Table I. Results of the χ^2 tests of the data for various assumed values of the parameter $a_{\tau'}$.

$a_{\tau'}$	χ^2	Probability	Remarks
4.6	13.6	0.017	τ value
0.0	8.8[a]	0.12	
-7.1	3.8	0.57	maximum likelihood value
-9.3	8.2	0.14	predicted value

[a]This value of χ^2 differs from that in reference 3, since here the relativistic boundary of the Dalitz plot was used, while reference 3 used the nonrelativistic approximation.

errors in measuring the kinetic energies has been tested by recalculating the maximum likelihood value of $a_{\tau'}$ using the upper and lower error limits of the most sensitive event of the data, which also happens to have large errors. This event has $T_3=49.0^{+4.3}_{-6.0}$ Mev. The maximum likelihood values so obtained are $a_{\tau'}=-6.8$ and $a_{\tau'}=-7.4$ for the upper and lower limits, respectively.

The measured value $a_{\tau'}=-7.1$ is in satisfactory agreement with the predicted value of -9.3. Hence, the present data, as tested by Weinberg's analysis, are consistent with a $\Delta T=\frac{1}{2}$ rule. It is unlikely that $a_{\tau'}$ is as large as a_{τ}, although the possibility cannot be completely excluded by the present data.

We thank Mr. T. Faith for his aid in carrying out the calculations.

---

*Supported in part by a grant from the National Science Foundation.

[1]S. Weinberg, Phys. Rev. Letters 4, 87 (1960).

[2]S. McKenna, S. Natali, M. O'Connell, J. Tietge, and N. C. Varshneya, Nuovo cimento 10, 763 (1958). Of the events studied, 419 were from M. Baldo-Ceolin, A. Bonetti, W. D. B. Greening, S. Limentani, M. Merlin, and G. Vanderhaege, Nuovo cimento 6, 84 (1957).

[3]S. Taylor, G. Harris, J. Orear, J. Lee, and P. Baumel, Phys. Rev. 114, 359 (1959).

AUGER EFFECT IN MESONIC ATOMS

Richard A. Ferrell*
University of Maryland, College Park, Maryland and CERN, Geneva, Switzerland
(Received March 21, 1960)

The Auger effect is important in numerous experiments dealing with the capture of negative particles by nuclei, because of the dependence of the interpretation of such experiments on the details of the atomic cascade process. It is often essential to know, in the case of a strongly interacting negative particle, in which state it undergoes interaction with the nucleus. The answer to this question depends in turn upon the transition rates out of the various states. Therefore it is necessary to have a reliable theory of the Auger transition rate. Unfortunately, although the usual perturbation theory of the Auger effect is straightforward, it has been put into doubt by the serious discrepancies found by Stearns and Stearns[1] for the x-ray yields in μ- and π-mesonic atoms. The purpose of the present note is to point out an alternative way of computing the Auger effect which makes it clear that the usual perturbation treatment is absolutely reliable. In fact, we demonstrate a quantitative relationship between the Auger rate on the one hand and the photoelectric cross section on the other. From this relationship, it is possible to calculate the Auger effect directly from the empirical x-ray absorption data. As the Auger rate is too small and no other significant radiationless process exists, it follows that the effect found by Stearns and Stearns must have its origin outside the target. This conclusion is supported by noting that the discrepancies for the μ-K, μ-L, and π-L series, although usually considered separately, are really the same and are a function of only the photon energy, and independent of the target.[2] For example, the targets of boron, fluorine, and sodium give, respectively, μ-K, π-L, and μ-L x rays all with photon energies within 5 kev of 47 kev, and all with a yield of only one-half of the expected yield.

The Auger rates for the μ-K and μ-L series have been computed by Burbidge and de Borde.[3] These rates are too small by factors of 300 and of 30, respectively, to explain the drop in x-ray yield found by Stearns and Stearns. It might be supposed that somehow the Auger rate calculation could be in error to this extent (although this would be surprising because of its straightforwardness).[4] For this reason we wish to emphasize here the reliability of the theoretical Auger rate, by pointing out the following entirely new way of making the calculation.

