In the interest of completeness we intend to improve the sensitivity of the nuclear resonance experiment at least one order of magnitude by obtaining a narrower line width. We shall study the nuclear resonance signal of nuclei consisting of closed shells plus or minus one nucleon. Also we shall search for an anisotropic mass which varies other than as $P_2(\cos\theta)$ by studying nuclear resonance signals of nuclei with spin greater than 3/2. Finally, since in the spirit of this investigation it is not necessarily excluded that mass anisotropy could be associated with a point in the universe other than the center of our galaxy, we shall study the nuclear resonance signal with respect to any arbitrary direction.

It is a pleasure to thank Professor H. Smith and Professor R. Wildt of the Yale Astronomy Department and Professor E. E. Salpeter for helpful and enlightening discussions.

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*This research has been supported in part by the National Aeronautics and Space Administration.

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SIGNIFICANCE OF ELECTROMAGNETIC POTENTIALS IN THE QUANTUM THEORY IN THE INTERPRETATION OF ELECTRON INTERFEROMETER FRINGE OBSERVATIONS

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The effects of electromagnetic potentials in quantum theory recently discussed by Aharonov and Bohm[1] and also by Furry and Ramsey,[2] and a decade ago by Ehrenberg and Siday,[3] have been the subject of some lively discussion[4] regarding experimental observations. Interference fringes have been observed[5,6] in electron interferometers in which electron beams are split into separate component beams which travel along spatially separated paths before being recombined to produce the interference. One of the effects in question would be an observable shift of interference fringes produced by a quite small amount of magnetic flux passing between the two component beams, but confined completely to regions not penetrated by the beams, for example by means of a long solenoid. The presence of such a magnetic field would thus be detected in spite of the fact that none of the detecting apparatus ever actually entered any of the region of that field.

The question has arisen whether the existence of such an effect can be ruled out already, in the light of the fact that electron interference fringes have actually been observed. Marton has informed us that stray 60-cycle magnetic fields were present in his electron interferometer such that the magnetic flux passing between the beams was quite large in comparison with the very small amount of flux predicted to be needed to change the relative phase of the two component beams by 2π. It was thought at first that if the effect predicted by quantum theory referred to above had indeed been present, the interference fringes would have been shifted back and forth sixty times each second during the exposure by such large amounts that they could not possibly have been seen.[7] After considerable discussion at Princeton, National Bureau of Standards, Swarthmore, New York, and elsewhere, it was finally realized, during the New York meetings of the American Physical Society, that the change in length of the two component electron beams, due to their being bent in actually passing through a magnetic field of this sort, surely must be taken into account. Furthermore, order of magnitude calculations at that time led to the conclusion that the effect of magnetic potential, rather than being absent, actually contributed significantly to the stability of the fringe pattern in spite of such beam bending.[8] A detailed treatment follows.

In the Marton-type interferometer schematically indicated in Fig. 1(a) the incoming beam at the top (solid line) is amplitude-split by a very thin crystal C_1, into a zero-order beam (vertical dashed line) continuing in the same direction, and a first order beam (slanting dotted line) which makes an angle θ with the original beam direction. After travel distances l_1 and m_1, respectively, these beams meet another very thin crystal C_2, where both beams are diffracted, partly into beams (slanting dashed line and vertical dotted line) which come together at a third very thin crystal C_3 after travelling additional distances l_2 and m_2, respectively. At C_3 the beams are again diffracted, each partly into the vertical direction of the outgoing beams at the bottom (dash-dotted vertical line). If the geometry and the crystals were ideal, these two component beams would superpose constructively with no angle between them. However, due to nonideal geometry and crystal warp the two beams fail to coincide exactly, differing not only in phase, but also in direction by the very small angle ϕ. In contrast to the first diffraction angle θ, this angle of deviation from parallelism ϕ can be made sufficiently small so that fringes resulting from interference of these two nearly parallel component beams can be observed.

In the presence of a weak uniform magnetic field H coming out from the plane of the paper in Fig. 1(b) the component beams will follow circle arcs, l_1', m_1', l_2', and m_2'. These circles all have the same radius,

$$R=\frac{cp}{|-e|H}=\frac{2\pi\hbar c}{e}\,\frac{1}{H\lambda},$$

where $(-e)$ is the electron charge, p the magnitude of momentum of the electrons, and λ the wavelength. The angle through which a beam is bent after travelling an arc length L is $\tau=L/R$. The angle between m_1 and the chord joining the ends of m_1' is

$$\tfrac{1}{2}\tau_1=\tfrac{1}{2}m_1'/R\doteq\tfrac{1}{2}m_1/R$$

to first order in the magnetic field strength $H=2\pi(\hbar c/e)/(\lambda R)$. As shown in more detail in the enlargement of Fig. 1(c), the line m_1' exceeds in length the line m_1 by the amount

$$\Delta m_1\equiv m_1'-m_1\doteq[m_1(\tfrac{1}{2}\tau_1)]\tan\theta\doteq\tfrac{1}{2}(m_1^2/R)\tan\theta.$$

