ANALYSIS OF THE SCATTERING OF K^+ MESONS IN EMULSION*

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Using data on the scattering of K^+ mesons in emulsion,[1] together with data on the scattering of K^+ mesons by hydrogen[2] ($T=1$), we have investigated the K^+-nucleon interaction in the $T=0$ state. The method used requires only simple hand calculations and is believed to be quite accurate at high energies. The emulsion data of Zorn[1] are summarized in Table I. The K^+-p data may be summarized in terms of the S-wave effective range expansion as follows[3]:

$$k_{\mathrm{c.m.}}\cot\delta_1=-\frac{1}{a_1}+\tfrac{1}{2}k_{\mathrm{c.m.}}{}^2r_1, \tag{1}$$

with $a_1=0.34$ f, $r_1=0.50$ f, where a_1 and r_1 are accurate to about 10%.[4] For the energy region under consideration here (incoming kinetic energy less than 350 Mev), the K^+-p angular distribution indicates that the $T=1$ phase shifts for $L\geq 1$ may be neglected.

If we assume that the elastic scattering of K^+ on nuclei is given by a complex square well potential of depth $V+iW$, then in the WKB approximation one finds[5] for the total inelastic scattering cross section:

$$\sigma_{\mathrm{inel}}=\pi R^2-\frac{2\pi}{\lambda^2}[1-e^{-\lambda R}(1+\lambda R)]. \tag{2}$$

Here R is the radius of the nucleus with $R=1.2A^{1/3}\times10^{-13}$ cm and $\lambda=-4E_KW/(k_{\mathrm{lab}}\hbar^2)$. Equation (2) is valid when $(V+iW)/E_K\ll 1$ and $k_{\mathrm{lab}}R\gg 1$. Equation (2) is, of course, averaged over the representative elements of the emulsion. The total inelastic cross section then determines λ and hence W. The values of λ de-

Table I. The emulsion data of Zorn,[a] the quantities X and λ, the K^+-nucleon phase shift parameters, and the optical potentials derived from these data. T_K is the average laboratory kinetic energy of the K meson in each energy interval.

	$T_K=180$ Mev	$T_K=255$ Mev	$T_K=340$ Mev
σ_{inel} (mb)	357 ± 27	412 ± 32	406 ± 35
$P=\left(\dfrac{\sigma_{\mathrm{c.x.}}^{\mathrm{emulsion}}}{\sigma_{\mathrm{inel}}-\sigma_{\mathrm{c.x.}}^{\mathrm{emulsion}}}\right)$	0.24 ± 0.04	0.51 ± 0.08	0.70 ± 0.12
$\sigma_{\mathrm{c.x.}}^{\mathrm{emulsion}}$ (mb)	70 ± 14	139 ± 25	167 ± 31
λ (f^{-1})	0.36 ± 0.06	0.51 ± 0.10	0.49 ± 0.11
X (f^2)	0.038 ± 0.011	0.089 ± 0.023	0.106 ± 0.029
δ_1 (rad)	-0.52	-0.64	-0.76
δ_0 (rad)	-0.035 ± 0.087	0.14 ± 0.31	0.26 ± 0.12
μ_0	0.03 ± 0.03	0.16 ± 0.13	0.37 ± 0.09
V_S (Mev)	17.3	14.5	12.1
W (Mev)	-12.1 ± 2.0	-19.0 ± 3.7	-19.5 ± 4.4

[a]See reference 1.

termined from the experimental data are given in Table I. In the impulse approximation the optical model potential is[6]

$$V+iW=-2\pi A\bar{f}(0)\frac{k_{\mathrm{lab}}}{k_{\mathrm{c.m.}}}\frac{\hbar^2}{E_K}\frac{1}{(\frac{4}{3}\pi R^3)},\tag{3}$$

for a square well, where A is the mass number of the nucleus, k_{lab} is the wave number in the laboratory system, $k_{\mathrm{c.m.}}$ is the wave number in the K^+-nucleon center-of-mass system, E_K is the total laboratory energy of the K meson (including rest energy), and $\bar{f}(0)$ is the forward scattering amplitude in the K^+-nucleon center-of-mass system averaged over isotopic spin. [Note that for $R\sim A^{1/3}$ Eq. (3) is independent of A.] Thus