The combination of the central Coulomb field of strength $Z-1$ centered at the nucleus and the perturbing dipole field of the meson is equivalent to a new central Coulomb field shifted in position to the charge center of the nucleon-meson system. Since the charge center oscillates with the transition frequency ω, it is convenient to work in the accelerated frame of reference attached to it. In this frame an oscillating inertia, or gravitational, force acts on the atomic electrons. The effect of this force is the same as that of a uniform fluctuating electric field, allowance being made for the charge-to-mass ratio. Thus the Auger rate is equal to the rate of photoelectric ionization, when the atom is subjected to an "equivalent photon flux" given by

$$I=\frac{c}{2\pi\hbar\omega}\left(\frac{m}{e}\right)^2\omega^4(Z-1)^{-2}\sum_f|\vec{r}_{fi}|^2$$

$$=(Z-1)^{-2}\sigma_T^{-1}w_R, \qquad (1)$$

where we made use of the standard formula for the radiation rate w_R, and have introduced the Thompson cross section $\sigma_T=(8\pi/3)(e^2/mc^2)^2$. $\vec{r}$ is the meson coordinate and the other symbols have their conventional significance. Since the ionization rate is given by multiplying the photon flux by the photoelectric cross section σ_{Z-1}, we find for the ratio of the Auger to radiative rates

$$w_A/w_R=(Z-1)^{-2}\sigma_{Z-1}/\sigma_T. \qquad (2)$$

It should be emphasized that despite this equivalence of the Auger rate to the photoeffect, the Auger effect is not in fact due to the conversion of photons, and does not take place in the outer radiation zone of the mesonic atom. It instead takes place in the near zone and is due only to the quasi-static electric field. Indeed, Eq. (2) is valid only when retardation is negligible; but this is true in the cases of interest. Of course, Eq. (2) is also based on treating the meson as a dipole. Since the meson orbits are one to two orders of magnitude smaller than that of the electron, this is a justifiable approximation.

One can use Eq. (2) to calculate the Auger rate directly from x-ray absorption data. Consider the L x ray emitted by a μ meson captured in fluorine, of energy 32 kev. The $Z-1$ atom, oxygen, has an interpolated mass absorption coefficient[5] of 0.302 at this photon energy, corresponding to a photon cross section of 8.06×10^{-24} cm^2. Subtracting from this eight times the Compton cross section[6] of 0.601×10^{-24} cm^2 we find

$$\sigma_{Z-1}=3.25\times10^{-24}\text{ cm}^2,$$

or from Eq. (2)

$$w_A/w_R=0.075. \qquad (3)$$

This can be compared with the value of Burbidge and de Borde[7] of 0.083. But the latter authors did not allow for the screening of one K electron by the other. The screening is approximately taken into account[8] by further reducing the effective nuclear charge by 5/16, so that the electron density at the nucleus must be reduced by the factor

$$[(Z-1-5/16)/(Z-1)]^3\approx1-15/16(Z-1)=0.88.$$

We thus obtain the theoretical value of 0.073, in good agreement with Eq. (3). As this example demonstrates, Eq. (2), by making use of the empirical x-ray absorption data to avoid the tedious calculations with the free-electron Coulomb wave functions, may have some practical advantage. Equation (2) has the further advantage of automatically including screening and the contribution of the higher electron shells, when these are important.

To continue with the example of fluorine, the μ-L yield found by Stearns and Stearns was only (24 ± 2)%. The μ-K series yield for the same atom indicates that approximately 20% of the muons jump directly to the ground state from the $n=3$ and higher shells, so that the L-yield would be expected to be about 80%.[9] To reduce this figure to 24%, the Auger rate would have to be 31 times greater than given by Eq. (3). In view of Eq. (2), this would require a photoelectric cross section greater by the same factor, which is, of course, excluded. On the other hand, as emphasized by Bernstein and Wu,[10] processes involving electrons of neighboring atoms make no significant contribution, since the only essential feature of the initial electron distribution which enters is the density at the nucleus. The further possibility of low-energy radiationless transitions between levels belonging to the same quantum number $n=3$, as suggested by Krall and Gerjuoy[11] for $n=2$, can be ruled out as there is no metastable state available.[12] In any case, from the existence of the K radiation the mesons are known to have passed through the $n = 2$ level. Hence, one can conclude that the L radiation must have been emitted but somehow eluded detection.