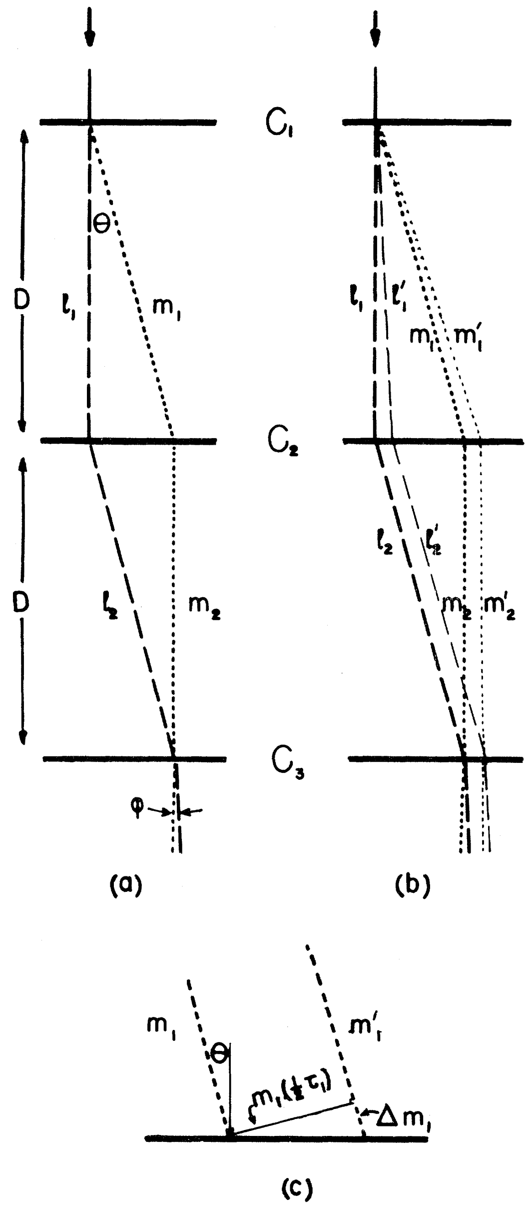

FIG. 1. Schematic diagram of component beam paths (dashed and dotted lines) in a Marton-type three-crystal electron interferometer: (a) in the absence of magnetic field; (b) with beams being bent by a uniform magnetic field coming out from the page. (c) Shown in enlarged detail is the region in (b) where original and displaced beam components m_1 and m_1' meet the second thin crystal C_2.

Since the angle of diffraction between l_1' and l_2' is the same as it was between l_1 and l_2, the angle between l_2 and the chord joining the ends of l_2' is

$$\frac{l_1'}{R}+\frac{1}{2}\frac{l_2'}{R} \doteq \frac{l_1+\frac{1}{2}l_2}{R}.$$

The excess of l_2' over l_2 is then

$$\Delta l_2 \doteq \frac{l_1+\frac{1}{2}l_2}{R} l_2 \tan\theta.$$

The differences in length between l_1' and l_1, and between m_2' and m_2 are both smaller by another order in H.

Substituting the distance D between the thin crystals for l_1 and $D/\cos\theta$ for m_1 and l_2, we have the change in the difference between path-lengths of the two component beams from C_1 through C_2 to C_3, resulting from bending of the beams by the uniform magnetic field H,

$$\Delta l_2 - \Delta m_1 \doteq \frac{1}{R}\left(D+\frac{1}{2}\frac{D}{\cos\theta}\right)\frac{D}{\cos\theta}\tan\theta - \frac{1}{2R}\frac{D^2}{\cos^2\theta}\tan\theta$$

$$\doteq \frac{e}{2\pi\hbar c} H\lambda D^2 \frac{\tan\theta}{\cos\theta}.$$

As pointed out by Aharonov and Bohm[1] and by Ehrenberg and Siday,[3] the change in phase difference between the two component beams from their differences in magnetic potential in the presence of magnetic flux passing between the two component beams is

$$\frac{(-e)}{\hbar c}\oint \vec{A}\cdot d\vec{x} = \frac{(-e)}{\hbar c} H \,(\text{area between beams})$$

$$\doteq \frac{(-e)}{\hbar c} HD^2 \tan\theta.$$

Taking into account the negative sign of the charge of the electron and combining this effect with the change in phase difference due to length change from beam bending, we have the net change in relative phase,

$$\delta \doteq \frac{e}{\hbar c} HD^2 \left(\frac{1}{\cos\theta} - 1\right) \tan\theta,$$

to first order in H. For small diffraction angle θ, this is approximated by

$$\delta \cong (e/\hbar c) HD^2\theta^3/2.$$

In terms of the distance of maximum separation d, and the difference in length ϵ between l_1 and m_1, this is

$$\delta \cong (e/\hbar c) H\epsilon d/2.$$

This is just $(e/\hbar c)$ times the magnetic flux through the very small triangle of base d and altitude ϵ in contradistinction to that through the very much larger parallelogram between the component beams. For practically perfect geometry, where the fringe width is very large, this means that for a uniform magnetic field the change in phase due to the integral of the magnetic potential is almost perfectly compensated by the change in phase due to the change in length of the component beams resulting from bending.

