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PARTIAL-WAVE DISPERSION RELATIONS FOR MESON-NUCLEON SCATTERING*

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It is of interest to compute the implications for the pion-nucleon scattering amplitude of possible resonance effects in the two-pion system.[1] In this note we propose a form of the π-N partial-wave dispersion relations[2] which is suitable for this purpose. Our formulas can be easily generalized for amplitudes involving other particles with unequal mass.

We write the covariant π-N amplitude in the form[3] $F = A - i\gamma\cdot\frac{1}{2}(k+k')B$, where $A_{\alpha\beta} = A^+\delta_{\alpha\beta} + A^-\frac{1}{2}[\tau_\alpha, \tau_\beta]$, etc. and $k+p = k'+p'$, $k^2 = k'^2 = -\mu^2$, $p^2 = p'^2 = -m^2$. In the following we discuss explicitly only the invariant amplitudes $A^\pm$; the extension to $B^\pm$ and to center-of-mass quantities is straightforward. Let us first consider the $A^\pm$ as functions of the invariant variables $z = -(k+p)^2$, $\zeta = -(k-k')^2$. We assume that they are analytic in both variables except for the absorptive singularities[4] due to the possible intermediate states of the three physical reactions associated with the π-N Green's function. Then we have the branch lines[3]

$$z = s \geq (m+\mu)^2,\quad \bar{z} = \bar{s} \geq (m+\mu)^2,\quad \text{and } \zeta = t \geq (2\mu)^2, \tag{1}$$

where $\bar{z} = 2m^2 + 2\mu^2 - z - \zeta$. The single nucleon poles $z = m^2$, $\bar{z} = m^2$ appear only in the amplitudes $B^\pm$. The variables z and ζ may be expressed in terms of q^2 (q = c.m. momentum) and θ (c.m. angle) by

$$z = m^2 + \mu^2 + 2q^2 + 2[(q^2+m^2)(q^2+\mu^2)]^{1/2}, \tag{2a}$$

$$\zeta = -2q^2(1-\cos\theta). \tag{2b}$$

Note that the complex z plane is mapped into a Riemann surface with two leaves which are connected through the cut $-m^2 \leq q^2 \leq -\mu^2$. We define sheet I (II) by the requirement that the root in Eq. (2a) is positive (negative) for real $q^2 > -\mu^2$. In the z plane, the exterior of the circle $|z| = m^2 - \mu^2$ corresponds to sheet I and the interior to sheet II.

Let us now consider the amplitude A as an analytic function of q^2 for $-1 \leq \cos\theta \leq +1$. It is regular on the Riemann surface mentioned above except for cuts along the real axes in both sheets. In particular, the partial-wave amplitudes,

$$A_J(q^2) = \tfrac{1}{2}\int_{-1}^{+1} d\cos\theta\, P_J(\cos\theta) A(q^2, \cos\theta), \tag{3}$$

have the following cuts: (πN) from 0 to $+\infty$ in I, due to $s \geq (m+\mu)^2$; $(\bar{\pi}N)$ from 0 to $+\infty$ in I and from $-\infty$ in II up to $-m^2$ and then down to $-\infty$ in I, both due to $\bar{s} \geq (m+\mu)^2$; $(\pi\bar{\pi})$ from $-\mu^2$ to $-\infty$ in I and II, due to $t \geq (2\mu)^2$. The weight functions associated with the unphysical branch lines $(\bar{\pi}N)$ and $(\pi\bar{\pi})$ can be expressed in terms of the absorptive parts $N(\alpha,\beta)$ and $M(\alpha,\beta)$ (α,β being the squares of energy momentum transfer), corresponding to the reactions $\pi + N \to \pi + N$ and $\pi + \bar{\pi} \to N + \bar{N}$, respectively. In the physical sheet I we can write partial-wave dispersion relations in the simple form

$$A_J(q^2) = \frac{1}{\pi}\int_0^\infty dq'^2 \frac{\mathrm{Im}A_J(q'^2)}{q'^2 - q^2} + \frac{1}{\pi}\int_{-\infty}^{-\mu^2} dq'^2 \frac{\mathrm{Im}A_J(q'^2)}{q'^2 - q^2}, \qquad (4)$$

which should be amended by the usual substractions. However, in the interval $-m^2 \leq q^2 \leq -\mu^2$ the real weight $\mathrm{Im}A_J$ can be expressed in terms of the absorptive parts N and M only indirectly by writing a similar dispersion formula in sheet II. Dispersion relations involving both sheets can be easily written down, but it seems to be advantageous to use the invariant variable z for those portions of the cuts which map onto the real axis in the z plane, and to retain the variable q^2 for that part of the branch line $(\pi\bar{\pi})$, which maps onto the circle $|z| = m^2 - \mu^2$. Omitting again appropriate subtractions, we can write

$$A_J(z) = \frac{1}{\pi}\int_{(m+\mu)^2}^\infty ds \frac{\mathrm{Im}A_J(s)}{s - z} + \frac{1}{\pi}\int_{-\infty}^{(m-\mu)^2} ds \frac{\mathrm{Im}A_J(s)}{s - z} + F_J(z) + F_J^*(z^*), \qquad (5)$$

where

$$F_J(z) = -\frac{1}{\pi}\int_{-m^2}^{-\mu^2} d\lambda \frac{2s(\lambda)}{s(\lambda) - m^2 - \mu^2 - 2\lambda}\,\frac{M_J(s(\lambda))}{s(\lambda) - z}, \qquad (6)$$

with

$$s(\lambda) = m^2 + \mu^2 + 2\lambda + 2i[(\lambda + m^2)(-\lambda - \mu^2)]^{1/2}, \qquad (7)$$

and, for $s \leq (m - \mu)^2$,

$$\mathrm{Im}A_J^{\pm}(s) = \theta(-s)\epsilon(s + m^2 - \mu^2)N_J^{\pm}(s) \mp M_J^{\pm}(s). \qquad (8)$$

The functions M_J and N_J are given in terms of the absorptive parts $M(\alpha, \beta)$ and $N(\alpha, \beta)$ by

$$M_J(s) = -\frac{1}{4q^2}\int_{4\mu^2}^{-4q^2} dt\, P_J(1 + t/2q^2)M(t, s),$$

$$N_J(s) = -\frac{1}{4q^2}\int_{2m^2+2\mu^2-s}^{c(s)} d\bar{s}\, P_J\left(1 + \frac{2m^2 + 2\mu^2 - s - \bar{s}}{2q^2}\right)N(\bar{s}, 2m^2 + 2\mu^2 - s - \bar{s}), \qquad (9)$$

where $q^2 = q^2(s)$ is given by the inverse of Eq. (2a) with $z = s$; and $c(s) = (m^2 - \mu^2)^2/s$ for $0_+ \leq s \leq (m - \mu)^2$, $c(s) = (m + \mu)^2$ for $s \leq 0_-$. The term $F_J + F_J^*$ in Eq. (5) contains the low-energy properties of the 2π system. Using Eqs. (6) and (9), it can be easily written in terms of real and imaginary parts of $M(t, s)$.[5] The latter quantity is complex due to the appearance of $s(\lambda)$ given by Eq. (7). In a partial-wave expansion of $M(t, s)$ the variable s will appear only in the argument of the spherical harmonics.

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