$$\bar{f}(0)=\alpha_1 f_1(0)+\alpha_0 f_0(0),\tag{4}$$

where the subscript denotes the isotopic spin state, and $\alpha_1=(A+Z)/2A$, $\alpha_0=(A-Z)/2A$. If only the S-wave phase shifts need be considered for $T=1$, and only the S- and P-wave phase shifts for $T=0$,

$$f_1(0)=\frac{1}{k_{\mathrm{c.m.}}}e^{i\delta_1}\sin\delta_1,$$

$$f_0(0)=\frac{1}{k_{\mathrm{c.m.}}}\{e^{i\delta_0}\sin\delta_0+e^{i\delta_{01}}\sin\delta_{01}+2e^{i\delta_{03}}\sin\delta_{03}\}.\tag{5}$$

Here δ_1 and δ_0 are the S-wave phase shifts in the $T=1$ and 0 states, respectively, and $\delta_{0,\,2J}$ are the $T=0$, $L=1$ phase shifts for states of total angular momentum J. In terms of these phase shifts λ is

$$\lambda=\frac{6A}{k_{\mathrm{c.m.}}{}^2R^3}\{\alpha_1\sin^2\delta_1+\alpha_0\sin^2\delta_0+3\alpha_0\mu_0\},\tag{6}$$

where

$$\mu_0=\tfrac{1}{3}\sin^2\delta_{01}+\tfrac{2}{3}\sin^2\delta_{03}.\tag{7}$$

If we assume that phase shifts for $L\geq 2$ may be neglected in the $T=0$ state, we may write the total cross section for the charge-exchange reaction, $K^+ + n \rightarrow K^0 + p$, as

$$\frac{\sigma_{\mathrm{c.x.}}}{4\pi}=X=\frac{1}{4k_{\mathrm{c.m.}}{}^2}[\sin^2(\delta_1-\delta_0)+3\mu_0].\tag{8}$$

A good approximation to the total charge-exchange cross section in emulsion at high energy is

$$\sigma_{\mathrm{c.x.}}{}^{\mathrm{emulsion}}=\bar{N}'\sigma_{\mathrm{c.x.}},\tag{9}$$

where $\bar{N}'$ is the average number of neutrons with which the K meson may interact per emulsion nucleus. If there were no attenuation of the incident beam, we would have $\bar{N}'=\bar{N}$, the average number of neutrons per emulsion nucleus. (In the usual way we treat the nuclear emulsion as a mixture of N:Br:Ag in the ratio 56:22:22, so that $\bar{N}=27.2$.) In general N' can be obtained by multiplying the density of neutrons at position $\vec{r}$ by the probability that the meson reaches $\vec{r}$ without being scattered inelastically, and integrating over all $\vec{r}$. For a uniform medium this gives

$$N'=\frac{3N}{2\pi\lambda R^3}\sigma_{\mathrm{inel}},\tag{10}$$

with σ_{inel} given by Eq. (2). The average value of N'/N is about $\frac{1}{2}$ at these energies. Values of X determined from the emulsion data are given in Table I.

Equations (9) and (10) may be used because of: (a) the incoherence of charge exchange scattering in nuclei; (b) the high incident kinetic energy; and (c) the very small probability of multiple charge exchange collisions. It turns out that the mean free path for a charge exchange collision in nuclear matter at the energies considered here is greater than ~10 f. There is also only a very small probability that an inelastic non-charge-exchange collision (involving a large energy loss) will be followed by a charge exchange collision because of the very rapid decrease of the charge exchange cross section with decreasing energy. Attenuation due to inelastic scattering may, of course, not be neglected. Comparison with the geometric cross section of the total inelastic cross section and the charge exchange cross section indicates readily why this is to be expected, since $0.1\leq\sigma_{\mathrm{c.x.}}{}^{\mathrm{emulsion}}/\pi R^2\leq 0.3$ whereas $0.6\leq\sigma_{\mathrm{inel}}/\pi R^2\leq 0.8$, in the energy region under consideration. For the geometric cross section we have used

$$\pi R^2=\pi(1.2\ \mathrm{f})^2[0.22(108)^{2/3}+0.22(80)^{2/3}+0.56(14)^{2/3}]=556\ \mathrm{mb}.\tag{11}$$