To test this hypothesis, it is natural to assume that the discrepancies found with the μ-K, μ-L, and π-L yields, although generally discussed separately, are in fact the same thing. In order to compare the various yield curves, it is necessary to plot them vs energy rather than vs atomic number.[13] Figure 1 shows the data of Stearns and Stearns[1] and of Stearns, Stearns, and Leipuner,[14] plotted in this manner. As a rough approximation all three sets of data, taken with the same apparatus, define the same curve. This indicates a yield independent of the target and depending only upon the quantum energy, or, "by definition," an instrumental effect. A reasonable fit is given by a linear fall-off of detector efficiency with decreasing energy, beginning at about 90 kev. Taking such a linear efficiency and dividing it into the π-K yields found by Stearns and Stearns[15] changes these into a decreasing monotonic function of atomic number, as expected if only nuclear absorption competes with radiation. Furthermore, dividing this linear efficiency function into the roughly constant background found below 90 kev converts it into a bremsstrahlung-type spectrum similar to that found for the background above this energy.[16]

It is well known that the decreased efficiency cannot be attributed to the sodium iodide crystals used for detecting the x rays. Neither can

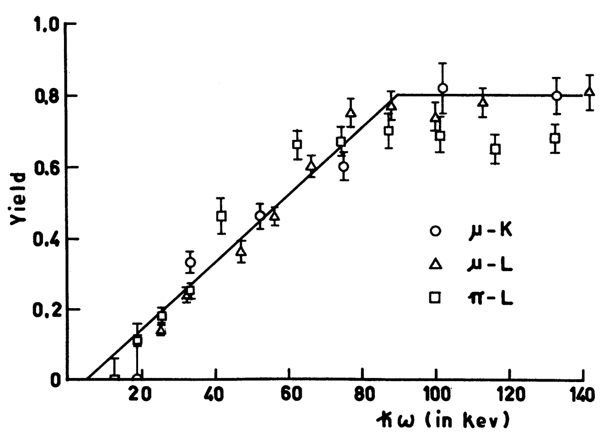

FIG. 1. μ-K, μ-L, and π-L x-ray yields vs photon energy, $\hbar\omega$. The three sets of data, taken with the same experimental apparatus but with different target materials, can be accounted for by the single linear efficiency curve shown.

such a linear efficiency curve be attributed to some absorber overlooked between the targets and the detector. Any x-ray absorber which gives a break at 90 kev gives too strong an absorption at the lower energies. But it is sometimes possible for an inefficiency to be introduced by the electronic circuitry for small pulse heights.[17] Finally, note should be taken of the decrease in yield reported in reference 14 for the higher cyclotron intensities. It would be desirable to have further experimental studies of this effect, especially in the crucial energy range of less than 90 kev.

---

*National Science Foundation Senior Postdoctoral Fellow.

[1]M. Stearns and M. B. Stearns, Phys. Rev. 105, 1573 (1951).

[2]This has already been noted by many persons. See, for example, reference 12.

[3]G. R. Burbidge and A. H. de Borde, Phys. Rev. 89, 189 (1953).

[4]The satisfactory agreement found in ordinary atomic applications also makes this unlikely. See E. H. S. Burhop, The Auger Effect (Cambridge University Press, New York, 1952).

[5]A. H. Compton and S. K. Allison, X-Rays in Theory and Experiment (D. van Nostrand Company Inc., New York, 1935), Appendix IX.

[6]W. Heitler, The Quantum Theory of Radiation (Oxford University Press, New York, 1954), 3rd ed., p. 221, Eq. (46).

[7]Since Table I of reference 3 does not include fluorine, we have extrapolated from the μ-L Auger rates for carbon, nitrogen, and oxygen.

[8]L. I. Schiff, Quantum Mechanics (McGraw-Hill Book Company Inc., New York, 1955), 2nd ed., p. 176.

[9]Actually, about one-quarter of the "higher transitions" must be addition peaks, so that the true L yield should be about 85%.

[10]J. Bernstein and T. Y. Wu, Phys. Rev. Letters 2, 404 (1959).

