EFFECTS OF PION-PION INTERACTION IN ELECTROMAGNETIC PROCESSES

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Resonances in pion-pion scattering have been proposed by some authors.[1] Recently Frazer and Fulco,[2] following the approach of Chew and Mandelstam, and assuming a resonance in the $J=1$, $I=1$ state of two pions, derived an electromagnetic form factor for the pion which has a resonance-like behavior as a function of the invariant momentum transfer. They show that the position and width of the resonance can be adjusted to give a good fit to the isotopic vector part of the nucleon electromagnetic structure.

We have calculated the effect of this pion form factor on the photon propagator, and have obtained corrections of order e^2 to several purely electromagnetic processes. In spite of the large mass of the intermediate pion pair, these corrections turn out to be perhaps detectable in electron scattering experiments at center-of-mass energies smaller than those expected from machines now under construction.[3] Such experiments would have the obvious advantage of giving information on the pion form factor independently of the complications of nuclear structure.

Our method of calculation parallels that used by Källén in his discussion of charge renormalization. Our modifications consist of using intermediate bosons instead of fermions and of inserting the pion form factor.

We will outline here only the scheme of calculation, reserving the details and further results for publication elsewhere. We write first, following Källén, the renormalized photon propagator in the momentum representation:

$$D_{\mu\nu}^{F'}(p)=(p^2-i\epsilon)^{-1}\{\delta_{\mu\nu}+(\delta_{\mu\nu}-p_\mu p_\nu/p^2)\times[R(p^2)+iI(p^2)]\},\qquad(1)$$

with

$$R(p^2)=\overline{\Pi}(0)-\overline{\Pi}(p^2),\qquad(2a)$$

$$I(p^2)=-\pi\Pi(p^2).\qquad(2b)$$

The imaginary part $I(p^2)$, which involves the product of two current operators, is expressed as a sum over physical intermediate states through[4]

$$\Pi(p^2)=-(3p^2)^{-1}\sum_{p^{(z)}=p}\langle 0|j_\mu|z\rangle\langle z|j_\mu|0\rangle.\qquad(3)$$

Restricting the sum to intermediate states consisting of two charged physical pions, we multiply the usual first-order matrix element by the pion form factor[5] $F_\pi(p^2)$ and carry out the sum. We then construct the real part

$$R(p^2)=P\int_0^\infty\frac{p^2\Pi(-a)}{a(p^2+a)}da,\qquad(4)$$

where P designates the principal value.

As the form factor of FF has a rather complicated form for analytical work, we use a simple analytic fit:

$$|F_\pi(p^2)|^2=|F_\pi'(p^2)|^2,\quad 0>p^2>p_0^2$$
$$=1,\quad p_0^2>p^2\qquad(5)$$

where

$$|F_\pi'(p^2)|^2=A[(p^2+C)^2+B^2]^{-1},$$
$$|F_\pi'(0)|^2=|F_\pi'(p_0^2)|^2=1.\qquad(6)$$

The numerical results which we give below for electron scattering are based on the choice

$$A=66.25,\ B=1.5,\ C=8,\qquad(7)$$

which corresponds to the FF form factor having a peak at $p^2=-8$, using always units of the pion mass. (The other FF form factors give slightly smaller corrections.)

The final result is

$$\Pi(p^2)=(\alpha/6\pi)z^{3/2}\theta(-p^2-4)|F_\pi(p^2)|^2,\qquad(8)$$

$$R(p^2)=(\alpha/6\pi)\{[1-|F_\pi'(p^2)|^2]z_0^{3/2}f(z/z_0)-f(z)+p^2|F_\pi'(p^2)|^2[-\gamma+\delta(C+p^2)/B]\},\qquad(9)$$

where

$$\alpha \approx 1/137,$$
$$z = 1 + 4/p^2, \quad z_0 = 1 + 4/p_0^2,$$
$$\theta(x > 0) = 1, \quad \theta(x < 0) = 0,$$
$$f(z) = \tfrac{2}{3} + 2z + z^{3/2}\ln\left(\frac{z^{1/2}-1}{z^{1/2}+1}\right). \qquad (10)$$

