PION FORM FACTORS FROM POSSIBLE HIGH-ENERGY ELECTRON-POSITRON EXPERIMENTS

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Theoretical work on nucleon structure[1,2] has revealed the important role of the photon-(n-pion) vertices in the theory of the nucleon form factors. In particular the $\gamma - 2\pi$ vertex is thought to be the most important one contributing to the isotopic vector part of the structure, and the $\gamma - 3\pi$ vertex the most important one for the isotopic scalar part. Recent technical developments showing the feasibility of colliding beam experiments[3] make it appealing to think of possible direct measurements of the photon-pion vertices through processes of the sort

$$e^+ + e^- \to n \text{ pions}. \tag{1}$$

We shall here discuss the reactions (1) in their lowest electromagnetic approximation. Examination of the higher order terms in the electromagnetic coupling constant may become necessary when detailed experiments are carried out.

Let us consider the reaction (1) in the center-of-mass frame. In the lowest electromagnetic approximation only the $^3S_1$ and $^3D_1$ states for the initial positron-electron state will contribute to (1) and the final n-pion state must have parity minus, charge-conjugation quantum number minus, total angular momentum one, and total isotopic spin one if n is even, zero if n is odd. In particular the process is forbidden in this order if the final pions are all neutrals. The S-matrix element for (1) is given by

$$S_{fi} = \delta(p^{(1)} + p^{(2)} + \cdots + p^{(n)} - e^{(+)} - e^{(-)}) \times \frac{2\pi e}{K^2}(\bar{v}(e^+)\gamma_\nu u(e^-))\langle p^{(1)}, p^{(2)}, \cdots p^{(n)} | j_\nu(0) | 0\rangle, \tag{2}$$

where $p^{(1)}, p^{(2)}, \cdots p^{(n)}$ are the final pion momentum four-vectors, $e^{(+)}$ and $e^{(-)}$ the positron and electron momenta, respectively, $K = e^{(+)} + e^{(-)}$, $\bar{v}(e^+)$ and $u(e^-)$ are the Dirac spinors, and the matrix element of the electric current operator $j_\nu(x)$ is taken between the vacuum state and the final state of n outgoing pions.

We define

$$J_\nu(p^{(1)}, p^{(2)}, \cdots p^{(n)}) = (2\pi)^{3n/2}\langle p^{(1)}, p^{(2)}, \cdots p^{(n)} | j_\nu(0) | 0\rangle. \tag{3}$$

It follows from gauge invariance that, in the center-of-mass system for (1), J_ν has only the space component $\vec{J}$. From parity conservation $\vec{J}$ is a polar vector formed from the n vectors $\vec{p}^{(1)}, \vec{p}^{(2)}, \cdots \vec{p}^{(n)}$ if n is even, an axial vector formed from these vectors if n is odd. Inserting (3) into (2) one finds for the total cross section

$$\sigma = \frac{\alpha}{32E^4(2\pi)^{3n-5}}\int d\vec{p}^{(1)}d\vec{p}^{(2)}\cdots d\vec{p}^{(n)}\delta(\omega^{(1)} + \omega^{(2)} + \cdots + \omega^{(n)} - 2E)\delta(\vec{p}^{(1)} + \vec{p}^{(2)} + \cdots \vec{p}^{(n)})|\vec{J}|^2\sin^2\theta, \tag{4}$$

where α is the fine structure constant (another factor α is contained in $|\vec{J}|^2$), E is the electron (or positron) energy, $\omega^{(i)}$ is the energy of the ith pion, and θ is the angle between the vector $\vec{J}$ and the line of collision. The dependence on θ, as $\sin^2\theta$, is therefore a direct consequence of gauge invariance independent of any knowledge of the γ-pion vertices. In the case of two final pions, with only one vector available, $\vec{J}$ must be proportional to $\vec{p}^{(1)} = -\vec{p}^{(2)}$ and the angular distribution is thus completely determined of the form $\sin^2\theta$. In the case of three final pions, $\vec{J}$ must be proportional to the only available axial vector $\vec{p}^{(1)} \times \vec{p}^{(2)} = -\vec{p}^{(1)} \times \vec{p}^{(3)}$, etc, and therefore the distribution of the angle θ between the normal to the plane in which the three pions are produced and the line of collision is uniquely given by $\sin^2\theta$.