[11]N. A. Krall and E. Gerjuoy, Phys. Rev. Letters 3, 142 (1959).

[12]This mechanism has recently been analyzed in detail by M. A. Ruderman (to be published).

[13]This has already been done by Ruderman, reference 12, Fig. 1. We nevertheless include such a plot here, since a linear rather than logarithmic scale is more convenient for a discussion of the origin of the effect.

[14]M. B. Stearns, M. Stearns, and L. Leipuner, Phys. Rev. 108, 445 (1957).

[15]M. Stearns and M. B. Stearns, Phys. Rev. 107, 1709 (1957).

[16]From later measurements made with the Carnegie Tech. facilities there is good reason to attribute most of the background to bremsstrahlung [G. Backenstoss (private communication)].

[17]It should, however, be noted that checks were made

in the experiments to guard against such a possibility [M. B. Stearns (private communication)]. It should also be noted that the Rochester group independently found a similar decrease in π-L x-ray yield, but with a somewhat weaker fall-off at the lower energies [M. Camac, M. L. Halbert, and J. B. Platt, Phys. Rev. 99, 905 (1955)]. One can only conclude that similar experimental conditions must have prevailed.

UPPER LIMIT FOR PRODUCTION OF $\Sigma^- n$ HYPERFRAGMENTS BY K^- CAPTURE IN DEUTERIUM*

Orin Dahl, Nahmin Horwitz, Donald Miller, and Joseph Murray
Lawrence Radiation Laboratory, University of California, Berkeley, California
(Received March 21, 1960)

Recently, Gondolfi et al. reported evidence for the existence of a bound state of the $\Sigma^- n$ system.[1] Pais and Treiman had already suggested that such a system might be produced when K^- mesons were captured in deuterium.[2] They assumed that the $\Sigma^- n$ would form a deuteron-like structure bound in either the ^1S or ^3S state and used an impulse-type model to estimate the relative rates for the processes

$$K^- + d \rightarrow \Sigma^- n + \pi^+, \quad (1)$$

$$K^- + d \rightarrow \Sigma^- + n + \pi^+, \quad (2)$$

for both odd and even K-Σ parities. Day and Snow have pointed out that the strong S-wave K^--nucleon interaction will dominate the capture process from either S or P atomic orbitals and have extended the calculations of Pais and Treiman to the latter case.[3] Typical values calculated for the fraction of all Σ^- productions leading to the formation of the bound state are summarized in Table I.

In order to determine the relative rates for reactions (1) and (2), we are examining 2100 $\Sigma^-(n)$ productions obtained during two exposures of the Alvarez 15-in. deuterium chamber to the separated 450-Mev/c K^- beam. Of the $\Sigma^-(n)$ productions, $85 \pm 5\%$ result from K^- absorptions at rest. Thus far we have analyzed in detail 227 events in which the $\Sigma^-(n)$ came to rest and was captured via the reactions[4]

$$\Sigma^-(n) + d \rightarrow \Lambda \text{ (or } \Sigma^0) + n + n (+n).$$

For the three cases giving best fits when these events are interpreted as examples of reaction (1), the probabilities for exceeding the observed values of χ^2 were 30%, 10%, and 1%. From this result it is concluded that the fraction of Σ^- productions leading to formation of the bound state is $< 1\%$.

Day, Snow, and Sucher have argued that it is extremely likely that K^- mesons, when stopped in liquid H_2 or D_2, are captured from high-lying S orbitals.[5] If their prediction is correct and if the existence of the $\Sigma^- n$ is confirmed, the present

Table I. Probability for formation of a $\Sigma^- n$ with binding energy E_b when a K^- meson is captured in deuterium via the S-wave K-nucleon interaction.

K-Σ parity	$\Sigma^- n$ bound state	S-orbit capture[a] E_b = 0.1 Mev	S-orbit capture[a] E_b = 0.5 Mev	P-orbit capture[b] E_b = 0.1 Mev	P-orbit capture[b] E_b = 0.5 Mev
-	^3S	0.20	0.37	0.03	0.06
-	^1S	forbidden		0	0
+	^3S	0.13	0.25	0.02	0.04
+	^1S	0.07	0.13	0.01	0.02

[a]See reference 2.
[b]See reference 3.