In fact, for proper adjustment of the geometry, the small angle of deviation from parallelism of outcoming component beams ϕ can be made such that this small relative phase change just compensates for the sidewise displacement (due to bending) of the place where the component beams join to form the interference fringes. In this case, when the magnetic field strength has reached the value H_C which results in bending the beams into arcs of radius R_C so that the point of recombination at C_3 is displaced by one whole fringe spacing, the net change in the relative phase of the beams δ, due to both beam length change and the magnetic vector potential integral, will reach the value 2π. Here the positions of the maxima and minima of the fringes are undisplaced to first order in H by the presence of the uniform magnetic field, which in this case only changes the relative intensity of illumination of the fringes.

Far from being absent, the effect of the change of relative phase from magnetic potentials is essential for the observation of fringes in a Marton-type electron interferometer in the presence of uniform magnetic fields which vary slowly with time.

We are grateful to E. P. Wigner, R. Brill, L. Marton, W. H. Furry, N. F. Ramsey, and R. N. Euwema for very stimulating discussions on this point.

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[8]After the presentation of the paper of Furry and Ramsey at the meeting in New York we were pleased to learn, in discussion with Professor Ramsey, that he had also reached the same conclusion independently in discussion with Dr. H. Mendlowitz. Dr. Marton kindly sent us comments by Dr. Mendlowitz on a preliminary draft of this Letter, indicating that in calculations carried out at the National Bureau of Standards Dr. Z. Bay and he also arrived (from a slightly different approach) at essentially the same conclusion.

MICROWAVE WHISTLER MODE PROPAGATION IN A DENSE LABORATORY PLASMA*

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If plane wave solutions which retain constant polarization are sought to the wave equation applicable to propagation at angular frequency ω in a homogeneous plasma in a magnetic field $\vec{H}_0$, one finds the indices of refraction for the characteristic ordinary and extraordinary wave given by Appleton's equation,[1,2]

$$n^2 = 1 - \frac{X}{1-\frac{1}{2}Y_T^{\,2}/(1-X) \pm [\frac{1}{4}Y_T^{\,4}/(1-X)^2 + Y_L^{\,2}]^{1/2}}, \quad (1)$$

where $X = \omega_p^{\,2}/\omega^2$; $Y = \omega_H/\omega$; $Y_L = Y\cos\theta$; $Y_T = Y\sin\theta$; $\omega_p = (4\pi Ne^2/\epsilon_0 m)^{1/2}$, the plasma frequency for electron density N; $\omega_H = \mu_0 H_0 e/m$, the electron gyro frequency; and θ is the angle between $\vec{H}_0$ and the wave normal. Here the collision frequency ν is neglected, and the ion plasma and ion gyro frequencies are assumed negligibly small. Propagation occurs for real positive n. The usual modes used for diagnostics are for $\omega > \omega_p$, requiring inconveniently large ω for even moderately dense plasmas. Another propagating mode occurs when $\omega < \omega_H$, for right-hand circular polarization, and $\cos\theta$ near unity. This mode has recently been used to explain extraterrestrial very low frequency propagation associated with lightning discharges[3] and very low frequency emissions in the exosphere of the earth[4] and has been called the "whistler" mode. It is characterized by an index of refraction greater than unity and confinement of the radiated energy to a narrow cone along $\vec{H}_0$.

One of us (R. M. G.) pointed out at the 1958 summer International School of Physics, Varenna, Italy, the usefulness of the whistler mode for plasma diagnostics particularly under conditions such that the refractive index becomes very large (of the order of 100 or more) and suggested that a large, slow thermonuclear machine such as the ZETA[5] at Harwell, England, would offer the best initial laboratory medium for observation of this mode. Through the efforts of Thonemann and Pease an invitation was extended by the Atomic Energy Research Establishment, Harwell, to the American authors to attempt such experiments on ZETA.

In ZETA, $\omega_p/2\pi \simeq 100$ kMc/sec, $\omega_H/2\pi \simeq 3$ to 40 kMc/sec, and $\nu \simeq 10^7$ sec^{-1}. If we choose $\omega/2\pi \simeq 3$ kMc/sec, then $X \simeq 1600$, $Y \simeq 1$ to 10, and ν is satisfactorily small. Under these conditions we can simplify the expression for n^2. For propagation along $\vec{H}_0$,

$$n^2 \simeq \frac{\omega_p^{\,2}/\omega^2}{(\omega_H/\omega)-1}, \quad (2)$$

provided $\nu \ll \omega_H - \omega$.

Measurement of n at two frequencies permits solution for ω_p and ω_H and hence N and H_0. As the propagation is sharply beamed, two receiving antennas may be used to define direction so that in fact $\vec{H}_0$ is measured. This technique could be used to map the magnetic field in a dense plasma. Both N and $\vec{H}_0$ are obtained as a function of time.