one would suspect that a theory analogous to the Butler theory should not predict the correct, rather narrow angular correlation since (*a*) it does not contain information about the momentum distribution in different regions of the nuclear surface, and (*b*) it does not distinguish between the regions of the nuclear surface as to which are more likely to contribute to the reaction. Calculations using such a theory confirm this prediction.

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NUCLEAR MATRIX ELEMENTS IN THE BETA DECAY OF Sb^{124}†

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After the recent clarification of the beta interaction it has become of interest to study the relative contributions of the various matrix elements to first forbidden β transitions. It is the purpose of this paper to demonstrate that an unambiguous determination of matrix elements in a nonunique β transition (e.g., Sb^{124}) is possible on the basis of precise β-γ directional and β-γ circular polarization correlation measurements, if the β transition shows appreciable deviation from the ξ approximation.[1-3]

The ξ approximation was first introduced by Konopinski and Uhlenbeck[1] to explain the statistical shape of most nonunique first-forbidden beta spectra. In this approximation the beta transition probability is expanded in powers of the nuclear radius R and only the leading terms are taken into account, which are associated with the Coulomb factor $\xi = \alpha Z/2R$. Deviations from this approximation may be caused by selection rule effects which inhibit contributions from matrix elements other than $\int B_{ij}$. The contribution of the $\int B_{ij}$ term, which is of rank $\lambda = 2$ and which describes the component of the lepton field carrying away two units of angular momentum, may then become very important. The spectra of such β transitions exhibit deviations from the statistical shape and their ft values ($\log ft > 10$) are considerably larger than the characteristic ft values of nonunique first forbidden transitions ($\log ft \cong 8$).

It was suggested[2,3] that such a selection rule effect rather than a mutual cancellation of matrix elements explains the large ft value of the 2.31-Mev β transition of Sb^{124} ($\log ft = 10.6$). The results of the present investigation confirm this hypothesis.

The angular and energy dependence of the β_1-γ_1 directional correlation of Sb^{124} involving the β_1 component of 2.31-Mev maximum energy (refer to inset of Fig. 1) was measured with the vacuum chamber described previously.[4] The directional correlation $W_{\beta\gamma}(\theta, \overline{W}_\beta = 4.8)$ of the β_1-γ_1 cascade measured at a fixed average energy of $\overline{W}_\beta = 4.8$ (in units of mc^2) is shown in Fig. 1. A least-squares fit of the experimental points to the correlation function:

$$W_{\beta\gamma}(\theta, \overline{W} = 4.8) = 1 + A_2(4.8)P_2(\cos\theta) + A_4(4.8)P_4(\cos\theta), \quad (1)$$

yielded the following values for the correlation coefficients:

$$A_2(4.8) = -0.390 \pm 0.011,$$
$$A_4(4.8) = +0.004 \pm 0.013. \quad (2)$$

The absence of a $P_4(\cos\theta)$ term provides further evidence against the decay scheme $4^+(\beta_1)2^+(\gamma_1)0^+$.

The dependence of the coefficient $A_2(W)$ on the β energy is shown in Fig. 2. A simultaneous measurement of the energy dependence of the β_2-γ_2 directional correlation made it possible to correct the data for the presence of the β_2-γ_1 directional correlation at β energies below the maximum energy of the β_2 spectrum ($W_0 = 4.15$). There is, however, some uncertainty in this correction due to the fact that the sign of the $E2$-$M1$ mixing ratio δ of the γ_2 transition is not known ($\delta = \pm 1.00 \pm 0.085$).[5] The error caused by this uncertainty is included in the error flags of the experimental points corresponding to $W<4.15$.

All available data indicate strongly that the β_1-γ_1 cascade of Sb^{124} follows the decay scheme $3^-(\beta_1)2^+(\gamma_1)0^+$. The four matrix elements which can contribute to a first forbidden β transition with $\Delta I = \pm 1$, yes, are the relativistic matrix element $y = C_V \int i\vec{\alpha}$, the moment type matrix elements $x = -C_V \int \vec{r}$, and $u = C_A \int i\vec{\sigma} \times \vec{r}$ (all of rank $\lambda = 1$) and the matrix element $z = C_A \int B_{ij}$ ($\lambda = 2$).

