protons/hr, the high-energy neutrino flux is ~ 5000 neutrinos sec^{-1} cm^{-2}. With a cross section $\sigma \sim 10^{-38}$ cm^2, the number of counts is $N \sim 1$ per hour in 10 000 kg of detector. The estimate here given is for neutrinos from high-energy pions. There is, as a matter of fact, a much greater flux of lower energy neutrinos from lower energy pions. However, because the neutrino cross section decreases rapidly with decreasing energy, the rate is not likely to be improved by more than a factor of two.

This estimate places the experiment outside the capabilities of existing machines by one or two orders of magnitude. Optimistic estimates for accelerators which are currently under construction, namely the 3-Bev machine at Princeton and the 10-Bev machine at Argonne, indicate that the experiments may be barely feasible in the near future. However, for really quantitative experiments it will be necessary to use high-intensity machines such as the FFAG machine proposed by MURA or the 10-Bev linear proton accelerator discussed by Blewett at Brookhaven. In these machines, one hopes to attain a beam intensity of the order of 10^{15} protons/sec at an energy of about 10 Bev.

The higher energy of the primary beam of protons makes the experiment easier because of the increased multiplicity of pions, the more concentrated forward distribution of the pions, and the increased cross section for neutrino reactions. Balanced against these is the fact that the percentage of higher energy pions that decay in 10 meters is smaller. The net result is likely to give a counting rate per primary proton that probably increases more than linearly with the primary proton energy.

Thus, a high-intensity 10-Bev proton machine with a beam intensity $\sim 10^{15}$ protons/sec may give a counting rate of more than 10^3 per hour, using the experimental setup described above. If that proves to be the case, it is perhaps desirable to have magnetic lenses to analyze and focus the pions so as to obtain more monoenergetic neutrino beams.

I would like to express my gratitude to Dr. T. D. Lee and Dr. C. N. Yang for many stimulating discussions which led to the above proposal.

Note added in proof. The author's attention has been called to a somewhat related paper which has just appeared: B. Pontecorvo, J. Exptl. Theoret. Phys. (U.S.S.R.) 37, 1751 (1959).

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THEORETICAL DISCUSSIONS ON POSSIBLE HIGH-ENERGY NEUTRINO EXPERIMENTS*

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The weak interaction so far most extensively studied is β decay, in which the momentum transfer is of the order of a few Mev. In μ decay and μ capture, momentum transfers of the order of 100 Mev are involved. In the theory of these processes, because of the limited region of momentum transfer studied, the phenomena can be described by a few parameters usually called coupling constants. For larger momentum transfers it is obvious that the weak interactions cannot continue to be described by these constants, because of the clothed structure of the nucleons due to the strong interaction, and also because of the reasonable expectation that the weak interactions, even without the interference of the strong interactions, may not be of the simple four-spinor product form in Fermi's theory.

In the preceding Letter,[1] Schwartz points out that the neutrinos from the decay of high-energy mesons can be used to study weak interactions. We have investigated the theoretical implication of such possible experiments. Efforts are made to separate and dissociate the inferences that can be drawn from different assumptions concerning the weak interactions. In this Letter we report briefly on this work.

1. The identity of the neutrinos. In the processes

$$\pi^+ \to \mu^+ + \nu_1, \qquad (\pi \text{ decay}) \qquad (1)$$

$$\mu^- + p \to n + \nu_2, \qquad (\mu \text{ capture}) \qquad (2)$$

$$Z \to (Z-1) + e^+ + \nu_3, \qquad (\beta^+ \text{ decay}) \qquad (3)$$

it is easy to see that ν_1 and ν_2 are the same particle. Experimentally it is known that ν_1 and ν_3 both have helicity -1. It is simplest to assume that ν_1 and ν_3 are also the same particle. However, a test of this assumption is clearly desirable. To obtain such a test it is necessary to do some kind of capture experiment on the neutrinos or antineutrinos. For example, if ν_1 and ν_3 are different particles, then the reaction

$$n+\nu_1\rightarrow p+e^- \tag{4}$$

does not occur.

2. Conservation of leptons. The conservation of leptons can be studied with neutrino capture experiments. For example, if both

$$\nu_1+p\rightarrow\Lambda^0+e^+ \tag{5}$$

and

$$\bar{\nu}_1+p\rightarrow\Lambda^0+e^+ \tag{6}$$

occur, there would be a violation of lepton conservation. While it is possible to study lepton conservation by helicity measurements, neutrino capture experiments seem to be the most direct and clean cut for such purposes.

