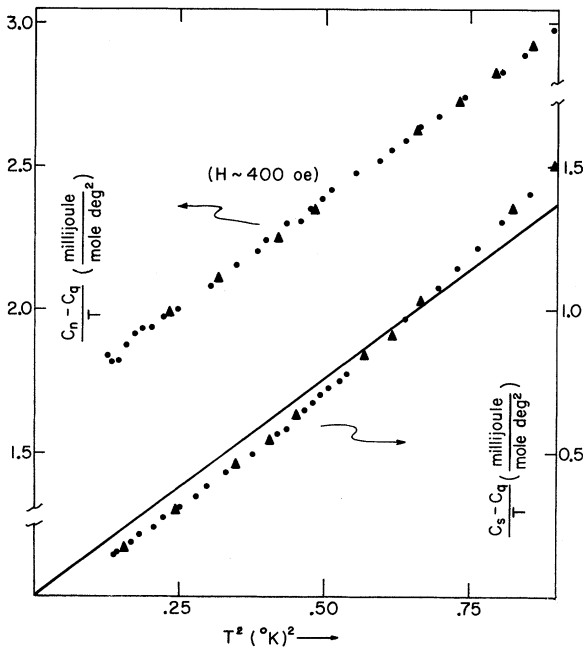


FIG. 1. Specific heat of two indium specimens in the normal and superconducting states after subtracting a nuclear contribution,  $C_q$ . The upper points, measured in a magnetic field of about 400 oersteds, are for the normal state. The solid line which is parallel to these points represents the lattice term in the normal state, and has been superimposed on the points measured in zero field, when the indium is superconducting.

cannot be greater than  $1.1T^3$  at the lowest temperature reached (or not greater than  $1.2T^3$  if the estimate of  $C_q$  were reduced to zero).

When these results were first obtained, the unexpectedly low value of  $\gamma$  and  $C_s$  led to a re-investigation of the conceivable sources of systematic error.<sup>4</sup> Besides errors in the readily checked factors, such as heat capacity of the addenda (at most, 2% of the total), heater current and resistance, and duration of heating pulse, there can be hidden errors in the thermometry and heat input for which limits must be set. The temperature scales used to calibrate the carbon resistance thermometer are based on the 1958 vapor pressure scale for He<sup>4</sup> above 1°K and on the He<sup>3</sup> scale of Sydoriak and Roberts, adjusted to the 1958 He<sup>4</sup> scale. An iron ammonium alum salt thermometer provided an additional check on the calibration over most of the range, and replaced He<sup>3</sup> vapor pressure as the standard below 0.45°K. The largest possible error in  $C$  due to calibration, including effects of contamination of the He<sup>3</sup> by about 1% He<sup>4</sup>, is estimated at 2%. The measurements below 1°K were done after midnight to avoid most of the sporadic heating from electrical and mechanical disturbances. The possibility was investigated that there may have been stray heat input during the heating periods. An extrapolation procedure corrected for the steady heat leak to the sample, which was not more than 3% of the input rate during a heating period. It is felt that the total systematic error in heat input is less than 1%. Afterward, a second sample was vacuum cast and annealed from 99.999% indium, and it produced the same results.

Below  $0.7T_c$ ,  $C_s$  can be represented rather well by the empirical formula

$$C_q + a\gamma T_c \exp(-bT_c/T) + \alpha_s(T)T^3,$$

with  $\alpha_s(T) \approx 1.77e^{-1.9/T}$  when  $a = 11$  and  $b = 1.6$ , which are estimates based on their values for other superconductors. Thus,  $\alpha_s(T)$  diminishes toward lower temperatures, as if the lattice were becoming stiffer as the electron excitation decreases. Alternatively,  $C_s$  may be written with a negative term attributed to the influence of the electrons on the phonon spectrum. In either representation, the separation of  $C_s$  into pure "electronic" and "lattice" terms is convenient but imprecise nomenclature.

It has been pointed out by Chester<sup>5</sup> that as a consequence of the similarity rule for the critical

magnetic field [i.e., the critical field has the functional dependence  $H_C = M^{-\alpha} F(M^\alpha T)$  on the isotopic mass  $M$ , where  $\alpha$  is a constant equal to about 1/2], the difference in the specific heats ( $C_n - C_s$ ) must have the form  $M^{-\alpha} f(M^\alpha T)$ . The electronic terms have this dependence, but  $\alpha_n \propto M^{3/2}$ , so that the lattice term in the normal state does not have the required form, from which Chester concludes that  $\alpha_n T^3$  must be cancelled by an identical term in  $C_s$ .

Though the similarity rule for isotopic mass holds accurately for some elements,<sup>6,7</sup> we are not aware of critical field measurements on the two natural isotopes of indium. However, Muench reported that another similarity rule, namely that the shape of the critical field curve is independent of pressure, does not hold for indium.<sup>7</sup> It would be desirable to have critical field measurements on the In isotopes, as well as extended heat capacity measurements on other soft superconductors having high  $T_C$  and rela-

tively large lattice terms.

A detailed discussion of our data up to 4°K will appear in a forthcoming article.

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## GIANT SPIN DENSITY WAVES

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The observations made in this paper call into serious question many details of the modern electron theory of metals. It will be shown below that, almost certainly, the Hartree-Fock ground state of a Fermi gas with Coulomb interactions is not the familiar Fermi sphere of occupied momentum states, but rather a state in which there are large static spin density waves, and in which large energy gaps exist in the single-particle excitation spectrum.

In order to emphasize the essential physical simplicity of the new low-energy states, we shall treat first a one-dimensional model. (Only translational freedom will be restricted to one dimension; the ordinary spin degrees of freedom will be retained.) The kinetic energy operator will be the usual one; and we shall assume that the repulsive interactions are delta functions:

$$V_{ij} = \gamma \delta(z_i - z_j). \quad (1)$$

The normal state of such a gas— $N$  electrons in a box of length  $L$ —has all (plane wave) states occupied for  $|k| \leq k_0 = \pi N/2L$ . The total kinetic

energy is  $\frac{1}{3}NE_F$ , where  $E_F = \hbar^2 k_0^2/2m$ ; and the expectation value of the interactions (direct plus exchange) is  $\gamma N^2/4L$ . It is of interest to compare the energy of the normal state with that of the ferromagnetic state (all spins parallel):

$$W_{\text{ferro}} - W_{\text{normal}} = NE_F(1 - n^{-1}), \quad (2)$$

where

$$n = (N/L)/(2m\gamma/\pi^2\hbar^2), \quad (3)$$

a dimensionless quantity proportional to the electron density,  $N/L$ . The critical density, at which a transition between the normal and ferromagnetic states would occur, corresponds to  $n = 1$ . We shall prove, however, that the normal state is never the Hartree-Fock ground state, and that the ferromagnetic state is stable only for  $n \leq \frac{3}{4}$ .

We shall "begin" by writing down the self-consistent Hartree-Fock potential for the solutions of interest.

$$U(z) = 2gE_F(\sigma_x \cos qz + \sigma_y \sin qz), \quad (4)$$