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SELF-DIFFUSION IN LIQUID He^3†

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The self-diffusion coefficient of liquid He^3 has been measured in the temperature range from 0.067°K to 2.97°K at pressures near the saturated vapor pressure. The results indicate that the diffusion coefficient goes through a minimum near 0.55°K and increases rapidly below 0.2°K.

The diffusion coefficient, D, was measured using the method of spin echoes.[1-3] A 90° - 180° - 180° rf pulse sequence was used with a fixed time delay between the 90° pulse and the first 180° pulse, and a variable time, τ, between the two echoes. The resulting spin echoes were displayed on an oscilloscope and photographed. The ratio, R, of the amplitudes of the two echoes is $R = \exp(-\tau/T_2 - \gamma^2 G^2 \tau^3 D/12)$, where G is the magnetic field gradient, typically 1 gauss/cm, $\gamma = 2.038\times10^4$ (gauss-sec)$^{-1}$, and T_2 is the transverse relaxation time, the effect of which is negligible in these experiments. An iron-free Helmholtz pair produced a resonance field, H_0, of 26.2 gauss. A Maxwell gradient pair produced, parallel to H_0, a uniform field gradient, whose magnitude was calculated from the geometry. The rf pulse amplitude, $2H_1$, was chosen small ($H_1 \cong 0.25$ gauss) in order to avoid excessive rf heating. Hence, the gradient was turned off during the duration of a pulse in order that H_1 would be much larger than the field inhomogeneity. The time τ was corrected accordingly. The signal-to-noise ratio was increased by magnetizing in a 125-gauss field which was turned off sufficiently slowly just before each diffusion measurement.

Temperatures below 1°K were obtained by the adiabatic demagnetization of approximately 75 grams of ferric ammonium alum and were measured to within about 2% by the susceptibility of 8.3 grams of cerium magnesium nitrate. Liquid He^3, 98.9% pure, was introduced through a Cu-Ni tube[4] into a 0.38-cm^3 cylindrical bulb molded of the epoxy resin, Epibond 104.[5] Thermal contact between the liquid He^3 and the two salts was obtained with a bundle of 2000 insulated copper wires, 0.0051 cm in diameter, 760 of which looped into the liquid He^3 cavity. The wire loops were rectangular, with a total length of 850 cm of wire perpendicular to the gradient and rather loosely packed next to the vertical walls of the cylinder, and with 320 cm of wire parallel to the gradient distributed vertically through the cylinder in three groups in order to inhibit convection.

The results are shown in Fig. 1. The 2.38-atmos data of Garwin and Reich[3] are shown for comparison. The diffusion measured in the spin-echo method is the diffusion of the magnetization of the nuclei. This diffusion may take place, at one extreme, by atomic diffusion, or, at the other extreme, by a spin diffusion, without atomic diffusion, due to the interaction of neighboring nuclei. Garwin and Reich, considering the magnetic dipole interaction, state that the D due to it is probably smaller than 3×10^{-12} cm^2/sec. Susceptibility measurements by Fairbank, Ard, and Walters[6] can be interpreted in terms of an exchange interaction between the spins which is large compared to the dipolar interaction. The resulting spin diffusion without

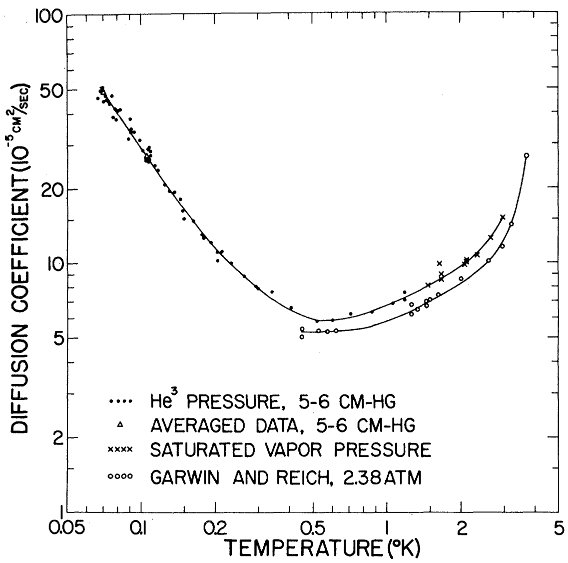

FIG. 1. Logarithm of the self-diffusion coefficient of liquid He^3 vs the logarithm of the temperature. The two triangles each represent the average of six data points too closely spaced to be shown separately. The data of Garwin and Reich are taken from reference 3.

atomic diffusion is still too small to account for the experimental results. Hence it is very likely that the increase of D at low temperatures is caused by a decreased probability of scattering of atoms, or groups of atoms, moving through the liquid. This decreased scattering was predicted for the elementary excitations of liquid He^3 by Pomeranchuk[7] and, more quantitatively, by Landau.[8] Experimental measurements of D by Garwin and Reich down to 0.45°K and of the thermal conductivity by Fairbank and Lee[9] down to 0.2°K give no indication of this effect. However, measurements[10] of the viscosity by Zinov'eva down to 0.35°K show an increase in the temperature dependence at the lowest temperatures. This fact has been interpreted by Abrikosov and Khalatnikov[8] as evidence for the decreased scattering at low temperatures. It should be noted that the effective nuclear interactions mentioned above may decrease the magnetization diffusion relative to the atomic diffusion in our experiments.