The final state of two pions (necessarily one positive, one negative) produced according to (1) is necessarily a p state of relative orbital angular momentum. The matrix element (3) for two pions is expressed in terms of the pion factor $M(K^2)$ by[2]

$$J_\nu(p^{(1)}, p^{(2)}) = e(4\omega^{(1)}\omega^{(2)})^{-1/2} M(K^2)(p^{(1)} - p^{(2)})_\nu. \quad (5)$$

The differential cross section, always in the center-of-mass frame, is given by

$$d\sigma = \frac{2\pi\alpha^2}{32E^5} |M(-4E^2)|^2 p^3 \sin^2\theta d(\cos\theta), \quad (6)$$

where $p = |\vec{p}^{(1)}| = |\vec{p}^{(2)}|$. Frazer and Fulco[4] propose a resonant form of $M(K^2)$ as a simplest explanation of the isotopic vector part of the nucleon structure. According to (6) one finds for $E = 230$ Mev, for which $M(-4E^2)$ comes close to its maximum, a total cross section for $e^+ + e^- \to \pi^+ + \pi^-$ of 4.6×10^{-31} cm^2, which is $\cong 17$ times bigger than the value one would get from perturbation theory ($M = 1$). The cross section then falls down very rapidly at higher energies.

The final state of three pions (necessarily one positive, one negative, one neutral) produced according to (1) is a superposition of states with $l = L = 1$, $l = L = 3$, $l = L = 5$, etc., where l is the relative $\pi^+\pi^-$ angular momentum and L the angular momentum of the π^0 relative to $\pi^+\pi^-$. The matrix element (3) for three pions can be expressed in terms of a form factor H^* depending on three independent scalars which can be chosen as E, ω_+ (the energy of the positive pion), and ω_- (the energy of the negative pion),[2]

$$J_\nu(\vec{p}^{(+)}, \vec{p}^{(-)}, \vec{p}^{(0)}) = -i(8\omega_+\omega_-\omega_0)^{-1/2} H^* \epsilon^{\nu\rho\sigma\tau} p_\rho^{(+)} p_\sigma^{(-)} p_\tau^{(0)}. \quad (7)$$

The vector $\vec{J}$ is then given by

$$\vec{J}(\vec{p}^{(+)}, \vec{p}^{(-)}, \vec{p}^{(0)}) = 2(8\omega_+\omega_-\omega_0)^{-1/2} H^* E(\vec{p}^{(+)} \times \vec{p}^{(-)}). \quad (8)$$

With our choice of the independent kinematical parameters the differential cross section can be written as

$$\frac{d^2\sigma}{d\omega_+ d\omega_-} = \frac{\alpha}{(2\pi)^2} \frac{|H|^2}{64E^2} (\vec{p}^{(+)} \times \vec{p}^{(-)})^2 \sin^2\theta d(\cos\theta). \quad (9)$$

Measurement of this cross section will thus inform us directly about $|H|^2$ which plays a relevant role in the nucleon structure problem. At present there is no knowledge at all about this form factor. Even a rough knowledge of its average magnitude would be very important for our understanding of the nucleon structure. Bosco and De Alfaro have assumed a constant approximation for H, $H = (\lambda e)/\mu^3$, and they claim that double pion photoproduction puts an upper limit $\cong 6$ for λ.[5] At a given E the maximum of $d\sigma$ is obtained for those configurations of the three final pions for which the area of the triangle formed with their momenta is maximum. For such configurations and for $E \cong 500$ Mev, we find, using a constant H and the above value for λ, $d^2\sigma/d\omega_+ d\omega_- \cong 1.3 \times 10^{-35}$ cm^2 Mev^{-2}.

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