

substantial enhancement of the resonance.

We wish to thank A. D. McGuire for help in the early portion of this experiment. We would like to thank M. Goldhaber for a helpful discussion. We take pleasure in acknowledging the help of the Omega reactor group in preparing sources.

[†]Work performed under the auspices of the U. S. Atomic Energy Commission.

¹R. L. Mössbauer, *Z. Physik* **151**, 124 (1958);

Naturwissenschaften **45**, 538 (1958); *Naturforsch.* **14a**, 211 (1959).

²P. P. Craig, J. G. Dash, A. D. McGuire, D. Nagle, and R. D. Reiswig, *Phys. Rev. Letters* **3**, 221 (1959).

³E. M. Bernstein and H. W. Lewis, *Phys. Rev.* **100**, 1345 (1955).

⁴A. W. Sunyar, *Phys. Rev.* **98**, 653 (1955).

⁵W. Visscher, *Ann. Phys.* (to be published).

⁶*American Institute of Physics Handbook* (McGraw-Hill Book Company, Inc., New York, 1957).

⁷W. Visscher (unpublished).

⁸F. A. Lindemann, *Physik. Z.* **11**, 609 (1910).

SPACE-TIME CORRELATION FUNCTION FORMALISM FOR SLOW NEUTRON SCATTERING

P. Schofield

Theoretical Physics Division, Atomic Energy Research Establishment, Harwell, England

(Received January 29, 1960)

Van Hove¹ has shown that the partial differential cross section for the coherent scattering of neutrons by an assembly of atoms is related to the space-time correlation function, $G(\vec{r}, t)$, by

$$\frac{\partial^2 \sigma}{8\Omega \partial \epsilon} = \frac{a^2 k'}{2\pi \hbar k} \Gamma(\vec{k}, \omega), \quad (1)$$

$$\Gamma(\vec{k}, \omega) = \iint \exp[i(\vec{k} \cdot \vec{r} - \omega t)] G(\vec{r}, t) d\vec{r} dt, \quad (2)$$

where a is the appropriate scattering length, \vec{k} and \vec{k}' the wave vectors of the incident and outgoing neutron, $\hbar \vec{k}$ the momentum transfer, and $\epsilon = \hbar \omega$ the energy lost by the neutron. The spatial part of the transform (2), $\gamma(\vec{k}, t)$, is given by

$$\gamma(\vec{k}, t) = N^{-1} \left\langle \sum_{l,j=1}^N \exp[-i\vec{k} \cdot \vec{r}_j(0)] \right. \\ \left. \times \exp[i\vec{k} \cdot \vec{r}_j(t)] \right\rangle_T, \quad (3)$$

where $\langle \dots \rangle_T$ denotes a thermal average over the states of the scattering system, and $\vec{r}_j(t)$ is the Heisenberg operator for the position of the j th atom at time t .

Now it can easily be verified, by writing the thermal average and $\vec{r}_j(t)$ in terms of the Hamiltonian of the system, that $\gamma(\vec{k}, t + i\hbar/2k_B T)$ is a real function of t . Let this function be denoted by $\gamma'(\vec{k}, t)$. The Fourier transform of $\gamma'(\vec{k}, t)$ with respect to t then defines a function $\Gamma'(\vec{k}, \omega)$ which is related to $\Gamma(\vec{k}, \omega)$ by

$$\Gamma'(\vec{k}, \omega) = \Gamma(\vec{k}, \omega) \exp[\hbar \omega / 2k_B T]. \quad (4)$$

From the properties of $\gamma'(\vec{k}, t)$ it follows that

$\Gamma'(\vec{k}, \omega)$ is even in both \vec{k} and ω . Thus (4) expresses the condition of detailed balance, which we know a priori must be satisfied by the scattering cross section.

By inverting the transform (2) and using (4), it is possible to obtain a dispersion relation relating the real and imaginary parts of $G(\vec{r}, t)$. This may be written in integral form:

$$\mathcal{I}\{G(\vec{r}, t)\} = (k_B T / \hbar) \\ \times \int_{-\infty}^{\infty} \csc(\pi k_B T t' / \hbar) \Re\{G(\vec{r}, t-t')\} dt', \quad (5)$$

or, more conveniently, in operator form:

$$\mathcal{I}\{G(\vec{r}, t)\} = -\tan\left(\frac{\hbar}{2k_B T} \frac{\partial}{\partial t}\right) \Re\{G(\vec{r}, t)\}. \quad (6)$$

This can be regarded as a constraint imposed upon the correlation function as a consequence of the thermal equilibrium of the system.