After Kotani[2] the energy dependence of the directional correlation coefficient $A_2(W)$ is expressed by

$$A_2(W) = \frac{W^2-1}{W} \frac{R_3 + eW}{1 + aW + cW^2 + (b/W)}. \quad (3)$$

The coefficients R_3, e, a, b, and c, which are complicated functions of the matrix element parameters x, u, z, and $Y = y - \xi(u+x)$ and of the maximum energy W_0 are given in reference 2.

By a least-squares method the values of the

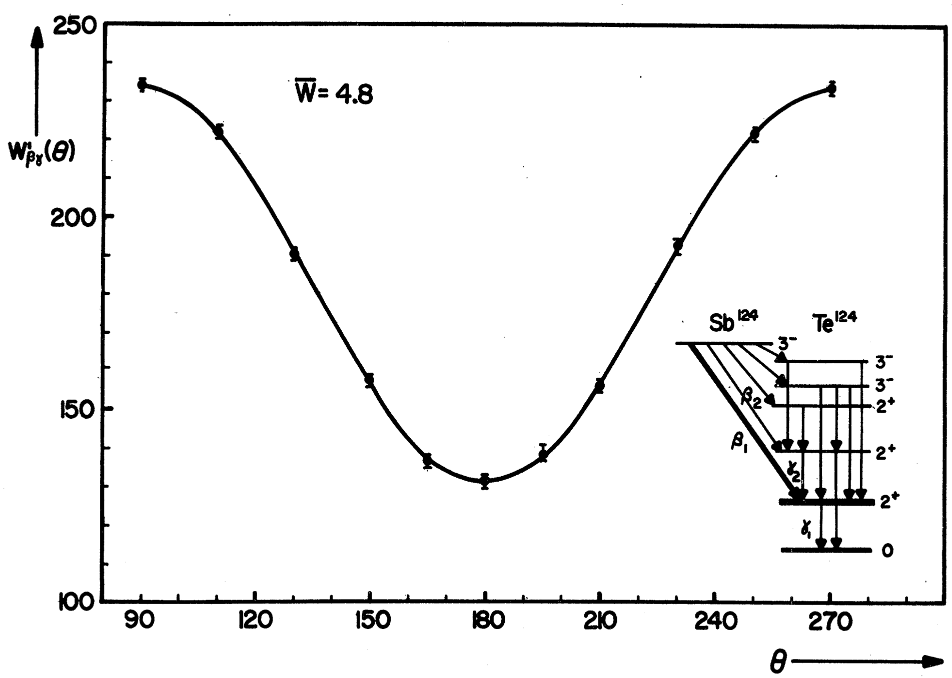

FIG. 1. Angular dependence of the β-γ directional correlation involving the 2.31-Mev β transition of Sb^{124}. The measurements were made at an average β energy of $\overline{W} = 4.8$ (in units mc^2).