For real positive z, the absolute value of the argument of the logarithm must be taken in (10). For negative z, one must replace $z^{1/2}\ln[(z^{1/2}-1)/(z^{1/2}+1)]$ by $-2y\,\mathrm{arc\,cot}\,y$, with $y = (-z)^{1/2}$. For complex values of z, $f(z)$ is defined in a plane cut in the negative real axis. The constants γ and δ are defined by[6]

$$\gamma + i\delta = \tfrac{1}{4}(1 - z')z_0^{3/2} f(z'/z_0) \qquad (11)$$

with

$$z' = 1 - 4/(C + iB).$$

We have applied this result to evaluate the correction to Møller and Bhabha scattering of electrons due to pion vacuum polarization. The fractional correction $k(E, \cos\theta)$, where E is the center-of-mass energy of each electron and θ the c.m. angle of scattering, takes the form in the extreme relativistic case:

$$\text{Møller:} \quad k = (1+X)R(p_1^2) + (1-X)R(p_2^2), \qquad (12)$$

$$\text{Bhabha:} \quad k = (1+Y)R(p_3^2) + (1-Y)R(p_4^2), \qquad (13)$$

with

$$X = 4\cos\theta(3 + \cos^2\theta)^{-1}, \qquad (14)$$

$$Y = (1+\cos\theta)(3-\cos\theta)(3+\cos^2\theta)^{-1}, \qquad (15)$$

and with

$$p_1^2 = p_3^2 = 2E^2(1-\cos\theta),$$
$$p_2^2 = 2E^2(1+\cos\theta),$$
$$p_4^2 = -4E^2. \qquad (16)$$

In Figs. 1 and 2 we have plotted some examples of these results.

The largest corrections are obtained for large-angle Bhabha scattering, which is dominated by the timelike momentum transfer p_4^2. For this reason the large-angle plot in Fig. 2 reproduces essentially the correction to the photon propagator for timelike p^2. From an experimental curve showing this behavior could be deduced unambiguously the position and width of the resonant part of the pion form factor, free of the uncertain interpretations of nuclear structure.

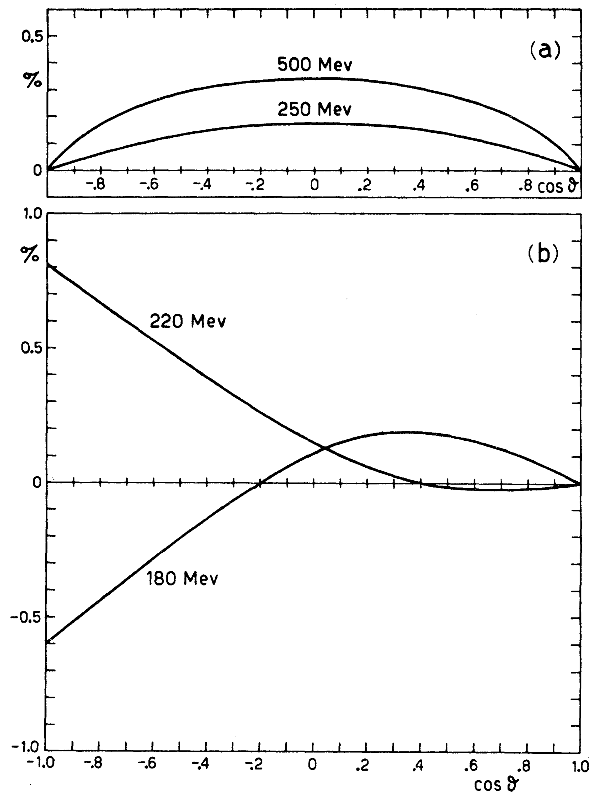

FIG. 1. Percentage correction as a function of c.m. angle for fixed c.m. energy of each electron: (a) Møller scattering and (b) Bhabha scattering.