In the rest of this Letter we shall assume that $\nu_1=\nu_2=\nu_3\equiv\nu$ and that the conservation of leptons holds.

3. Possible existence of a neutral lepton current. By using high-energy neutrinos it becomes possible to study whether reactions such as

$$\nu+p\rightarrow\nu+p \quad\text{and}\quad \nu+n\rightarrow\nu+n$$

exist or not, and if they exist, whether there is any similarity between these "neutral lepton currents" and the electromagnetic field. [See also the discussion in Sec. 8.]

4. Point structure of the lepton current. In the present theory of β decay, μ capture, etc., one assumes that all the weak reactions that contain both leptons and heavy particles can be represented by an effective Lagrangian of the type

$$-\mathcal{L}_{\text{eff}}=\sum_{\lambda=1}^{4}[J_\lambda(x)j_\lambda(x)+J_\lambda'(x)j_\lambda'(x)], \tag{7}$$

where

$$j_\lambda(x)=-i[\psi_l^\dagger\gamma_4\gamma_\lambda(1+\gamma_5)\psi_\nu], \tag{8}$$

$$j_\lambda'(x)=-i[\psi_\nu^\dagger\gamma_4\gamma_\lambda(1+\gamma_5)\psi_l], \tag{9}$$

l stands for either e^- or μ^-, ψ_ν and ψ_l are the field operators for ν and l, and $J_\lambda(x)$ and $J_\lambda'(x)$ are operators that act on heavy particles (including the pions, K mesons) only. Because of the Hermiticity of $\mathcal{L}_{\text{eff}}$, we have

$$J_\lambda'=\eta_\lambda J_\lambda^\dagger, \tag{10}$$

where

$$\eta_\lambda=+1 \quad\text{for}\quad \lambda=1,2,3,$$

$$\eta_\lambda=-1 \quad\text{for}\quad \lambda=4. \tag{11}$$

The nature of J_λ and J_λ' is known so far only in the nonrelativistic region. In the low-energy limit, the matrix elements of these heavy-particle current operators in the case of β decay are of the form

$$\langle p|J_\lambda|n\rangle=(i/\sqrt{2})u_p^\dagger\gamma_4\gamma_\lambda(G_V-G_A\gamma_5)u_n, \tag{12}$$

$$\langle n|J_\lambda'|p\rangle=(i/\sqrt{2})u_n^\dagger\gamma_4\gamma_\lambda(G_V^*-G_A^*\gamma_5)u_p, \tag{13}$$

where u_n and u_p are the spinor solutions of the free Dirac equations with the same 4-momenta as the physical neutron and proton; G_V and G_A are the Fermi and Gamow-Teller coupling constants. Due to the presence of strong interactions it is expected that in the high-energy region (12) and (13) do not hold. But one expects (7), which represents a "point interaction" for the leptons, to have a wider range of applicability.

The mere assumption that in the effective Lagrangian (7) the lepton current acts only at a single space-time point introduces rather strong restrictions on the forms of the cross sections for all neutrino and antineutrino reactions. For example, in either

$$\nu+n\rightarrow p+l^-+\text{pions} \tag{14}$$

or

$$\bar{\nu}+p\rightarrow n+l^++\text{pions}, \tag{15}$$

suppose one measures in the laboratory system the incoming momentum $\vec{k}_\nu$ and the outgoing lepton momentum $\vec{k}_l$ and does not measure the other kinematic quantities describing the reaction. The experimental cross section is then in a general case a function of the three real variables k_ν, k_l, and θ (= the angle between $\vec{k}_l$ and $\vec{k}_\nu$). Independently of the form of J_λ, assumption (7) restricts this function to a sum of three structure functions each of which has an unknown dependence on only two real variables: $E=k_\nu-k_l$ and $P=|\vec{k}_\nu-\vec{k}_l|$, which represent the energy transfer and the magnitude of the momentum transfer between the leptons and the strongly interacting particles. More explicitly, assumption (7) implies[2] that the cross section for (14) [also for (15)] is of the form

$$d\sigma = dk_l d(\cos\theta)(4\pi k_\nu)^{-1} k_l[(k_l+k_\nu)^2 - P^2] \times [xA_+ + x^{-1}A_- + B], \quad (16)$$

where $x = (k_l + k_\nu - P)(k_l + k_\nu + P)^{-1}$ and A_+, A_-, and B are functions of E and P only.

