is amusing to note that in the absorption by Fe_2O_3 of the emission lines of Fe^{57} in ordinary iron, two such accidental coincidences do indeed occur to give substantial absorption at zero velocity.

We wish to thank many of our colleagues, particularly M. Goldhaber and J. Weneser, for interesting discussions, and G. K. Wertheim for providing us with a sample of the particular stainless steel used in his measurements.[5]

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STRUCTURE OF NUCLEAR MATTER

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It is usually assumed that the Hartree (or Hartree-Fock) ground state of a Fermi gas is the familiar sphere of occupied momentum states. For a noninteracting gas such is, of course, the ground state; and for an interacting gas, this state is, to be sure, an exact solution of the Hartree-Fock equations. But to regard it as the lowest single-particle state has always been an unproved assumption, and indeed a false one. Explicit Hartree states of lower energy will be displayed below for a Fermi gas with attractive interactions. This problem is of practical interest with regard to nuclear matter, and will be discussed within that context. It will be shown that there are large static density waves in nuclear matter, so that (to an adequate approximation) the total nucleon density has the following spatial variation:

$$\rho = \rho_0(1+\gamma\cos qx)(1+\gamma\cos qy)(1+\gamma\cos qz). \quad (1)$$

In another paper it will be shown that a low-energy Hartree-Fock state of a Fermi gas with repulsive interactions will have large spin density waves. It should not be necessary to emphasize that these new states have not been proved to be the ground state in the single-particle approximation, although they do have considerably lower energy than the "normal ground state." However, it seems clear that any attempt to improve upon the single-particle approximation, such as the many-body techniques developed in recent years, should begin with the lowest Hartree-Fock state available.

It is not our purpose here to present a general argument, applicable to all conceivable interactions, but rather to illustrate the physical structure of the new low-energy states within the framework of a simple model. We shall treat first a one-dimensional problem, and afterwards a three-dimensional nucleon gas. Consider a large number N of spinless Fermi particles in a one-dimensional box of length L. The kinetic energy operator will be the usual one. We shall assume the total interaction energy of the gas to be

$$V = -(\beta L/2N)\int_0^L \rho^2 dx, \quad (2)$$

where $\rho(x)$ is the total particle density and β a constant. If the difference between N and N-1 is neglected, the Hartree potential $U(x)$ experienced by any one of the particles is

$$U(x) = -(\beta L/N)\rho(x). \quad (3)$$

[The results to be derived are by no means limited to the particular interaction (2). For example, any power of ρ may be used in the integrand of (2). However, the requirement that the equilibrium density be finite restricts that power to values between 1 and 3.] The single-particle quantum states will be indexed according to the wave vectors allowed by periodic boundary conditions. The usual momentum states will have energy $2\omega_k E_F$, where

$$\omega_k = \tfrac{1}{2}(k/k_0)^2, \quad (4)$$

and E_F is the Fermi energy, $\hbar^2 k_0^2/2m$, with $k_0 = \pi N/L$, the wave number at the Fermi surface.

Consider now the following single-particle wave functions:

$$\phi_k = \left[g e^{ikx} + (E_k - \omega_k) e^{i(k-q)x} \right] \Big/ L^{1/2} \left[g^2 + (E_k - \omega_k)^2 \right]^{1/2}, \quad (5a)$$

for $0 \leq k \leq k_0$; and

$$\phi_k = \left[g e^{ikx} + (E_k - \omega_k) e^{i(k+q)x} \right] \Big/ L^{1/2} \left[g^2 + (E_k - \omega_k)^2 \right]^{1/2}, \quad (5b)$$

for $-k_0 \leq k \leq 0$. The energy E_k, appearing parametrically, is

$$E_k = \tfrac{1}{2}(\omega_k + \omega_{k-q}) - \left[\tfrac{1}{4}(\omega_k - \omega_{k-q})^2 + g^2\right]^{1/2}, \quad (6)$$

for positive k, and the corresponding expression with $q = 2k_0$ replaced by $-q$ for negative k. The foregoing wave functions and energies are the solutions of a single particle in a potential,

$$U = 4gE_F \cos 2k_0 x, \quad (7)$$

provided admixture of states with $|k| > 2k_0$ is ignored. (These are the wave functions and energies which arise in the weak-binding theory of energy bands. The energy gap at $\pm k_0$ is $4gE_F$.) The qualitative feature of the wave functions (5) is that their square magnitudes have an oscillating component out of phase with the potential (7), and give rise to a density wave which can in turn yield (7) as the Hartree potential of the attractive interactions. States corresponding to $|k| > k_0$ may be defined similarly, but should not be occupied, since their square magnitudes oscillate in phase with (7). This is the reason why the wave number of the density wave should equal the diameter, $2k_0$, of the Fermi surface. (Only those states below the energy gap should be filled.) It can easily be proved that this choice leads to the lowest total energy. But we must first show that a Fermi sea filled with the states (5) has lower energy than one filled with plane waves.