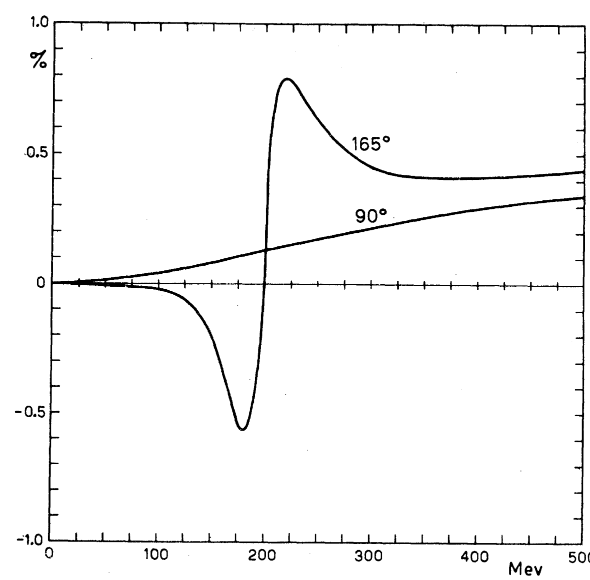

FIG. 2. Percentage correction to Bhabha scattering as a function of c.m. energy of each electron for fixed c.m. angle.

Of course annihilation processes leading to muon pairs or to two or more pions will become of interest at these energies. For the muon case we get a small correction similar to that for large-angle Bhabha scattering. For the very important case of annihilation into pion pairs one is studying directly the pion form factor. For example, at 90° in the c.m. system one gets

$$\frac{d\sigma(\pi\pi)}{d\sigma(\text{Bhabha})}=\frac{1}{18}\left(\frac{\vec{p}^2}{E^2}\right)^{3/2}|F_\pi(-4E^2)|^2, \qquad (17)$$

where $\vec{p}$ and E are the momentum and energy of each pion. This is certainly, therefore, the best means of investigating the pion form factor. If a resonant pion form factor is found in this way, then the small effects we have calculated become of relevance in the interpretation of scattering experiments in terms of a possible failure of quantum electrodynamics at small distances.[7]

However, while the positron-electron experiment appear to be the most promising for elucidating the pion form factor, electron-electron experiments will probably be done first and the above corrections for Møller scattering may be of interest. We are investigating other electromagnetic processes in which pion-pion interaction plays a role.

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[5]G. F. Chew, R. Karplus, S. Gasiorowicz, and F. Zachariasen, Phys. Rev. 110, 265 (1958).

[6]For the A, B, and C given in Eq. (7), we get $\gamma=0.047$, $\delta=0.106$.

[7]It must be emphasized that the usual electromagnetic radiative corrections, not considered here, are large and must be included.

INVARIANT COMMUTATORS FOR THE QUANTIZED GRAVITATIONAL FIELD*

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The problem of developing a formalism within which a union of the general theory of relativity with the quantum theory might successfully be brought about has been attacked by many authors.[1] The difficulties encountered in this work have seemed to be twofold: (1) the nonlinearity of the general theory, and (2) its coordinate invariance, which leads to constraints on the Cauchy data for the dynamical equations. The real difficulty may lie elsewhere, however. Approaches to the problem have almost without exception been made via a Hamiltonian or quasi-Hamiltonian canonical formalism, with an attendant loss of manifest covariance. It is the purpose of this Letter to raise the question of the suitability of canonical procedures and to suggest, by obtaining, without their aid, an explicit covariant expression for the commutators of the theory, that it may be possible to avoid them entirely.

We begin by pointing out that, apart from problems of factor ordering, the nonlinearity of the dynamical equations offers no complications for the commutators, contrary to a widespread impression. As Peierls[2] has shown, the commutator of two dynamical variables is determined by the variation in one due to an infinitesimal change in the action proportional to the other. But infinitesimal variations are propagated by means of linear equations, and standard techniques are available for handling the associated propagation functions.

Secondly, we note that in the general theory of relativity, well-defined commutators can be obtained only between absolute invariants. The infinitesimal coordinate transformation law,

$$\delta g_{\mu\nu}=\xi_{\mu\cdot\nu}+\xi_{\nu\cdot\mu}, \quad (\xi_\mu \text{ infinitesimal}) \qquad (1)$$