To test the validity of (16) it is not necessary to perform a detailed experiment for specific values of E and P. One could perform a capture experiment with a neutrino beam with a known spectrum $I(k_\nu)dk_\nu$ and measure for each event the values of x, P, and E. If $N(x,P,E)dxdPdE$ is the number of events, then (16) implies that

$$I^{-1}k_\nu^{\,2}N(x,P,E) = [A_1 + A_2x + A_3x^2](1-x)^{-4}, \quad (17)$$

where A_i $(i = 1, 2, 3)$ are functions of P and E. Integrating (17) over P and E, one obtains

$$\sum I^{-1}k_\nu^{\,2} = (a_1 + a_2x + a_3x^2)(1-x)^{-4}, \quad (18)$$

where a_1, a_2, a_3 are numerical constants, and the sum extends over all events with fixed x. To test the validity of (18), less than a thousand events could be enough provided that they do not cluster around one value of x.

5. Universality of weak interactions involving $e^\pm$ and $\mu^\pm$. Neglecting the mass of the μ meson, the assumption of the universality of the weak interactions implies equal differential cross sections for μ^+ and e^+ production and for μ^- and e^- production. If the mass of the μ meson is not neglected, comparison should be made between μ and e production processes in which the energy transfer E and the magnitude P of the momentum transfer from the leptons to the strongly interacting particles are fixed, provided the point structure of the lepton current discussed in Sec. 4 is valid.

6. S-symmetry. In (16) the structure functions determined by using ν are related to the appropriate matrix elements of J_λ in Eq. (7) while those determined by using $\bar{\nu}$ are related to that of J_λ'. Thus, unless J_λ and J_λ' obey some further symmetry property the reaction rates of, e.g., (14) and (15) in general are not related to each other in any simple way. A symmetry which will be called S-symmetry is of a type so as to link J and J'.

To explain the meaning of S-symmetry we observe that by using (12) and (13) at the low-energy limit it is readily verified that J_λ and J_λ' satisfy the following relation:

$$J_\lambda' = SJ_\lambda S^{-1}, \quad (19)$$

where S is the product of a 180° rotation along the y axis $[\exp(i\pi I_y)]$ in the isotopic spin space multiplied by the time-reversal operator T, i.e.,

$$S = [\exp(i\pi I_y)]T. \quad (20)$$

Condition (20) will be defined as the condition for S-symmetry.[3] If it is satisfied then there should exist identities among the matrix elements of J and J' and consequently relations between reaction rates caused by ν and $\bar{\nu}$. For example, if the mass of the lepton is neglected, the structure functions [defined in (16)] for ν and $\bar{\nu}$ processes are related to each other by

$$(A_+)_\nu = (A_-)_{\bar{\nu}}, \quad (A_-)_\nu = (A_+)_{\bar{\nu}}, \quad (B)_\nu = (B)_{\bar{\nu}}. \quad (21)$$

Experimentally it is not known at present whether S-symmetry is satisfied at energies of the order of 100 Mev and up. Theoretically it has been customary[4] to assume bare particle universal Fermi interactions for all Fermions. If such an assumption is made, S-symmetry follows.

7. Conserved vector current and proportionality with the electromagnetic current. The heavy-particle current J_λ can be written as a sum,

$$J_\lambda = V_\lambda + A_\lambda, \quad (22)$$

where V_λ and A_λ are, respectively, its vector part and axial-vector part. Recently, Feynman and Gell-Mann[5] proposed that the vector part V_λ satisfies the conservation law

$$\partial V_\lambda / \partial x_\lambda = 0. \quad (23)$$

Furthermore it is proposed that V_λ is equal to the corresponding isotopic vector part of the electromagnetic current times a constant (hereafter called the proportional vector current hypothesis). A sensitive test of the proposal can be given by studying reactions such as

$$\nu + n \to e^- + p, \quad (24)$$

$$\bar{\nu} + p \to e^+ + n, \quad (25)$$

and comparing them with existing data on electron scattering by nucleons[6] at the same momentum transfer to the nucleons.