The kinetic energy T of a Fermi gas filled with the distorted states (5) can be (tediously) evaluated in a straightforward manner:

$$T = \tfrac{1}{3} N E_F [1 + 3F(g)], \quad (8)$$

where

$$F(g) = 1 - (1+g^2)^{1/2} + g^2 \ln[g^{-1} + (1+g^{-2})^{1/2}]. \quad (9)$$

The first term of (8) is the kinetic energy of a normal gas, whereas the second is the increase arising from the assumed distortion of the N-particle (simple product) wave function. The density of the gas can also be calculated directly, and is

$$\rho = (N/L)[1 - G(g)\cos qx], \quad (10)$$

where

$$G(g) = g \ln[g^{-1} + (1+g^{-2})^{1/2}]. \quad (11)$$

As surmised, the state has a static density wave. This result together with (2) determines the interaction energy:

$$V = -\tfrac{1}{2}\beta N(1 + \tfrac{1}{2}G^2). \quad (12)$$

If F and G are expanded in powers of g, it can be made obvious that V will always decrease faster than the increase of T near $g = 0$. Actually, the sum of (8) and (12) can be minimized without approximation. The optimum g is

$$g = 1/\sinh(4E_F/\beta). \quad (13)$$

The resulting change in total energy relative to the normal state is

$$\Delta W = -NE_F[\coth(4E_F/\beta) - 1]. \quad (14)$$

This result is negative definite. It is also of interest to note that if (10) and (13) are inserted into (3), one obtains a constant term plus the oscillating potential (7), the coefficient of the latter being in agreement with (13). Consequently, the N-particle wave function is a self-consistent solution in the Hartree sense (states with $|k| > 2k_0$ being ignored). A further observation is that within the field of variation employed here, the "normal ground state" is an energy maximum.

The generalization to a three-dimensional gas is almost trivial. In order to optimize the effect of density waves, the occupied (unperturbed) states should fill a cube in momentum space. Three density waves will result, according to (1), and there will be a large energy gap around the surface of the Fermi cube. (The wave number q of the waves must equal the edge length of the cube.) The single-particle wave functions are merely the products of three factors, each having the form (5). To a first approximation each of the density waves can be treated independently, and the analysis is identical to that derived above. However, employment of an interaction term such as (2) does not lead to an equilibrium density for nuclear matter.

The energetics of nuclear density waves for a model which exhibits nuclear saturation may be explored easily with the method of Karplus and Watson.[1] According to their treatment, the binding energy per nucleon of the normal state is (in Mev)

$$B = -(23/\eta^2) + (V_0/2\eta^3)[1-(3\alpha/10\eta^2)], \qquad (15)$$

where η is a radius parameter, $\eta^3 = \rho_0/\rho$, ρ_0 being the observed nuclear density. The parameters V_0 and α may be estimated from nucleon scattering experiments or, for our purposes, may be chosen to yield the observed binding energy (~ 16 Mev) and density ($\eta \sim 1$) of nuclear matter. The first term of (15) is the kinetic energy per nucleon, and the second is the potential. The negative term in square brackets represents the decrease in attractive interaction associated with the mean square relative momentum P of the nucleons:

$$P \sim \sum_{i,j} |\vec{k}_i - \vec{k}_j|^2. \qquad (16)$$

[The physical origin of this term is of course the penetration of the repulsive core by nucleons of high relative momenta. The potential terms of (15) and (2) are equivalent if β is allowed to have this dependence. Nuclear saturation is thereby guaranteed.] The appropriate modification of (15) to include the effects of three perpendicular density waves is

$$B' = -\frac{23}{\eta^2}(1+3F) + \frac{V_0}{2\eta^3}(1+\tfrac{1}{2}G^2)^3\left[1-\frac{3\alpha}{10\eta^2}(1+\tfrac{3}{2}F)\right], \qquad (17)$$

where $F(g)$ and $G(g)$ are as defined by (9) and (11). For excitations near the Fermi surface, the fractional increase in P is one half the fractional increase in kinetic energy. Equation (17) should first be minimized with respect to g. If only linear terms in F and G^2 are employed in finding the optimum g, the result is analytic:

$$g = 1/\sinh\theta, \qquad (18)$$

where

$$\theta = (2760\eta^3 + 9\alpha V_0)/V_0(30\eta^2 - 9\alpha). \qquad (19)$$

We have computed B and B' vs η for various sets of V_0 and α. The set, $V_0 \sim 120$ Mev, $\alpha \sim 1.4$, yields the observed density and binding energy for B'. The extra binding energy ($B'_{max} - B_{max}$) attributable to the density waves is 4 Mev per nucleon. The energy gap, $g\hbar^2q^2/2m$, at the Fermi surface is 30 Mev. The amplitude γ of each density wave is 0.6. These quantitative results are relatively insensitive to the values of V_0 and α. One may conclude that density waves in nuclear matter are of great importance.