The values of D in Fig. 1 were derived neglecting T_2 decay, convection, residual field gradients, and artificial diffusion barriers. These possible sources of error will now be discussed. (*i*) Measured lower limits on T_2 are 0.55 second at 0.072°K, 1.5 seconds at 0.18°K, and longer at higher temperatures. The resultant systematic error in D is less than $+1.5\%$. (*ii*) Convection was produced intentionally at 1.75°K by changing the temperature at a rate $\dot{T}=0.018$°K/sec. The effective D in this case was 14×10^{-5} cm^2/sec compared with 9×10^{-5} cm^2/sec for $\dot{T}<10^{-6}$ °K/sec. Near 1°K the difference between D for $\dot{T}=3.6\times10^{-4}$ °K/sec and D for $\dot{T}<10^{-5}$ °K/sec was negligible. These results, together with convection theory[11] and the properties of He^3,[12] indicate that the error due to convection is negligible at our usual values of $\dot{T}$, less than 5×10^{-5} °K/sec. (*iii*) D was measured as a function of the polarity and magnitude of the field gradient G and no systematic changes of D with G were observed in the relevant region of G. (*iv*) The cooling wires and bulb walls limit the diffusion of some of the He^3 atoms, lowering the measured value of D. The He^4 impurities also lower the value of D relative to that of pure He^3.[13] Since both of these effects increase with D, they decrease the temperature dependence of D at low temperatures, which, in the present experiment, is $T^{-3/2}$ at 0.08°K. The decrease in the measured values of D due to the boundaries is probably a few percent at the low temperatures, but it cannot be estimated quantitatively.

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DOPPLER-SHIFTED CYCLOTRON FREQUENCY RADIATION FROM PROTONS IN THE EXOSPHERE

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In a recent publication[1] MacArthur has suggested that the phenomenon known as "Dawn chorus" may be due to Doppler-shifted radiation from protons which are rotating about the lines of force of the earth's magnetic field with the cyclotron frequency: $\nu_c = 1.54\times10^3 B$, where B is the field strength in gauss. The suggestion is quite attractive, because it involves a simple physical process, and because if it should prove correct, examination of the dispersion curves of the chorus radiation might furnish considerable information about the outer ionosphere. Accordingly, it seems worthwhile to investigate the suggestion in more detail.

The cyclotron frequency for protons in the earth's field is below 10^3 cps everywhere, and the frequencies observed in chorus reach several times this value. From this it is evident that the Doppler shift must be large, corresponding to protons with radial velocities very near the local velocity of electromagnetic radiation. The actual proton velocities, however, are low and consequently one is justified in neglecting the term in β^2 in the Doppler-shift equation

$$\nu' = \nu_0(1-\beta^2)^{1/2}/(1-\beta n).$$

Here ν' is the Doppler-shifted frequency, ν_0 is the proton gyrofrequency, and β is given by $\beta = V/c$, where V is the radial component of the velocity of approach of the particle, c is the velocity of light in free space, and n is the index of refraction for the frequency ν_0 (in the extraordinary mode). For frequencies of the order of a thousand cps the index of refraction is closely approximated by $n = (1+80.5N\times10^6/\nu_e\nu)^{1/2}$, where N is the electron density in electrons cm^{-3} and ν_e is the electron gyrofrequency.

A group of protons entering the earth's field at velocity V will then emit the Doppler-shifted frequency:

$$\nu' = \nu_c \Bigg/ \left[1 - \frac{V}{c}\left(1+\frac{80.5N\times10^6}{\nu_e\nu_c}\right)^{1/2}\right],$$

which will vary with the magnetic field strength (through ν_e), electron density, and particle speed. Whether or not the mechanism discussed can be considered as a possible source of chorus depends in part upon whether or not it can produce the proper frequency-time curve. Consideration of the various factors involved makes it seem likely that the most promising region where this might occur is the outer ionosphere. Accordingly, a model frequency-time curve for protons entering the earth's field at a radial velocity of 10^3 km sec^{-1} at a geomagnetic latitude of 65° has been calculated and is shown in Fig. 1. The maximum ion density is assumed to be 10^6 cm^{-3} at 300 km height and above this to follow an exponential distribution $N = N_0 e^{-0.0032h}$ (h is the distance in kilometers above the 300-km level). This corresponds to a scale height of a little over 300 km and is quite close to the model quoted by Johnson,[2] and is thought to be reasonable for the top of the ionosphere above College, Alaska. For the frequencies of interest here, the travel times through the ionosphere are quite long and have a considerable effect on the shape