Making the proportional vector current hypothesis, the electron scattering experiments[6] show that V_λ is proportional to

$$F(q^2) = [1 + \frac{1}{12} q^2 \alpha^2]^{-2}, \quad \alpha = 0.8 \times 10^{-13} \text{ cm},$$

where q^2 is the invariant four-momentum transfer squared. We determine the constant of proportionality by the condition $G_V = 10^{-5}/M^2$. For orientation purposes we assumed that A_λ is given by

$$\langle p | A_\lambda | n \rangle = 1.2\ G_V F(i/\sqrt{2}) u_p{}^\dagger \gamma_4 \gamma_\lambda \gamma_5 u_n,$$

and calculated the cross sections for (24) and (25). The results are exhibited in Fig. 1.

8. Possible existence of a weakly coupled Boson $W^\pm$. The question whether the weak interactions are "transmitted" by a Boson field was already discussed in Yukawa's original work on the meson. If such a Boson field $W^\pm$ exists, it must have spin 1, and one can also conclude that its mass m_W is $\gtrsim m_K$. The nonlocality of the weak interactions implied by the finite mass of the transmitting field W has been discussed before.[7] For μ decay, the change in the Michel parameter ρ is given by

$$\rho - 0.75 \cong \tfrac{1}{3}(m_\mu / m_W)^2.$$

A value of $m_W > m_K$ is thus consistent with existing experiments.

The coupling of the $W^\pm$ to the leptons is characterized by a coupling constant,

$$g^2/4\pi = (\pi\sqrt{2})^{-1} G_V m_W{}^2 < 6.4 \times 10^{-7}.$$

The decay rate of $W \to e + \nu$ is found to be

$$\lambda_{W \to e + \nu} = G_V m_W{}^3 (6\pi\sqrt{2})^{-1}, \qquad (26)$$

and the ratio of the decay rate of $W \to \mu + \nu$ to that of $W \to e + \nu$ is

$$2v_\mu{}^2(3 + v_\mu)/(1 + v_\mu)^3, \qquad (27)$$

where v_μ is the velocity of the muon in the rest system of W. Decays of W into pions are also possible. The lifetime of W is $< 10^{-17}$ sec.

While the existence of a $W^\pm$ as a virtual particle does not change in any essential way the considerations of the previous sections, its production makes possible a much higher cross section for neutrino reactions through the pair creation of W^+ and l^- in the Coulomb field of a target nucleus:

$$\nu + Z \to W^+ + l^- + Z. \qquad (28)$$

The cross section for (28) is large compared with those without W production. At values of $k_\nu >> m_W{}^2/2q_0 \approx 2$ Bev, where $q_0 = \hbar/$nucleon radius, the cross section is given by

$$\sigma \cong (6\pi\sqrt{2})^{-1}(137)^{-2} Z^2 G_V [\ln(2k_\nu q_0/m_W{}^2)]^3, \qquad (29)$$

which, for $Z = 26$, is of the order of 10^{-35} cm^2. The $W^\pm$ produced are easily identifiable through their decay products $e + \nu$ or $\mu + \nu$. If experimentally no $W^\pm$ is found, it would be possible to set a lower limit on the value of m_W.

The existence of the intermediate Boson $W^\pm$ has[8] been discussed in connection with the question of the absence of $\mu^\pm \to e^\pm + \gamma$. On reasonably sure grounds one could conclude that the existence of $W^\pm$ requires that in the notation of (1), (2), (3), $\nu_1 \neq \nu_3$. In other words, the existence of W would imply that reaction (4) does not occur.

For processes without W production, the discussions of Sec. 4 above concerning a point structure of the lepton current remain unchanged, but the current J_λ now includes the effect of the propagator of W. Furthermore a corresponding change in the proportional vector current hypothesis is necessary.

The question of a neutral W^0 will not be examined here.