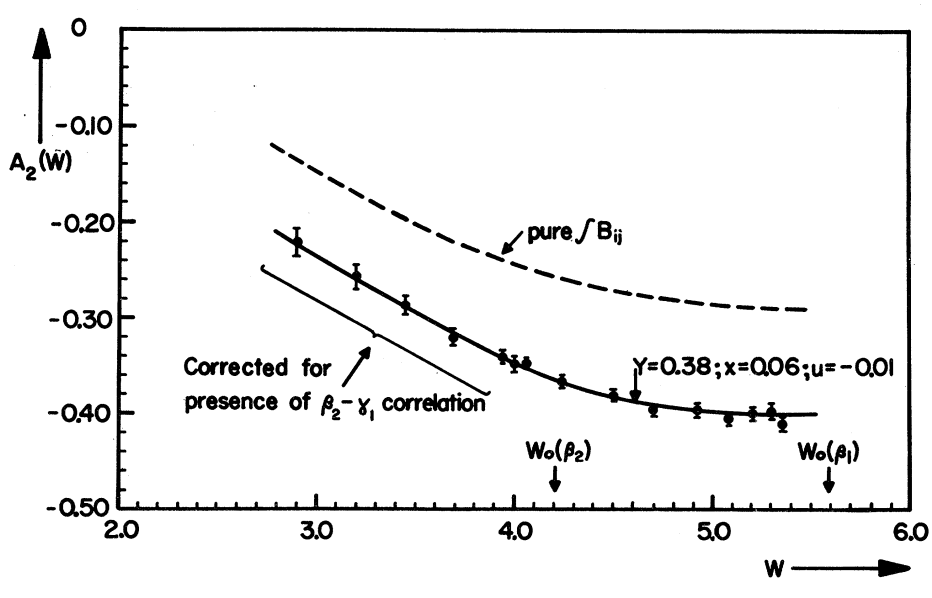

FIG. 2. Energy dependence of the anisotropy factor $A_2(W)$ of the β_1-γ_1 directional correlation. The solid line represents $A_2(W)$ calculated with the parameters $u = -0.01$, $x = 0.08$, $Y = 0.38$. The dashed line corresponds to a pure $\int B_{ij}$ (unique) transition.

parameter ratios $u/z, x/z$, and Y/z were determined which resulted in a best fit of the data of Fig. 2 to a curve given by Eq. (3). As an additional condition it was imposed that the set of parameters also satisfy the circular polarization correlation measurements of Hartwig and Schopper,[6] i.e., $P_C(126°)/P_C(160°) = -2$, where $P_C(\theta)$ is the degree of circular polarization of the γ_1 radiation measured at the angle θ. The fit of the data yields for the nuclear parameters:

$$u = -(0.01 \pm 0.04)z,$$
$$x = (0.08 \pm 0.08)z,$$
$$Y = (0.38 \pm 0.12)z. \tag{4}$$

This set of parameters agrees satisfactorily with the set obtained by Hartwig and Schopper,[6] which was determined on the basis of a somewhat different approach. The function $A_2(W)$ calculated with the parameters of Eqs. (4) is represented as solid line in Fig. 2. For comparison the curve $A_2(W)$ corresponding to a unique β transition (pure $\int B_{ij}$) is included.

By taking into account the corrected ft value of the 2.31-Mev β transition, $ft = 10^{10.6}$ sec, or $ft = 3.1 \times 10^{31}$ in units $\hbar = m = c = 1$, the absolute values of the matrix elements involved in this β decay can be computed[7]:

$$|\int B_{ij}|/R = (1.20 \pm 0.15) \times 10^{-2},$$

$$|\int \vec{r}|/R = (1.2 \pm 1.2) \times 10^{-3},$$

$$|\int i\vec{\sigma} \times \vec{r}|/R = (0.1 \pm 0.4) \times 10^{-3},$$

$$|\int i\vec{\alpha}| = (3.1 \pm 2.4) \times 10^{-4},$$

$$(\int i\vec{\alpha}/\int B_{ij}) > 0. \tag{5}$$

The values of the matrix elements are given in a form which is independent of the chosen system of units (R = nuclear radius). In addition, the lack of overlap of the nuclear wave functions which occur in the matrix elements is more evident in the form of Eqs. (5). If the wave functions of the initial and final nuclear states would overlap perfectly, the values of $\int B_{ij}/R$, $\int \vec{r}/R$, and $\int i\vec{\sigma} \times \vec{r}/R$ would be of order unity whereas the relativistic matrix element $\int i\vec{\alpha}$ would be of order $v_{\text{nucleon}}/c \cong 0.1$. It is interesting to note that all matrix elements involved in the Sb^{124} β_1 transition are considerably reduced. Compared to unique β transitions ($\Delta I = \pm 2$), where $|\int B_{ij}|/R$ is of the order ~0.1, the $\int B_{ij}$ involved in the Sb^{124} β_1 transition is reduced by a factor of about 10. The reduction of $\int B_{ij}/R$, however, is orders of magnitude smaller than the reduction of the other matrix elements. The cause of the unusual predominance of the $\int B_{ij}$ matrix element seems to be a selection rule effect as suggested by Kotani[2] and by Morita and Morita.[3]

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