By inserting in Eqs. (6) and (8) the values of λ and X determined from experiment (Table I), we may solve for δ_0 in terms of δ_1. The result is

$$\sin(2\delta_0 - \delta_1) = \frac{k_{\mathrm{c.m.}}{}^2[(R^3/6A)\lambda - 4\alpha_0 X] - \alpha_1 \sin^2\delta_1}{\alpha_0 \sin\delta_1} \qquad (12)$$

This gives the values for δ_0 quoted in Table I. These values for δ_0 are consistent with the form

$$\delta_0 = -k_{\mathrm{c.m.}} a_0, \text{ with } a_0 = -0.080 \pm 0.068 \text{ f}, \qquad (13)$$

where a_0 has been obtained by a least-squares fit. We may then solve Eqs. (6) and (8) for μ_0. These values also appear in Table I. It should be noted that the errors in δ_0 and μ_0 are correlated. The independent experimental parameters are σ_{inel} and P (Table I), so that λ and X are correlated; these depend on both δ_0 and μ_0. The result is a strong correlation between the errors in δ_0 and μ_0 so that a small value of μ_0 will be associated with a large value of δ_0, and vice versa.

To get some idea of the magnitude of the P-wave phase shifts, we may write $\mu_0 = \sin^2\bar{\delta}_{0P}$, where $\bar{\delta}_{0P}$ is an "average" P-wave phase shift. (In the absence of spin-orbit splitting, $\bar{\delta}_{0P}$ is the P-wave phase shift.) If we fit $\bar{\delta}_{0P}$ to the form

$$k_{\mathrm{c.m.}}{}^3 \cot\bar{\delta}_{0P} = -(1/\bar{a}_{0P})^3, \qquad (14)$$

we find

$$|\bar{a}_{0P}| = 0.44 \pm 0.03 \text{ f}.$$

To determine the individual P-wave phase shifts, one can use the real part of the optical potential, which can be written as the sum of an S-wave and a P-wave part. Thus

$$V = V_S + V_P,$$

with

$$V_S = \Gamma[\alpha_1 \sin 2\delta_1 + \alpha_0 \sin 2\delta_0],$$

$$V_P = \Gamma\alpha_0[\sin 2\delta_{01} + 2\sin 2\delta_{03}],$$

$$\Gamma = -\frac{3A}{4R^3}\frac{\hbar^2}{E_K}\frac{k_L}{k_{\mathrm{c.m.}}{}^2}. \qquad (15)$$

The values of V_S obtained using Eqs. (1) and (13) are given in Table I. If the optical model potential were known, V_P could be found and the two P-wave phase shifts determined by solving Eqs. (7) and (15) simultaneously. From the present experimental values,[7]

$$V + iW = [20.7 \pm 2.3 - (7.9 \pm 1.2)i] \text{ Mev}$$

at $T_K = 150$ Mev and[8]

$$V + iW = [\ 22 \pm 6 - (19.3 \pm 2.0)i] \text{ Mev}$$

at $T_K = 260$ Mev, we would conclude that V_P is small (indicating a large spin-orbit splitting) and repulsive. However, there is great uncertainty in the value of the optical model potential determined from experiment, so that these conclusions are subject to considerable doubt. Further, there is some indication from a comparison of inelastic scattering at low energies with Monte Carlo calculations [see, for instance, M. Grilli et al., Nuovo cimento 10, 205 (1958)] that the K^+-nucleon cross section has a significant backward peaking, implying an attractive P-wave amplitude. Future improvements in the elastic scattering data, with consequent improvement in the optical model parameters, and improvements in the analysis of low-energy data, may help to resolve this disagreement.

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[1]Although much emulsion data exist, in this analysis we have used only those supplied to us by Dr. Gus T. Zorn (private communication). We are very grateful to Dr. and Mrs. Zorn for making their data available to us prior to publication.

[2]T. F. Kycia, L. T. Kerth, and R. G. Baender, Phys. Rev. (to be published).

[3]The angular distribution[2] at 225 Mev may be fit using a large P-wave phase shift in the $J=\frac{1}{2}$ state with a negligible S-wave phase shift. However, the total K^+-p cross section is nearly constant from low energies up to 275 Mev; this energy dependence is not compatible with a dominant P-wave interaction.