9. Interactions with extremely large momentum transfers. For momentum transfers of the order of $(G_V)^{-1/2} \sim 300$ Bev/c, quantities other

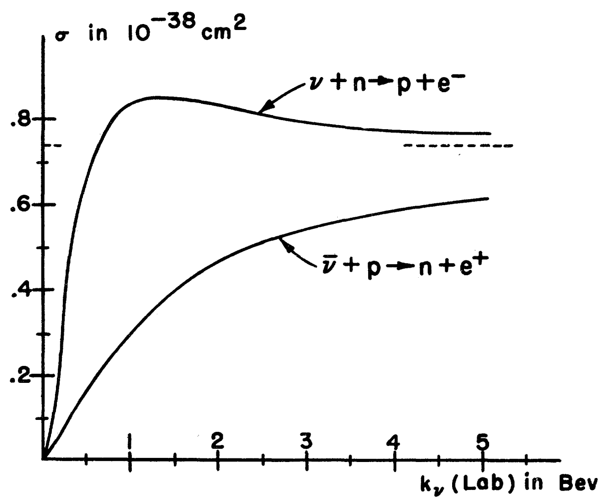

FIG. 1. "Elastic" neutrino cross sections. The dashed line represents the limit of σ as $k_\nu \to \infty$.

than the lowest power of G_V become important. For example, for $e^- + \nu \rightarrow \mu^- + \nu$, the cross section predicted by the lowest perturbation formula approaches the limit set by the unitary condition as the neutrino momentum in the center-of-mass system, $p_\nu \rightarrow 300$ Bev. For processes involving pion clouds, high-order terms in G_V presumably also become important as $p_\nu \rightarrow 300$ Bev. The description of the weak interaction at such high energies would seem to need ideas radically different from the current picture.

We wish to thank Professor M. Schwartz for many discussions.

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[1]M. Schwartz, preceding paper [Phys. Rev. Letters, 4, 306 (1960)].

[2]In Eq. (16), we neglect the mass of the lepton. For reactions in which the mass of the lepton may not be neglected, equations similar to but slightly more complicated than (16) result. For reactions in which strange particles are produced, (16) remains valid.

[3]By using the *CPT* theorem it is easy to see that the *S*-symmetry is closely related to the classification of weak interactions given by S. Weinberg, Phys. Rev. 112, 1375 (1958).

[4]See, e.g., M. L. Goldberger and S. B. Treiman, Phys. Rev. 111, 354 (1958).

[5]R. P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958).

[6]See, e.g., R. Hofstadter, F. Bumiller, and M. R. Yearian, Revs. Modern Phys. 30, 482 (1958).

[7]T. D. Lee and C. N. Yang, Phys. Rev. 108, 1611 (1957).

[8]G. Feinberg, Phys. Rev. 110, 1482 (1958); J. Schwinger, Ann. Phys. 2, 407 (1957).

ENHANCEMENT OF BREMSSTRAHLUNG PRODUCED BY 575-Mev ELECTRONS IN A SINGLE CRYSTAL OF SILICON*

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Experiments have been performed[1-3] to observe the expected enhancement of the bremsstrahlung produced by high-energy electrons when they pass through a single crystal.[4] The theory of Überall has been critically examined, and improved calculations have been made by Schiff.[5] The purpose of this report is to present the preliminary results of the revised experiment performed at Stanford, using a more sensitive technique of detection of the enhancement.

A single crystal of silicon, 0.013 in. thick and cut along the [110] plane, was aligned normal to the x-ray beam as defined by a 0.025-in. collimator, by the Laue back-reflection method. The crystal in its holder was mounted in a remote-controlled double-goniometer in which it could be rotated accurately about two coplanar perpendicular axes normal to the electron beam of the accelerator. The normal position of the [110] plane of the crystal with respect to the electron beam was established by using optical methods to transfer the x-ray alignment. Electrons of 575 Mev produced bremsstrahlung in the single crystal and were then deflected. The deflected electron beam was monitored by a standard secondary emission monitor.[6] The bremsstrahlung was monitored by a double-ionization chamber consisting of two identical ionization chambers constructed in one envelope, each preceded by appropriate beryllium converters. This unit was mounted accurately so that the axis of both the ionization chambers was in line with the incident beam direction. Continuous flow of purified hydrogen was maintained in the chamber at a constant pressure. The sensitivity of the first ionization chamber was a maximum around a photon energy of 30 Mev. The second ionization chamber looked at pairs which on the average have penetrated a larger range of beryllium and are therefore produced by a relatively harder component of the bremsstrahlung. Since the enhancement is large in the soft component of the bremsstrahlung, the relative increase of the charge in the first ionization chamber over that in the second ionization chamber would be quite pronounced. The charges collected in the two ionization chambers were fed separately to two dc amplifiers, the output of which was balanced using two potentiometers. The increase of the measured ratio as the crystal plane approached the correct angle with respect to the electron beam, would indicate the enhancement