Vineyard² has suggested that in many systems it is possible to replace $G(\vec{r}, t)$ in (2) by its classical equivalent $G_C(\vec{r}, t)$. Since the classical correlation function is real, the resulting cross section does not satisfy the detailed balance condition. The above analysis suggests that $G_C(\vec{r}, t)$ should be used to define $\Gamma'(\vec{k}, \omega)$, rather than $\Gamma(\vec{k}, \omega)$, so that this condition is satisfied. Although in the limit $\hbar \rightarrow 0$, $\gamma'(\vec{k}, t)$ and $\gamma(\vec{k}, t)$ are identical, since the variable of transformation is the energy transfer divided by \hbar , the transforms are not identical.

The use of the classical correlation function in this way takes some account of the recoil of the

scattering atom, and, as may be seen by considering the simple examples of the monatomic gas and the isotropic oscillator, gives a cross section which is formally correct to first order in \hbar .

Another case discussed by Vineyard² is that in which the atoms diffuse according to the Langevin equation. By means of the real function $\gamma'(\vec{k}, t)$ it is possible to obtain an explicit expression for the cross section:

$$\Gamma'(\vec{k}, \omega) = \frac{2}{\eta} e^{\alpha} \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \alpha^{n+1} \times \left[\frac{1}{(\alpha+n)^2 + \beta^2} - \frac{1}{(\alpha+n+1)^2 + \beta^2} \right], \quad (7)$$

where η is the coefficient of viscous damping, and

$$\alpha = k_B T k^2 / M \eta^2, \quad \beta = \omega / \eta.$$

The result obtained if the simple diffusion

equation is used to derive $G(\vec{r}, t)$ corresponds to the first part of the first term in (7) so that unless both α and β are small, this approximation is not valid. In fact, (7) gives a width for the scattered energy distribution less than that given by both the simple diffusion model and the gas model. This is in agreement with the observations of Pelah et al.³ on liquid lead. However, (7) does not give a width sufficiently small to explain the results of Turberfield⁴ on lead with cold neutrons. These experiments indicate that the atoms of the liquid must make many vibrational oscillations between diffusive jumps, and the results cannot be interpreted on a basis of continuous diffusion.

¹L. Van Hove, Phys. Rev. **95**, 249 (1954).

²G. H. Vineyard, Phys. Rev. **110**, 5 (1958).

³I. Pelah, W. L. Whittemore, and A. W. McReynolds, Phys. Rev. **113**, 767 (1959).

⁴K. C. Turberfield (to be published).

LEAD K ABSORPTION EDGE FOR μ -MESON MASS DETERMINATION

Alan J. Bearden*

Department of Physics, The Johns Hopkins University, Baltimore, Maryland

(Received January 26, 1960)

The recent accurate determination of the f_{μ}/f_p ratio for the μ meson¹ and the comparison of the magnetic moment of the μ meson based on this measurement with the value predicted by quantum electrodynamics² depends critically on the mass of the μ meson. It has been shown experimentally that the $3D-2P$ transition in μ -mesonic phosphorus lies near the Pb K edge energy.³ In fact, when considered in detail, it appears that the fine structure splitting of the $3D-2P$ transition in phosphorus will cause the lines to straddle the Pb K edge energy.⁴ Therefore, it seems possible that one may use the variation of the mass-attenuation coefficient in lead as a function of energy in the K edge region as a very accurate measurement of the mesonic x-ray energy. The detailed shape of the lead K edge must be known precisely both in energy scale and mass-attenuation coefficient. Knowing these transition energies, and accounting for corrections due to vacuum polarization, nuclear size effects, etc., it would be possible to make a precise determination of the μ -meson mass. The transition energies for the mesonic

phosphorus lines under discussion are given by⁴

$$3D_{3/2} - 2P_{3/2}: 425.42 m_{\mu}/m_e, \text{ relative intensity} = 1;$$

$$3D_{5/2} - 2P_{3/2}: 425.65 m_{\mu}/m_e, \text{ relative intensity} = 9;$$

$$3D_{3/2} - 2P_{1/2}: 427.75 m_{\mu}/m_e, \text{ relative intensity} = 5.$$

From the quoted "best value" of the μ -meson mass,⁵ $m_{\mu}/m_e = 206.86 \pm 0.11$, the $3D_{5/2} - 2P_{3/2}$ transition would lie at 88 041 ev. The best estimate of the inflection point of the Pb K edge is at 88 014 ev,⁶ and its width is of the order of 100 ev. The intent of this Letter is to present then an accurate measurement of the Pb K absorption edge.

A two-crystal spectrometer using the 20 $\bar{2}$ 3 planes of quartz as diffracting surfaces was constructed. Angles were measured by use of the precision circle of the Societ  Genevoise goniometer of the Johns Hopkins x-ray laboratory.⁷ In order to gain comparative freedom from scattering, distances of one meter were used between x-ray source, first crystal, second crys-