[4]These values are obtained if it is assumed that the data can be fit by an expression of the form of Eq. (1); the results of this Letter do not depend on this assumption, since δ_1 is used only in an energy region where it is known from experimental measurements.[2]

[5]S. Fernbach, R. Serber, and T. B. Taylor, Phys. Rev. 75, 1352 (1949).

[6]This equation and the conditions under which its use is justified have been written down in many places, see, for example, W. Riesenfeld and K. Watson, Phys. Rev. 102, 1157 (1956); H. A. Bethe, Ann. Phys. 3, 190 (1958); A. K. Kerman, H. McManus, and R. M. Thaler, Ann. Phys. 8, 551 (1959).

[7]G. Igo, D. G. Ravenhall, J. J. Tiemann, W. W. Chupp, G. Goldhaber, S. Goldhaber, J. E. Lannutti,

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[8]M. A. Melkanoff, O. R. Price, D. J. Prowse, D. H. Stork, and H. K. Ticho, Proceedings of the International Conference on the Nuclear Optical Model, Florida State University Studies, No. 32 (The Florida State University, Tallahassee, 1959).

FINITE SELF-ENERGY OF CLASSICAL POINT PARTICLES

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The infinite mass self-energy difficulties of quantum field theory already occur, as is well-known, in the corresponding classical theories. Although cutoffs may be introduced to effect renormalization in both the classical and quantum cases, such procedures are physically unsatisfactory. We wish to point out in this note that at least for the static (Coulomb-type) contribution, one obtains finite results for the classical self-energies if the gravitational contribution to the total energy is included. Furthermore, it will be shown rigorously (in the static case) that the natural cutoff furnished by general relativity implies that all the mass of a point charge arises from its total self-field and that a neutral particle has no mass.

It has previously been shown[1] that the energy of the gravitational field is given by[2]

$$E=\int(g_{ij,j}-g_{jj,i})\,dS_i, \tag{1}$$

where dS_i is a two-dimensional surface element at spatial infinity. When point particles or other fields (such as the electromagnetic field) are coupled to the gravitational field, Eq. (1) represents the total energy of the combined system.[3] We begin by considering the metric field arising from the coupling of a neutral static point particle. In isotropic coordinates $[g_{ij}=\chi^4(r)\delta_{ij}]$ the relevant field equation is

$$\begin{aligned}-2(R^0{}_0-\tfrac{1}{2}R)(^3g)^{1/2}&\equiv-8\chi\nabla^2\chi\\ &=-T^0{}_0(^3g)^{1/2}=m_0\delta^3(\vec{r}),\end{aligned} \tag{2}$$

where m_0 is the bare mass of the particle and $\delta^3(\vec{r})$ is invariantly defined in three-space as a scalar density,[4] i.e., $\int\delta^3(\vec{r})d^3r=1$. The solution of Eq. (2) which is asymptotically flat is seen to be

$$\chi(r)=1+m_0/[32\pi r\chi(0)]. \tag{3}$$

The parameter $m\equiv m_0/\chi(0)$ is given in terms of m_0 by

$$m=\lim_{\epsilon\to 0}2m_0[1+(1+m_0/8\pi\epsilon)^{1/2}]^{-1}. \tag{4}$$

In Eq. (4), ϵ is essentially the "radius" of the δ^3 function. This relation between m and m_0 is a consequence of explicitly considering the source term in Eq. (2). From Eq. (1) one sees that $E=m$. That this energy is to be correctly interpreted as the total mass of the particle follows from the fact that an isotropic time-symmetric metric possesses no dynamical gravitational modes.[5] Thus E represents the mass of a gravitationally clothed one-particle system (and no dynamical gravitational excitations). From Eq. (4) then, this total mass approaches zero as $(32\pi m_0\epsilon)^{1/2}$. The gravitational self-mass for a neutral particle therefore cancels the bare mass m_0. The physical origin of this result ($m=0$) is connected to the well-known fact that there is an upper limit, in general relativity, to the amount of energy that can reside in a given region. As the size of the particle (here ϵ) goes to zero its mechanical energy content must vanish. We may note that the (incorrect) weak-field result for the self-mass could be obtained from Eq. (4) by taking the limit m_0 small before letting ϵ tend to zero. Here one would find $m=m_0-\frac{1}{2}m_0{}^2/\epsilon+O(1/\epsilon^2)$, the standard linearly infinite Coulomb-type self-