The foregoing arguments have the force of the variational theorem. The density waves are so large that the variational solution should be improved appreciably by allowing states with wave vector components greater than q to be mixed into the single-particle wave functions. Also, the optimum value of g will be different from the crude estimate (18). Such refinements can only enhance the importance of density waves and increase their contribution to the binding energy. In writing (17), we have neglected the 8% increase in kinetic energy required to fill a cube rather than a sphere in momentum space. Most of this increase arises from filling the corners of the cube. It seems likely that the cube will be truncated by four smaller density waves parallel to the [111] type directions. The effect on the three major density waves should be small, but a major fraction of the 8% increase in kinetic energy can thereby be eliminated.

In the limit of weak attractive interactions, the Fermi surface will become a many-faced polyhedron, each pair of opposite faces arising from a density wave. All single-particle states near the Fermi surface will be essentially 100% perturbed compared to the normal state. It follows that the low-energy states proposed in this paper are not related to the normal state through a perturbation expansion in the interaction constant. The stability of static density waves arises from the fact that the greater binding caused by density fluctuations is achieved with less expenditure of kinetic energy, when compared with a mere uniform compression of the normal state. This observation points out the inadequacy of the Fermi-Thomas approximation for such considerations.

Glassgold, Heckrotte, and Watson[2] found that nuclear matter is unstable with respect to compressional collective modes. They assumed, of course, that the parentage of the true ground state is the normal free particle state. The interpretation of their result is now clear. Compressional modes relative to a highly excited "ground state" are unstable if they have sufficient admixture of low-energy states, similar to those considered here.

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REGENERATION AND MASS DIFFERENCE OF NEUTRAL K MESONS*

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A very significant feature of the Gell-Mann–Pais particle mixture theory[1,2] is the regeneration of the $K1$ from the $K2$ neutral meson. We examine the three possible types of regeneration and give the results of an experiment that exhibits the expected transformations as demanded by the theory. The experiment also allows an estimate of the difference between the masses of $K1$ and $K2$.

One of the three types of regeneration has been described previously[3]: A plate inserted into a parallel beam of $K2$ particles produces a parallel beam of $K1$ particles. This phenomenon, which we will henceforth call transmission-regeneration, is in striking contrast with other known processes whereby a particle transforms into another one: a parallel beam of charged pions obviously cannot produce a parallel beam of neutral pions by interacting with a plate.

Here we point out another process that typically follows from the theory, namely the regeneration by diffraction. Because the $\bar{K}^0$ and the K^0 waves are diffracted by a nucleus with different amplitudes, the diffracted wave contains $K1$ as well as $K2$ particles. Thus $K1$ mesons are regenerated by a nucleus with a typical diffraction angular distribution.

Regeneration of $K1$ can also occur by interaction of $K2$ with single nucleons. The angular distribution of this nucleon-regeneration is broad, not essentially different from that obtained in K-nucleon scattering, and therefore it is not a crucial consequence of the particle mixture theory.

All three of these components will emerge from a plate traversed by a parallel beam of $K2$'s. The angular distribution should permit one to recognize each component separately.

Case[4] and Good[5] have shown that the intensity of the transmitted component is a very sensitive function of the mean life τ_1 of the $K1$ and of the difference δm between the masses of $K1$ and $K2$. The mass difference appears in the final expression because of the phase difference it introduces between the $K1$ and the $K2$ waves, an effect which was first noted by Serber[6] in connection with K^0 production. Moreover, Good pointed out that the intensities of both the transmitted and "scattered" component (in the forward direction) are proportional to $|f_{21}{}^0|^2$, $f_{21}{}^0$ being the amplitude of the regenerated $K1$, at zero angle, in a $K2$-nucleus collision. Good's "scattered" component must be identified with the diffracted component described above. Thus the intensity ratio of the transmitted wave to diffracted wave is a function only of δm and τ_1. We derive here in a more concise way the expression for this ratio.

The computation of the expected transmitted and diffracted intensities can be greatly simplified by neglecting, from the start, the regeneration of $K2$ from $K1$. As the number of $K1$'s is always less than one thousandth of the number of $K2$'s, this approximation is very good. We consider then a plane wave of $K2$ particles, of wavelength λ, crossing our plate, which contains N nuclei per cubic centimeter. If each nucleus produces $K1$'s with a forward amplitude $f_{21}{}^0$, an infinitesimal thickness dx of the plate at depth x