PLASMA TRAPPING IN CUSPED GEOMETRIES*

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The question of the trapping of particles in cusped magnetic containment configurations has been recently examined.[1] The purpose of this Letter is to make accessible and extend certain early results which bear on this problem.[2] The trapping problem is, clearly, very closely related to the question of containment. The latter problem has been treated in some detail for the cusped geometry[3] (high $\beta \equiv 2\mu_0 p/B^2$) and to a lesser extent for the picket fence[4] (low β). The possibility of creating a cusped plasma by injection using a plasma gun has been described several times[5] and has been tried experimentally on a small scale with inconclusive results.[6]

The mechanisms are different for trapping of a single particle (i.e., a low-β stream) or a high-β cohesive plasma burst; we consider first the simpler single-particle mechanism (this has been termed impossible; see reference 1).

We consider a two-dimensional cusped configuration as shown in Fig. 1. In this simplified model the magnetic field is assumed to be totally excluded and a particle orbit consists of straight-line segments joined by helical arcs. Sufficiently close to a cusp, the orbit has the adiabatic invariant[7] (see Fig. 1 for notation),

$$\mu^* = 2mvd + \pi m^2(v^2 + w^2)/eB. \qquad (1)$$

The value of μ^* will be essentially constant while a particle is close to a cusp, but it will change its value when the particle crosses the device and approaches a second cusp. The simplest containment theory[8] is based on the assumption of a complete loss of memory on traversing the apparatus. In other words, the distribution function of particles approaching a cusp is Maxwellian without any gap in a loss-cone. A refinement of this theory takes into account a certain amount of persistence in two successive values of μ^*.[2] Turning to the injection problem, we see that a complete loss in memory of the value of μ^* yields complete trapping of all injected particles. They will enjoy a lifetime which is, on the average, the same as that of any particle in the plasma, whether recently injected or not. It is only those particles with a good memory which cross to the opposite cusp and escape after but a single transit.

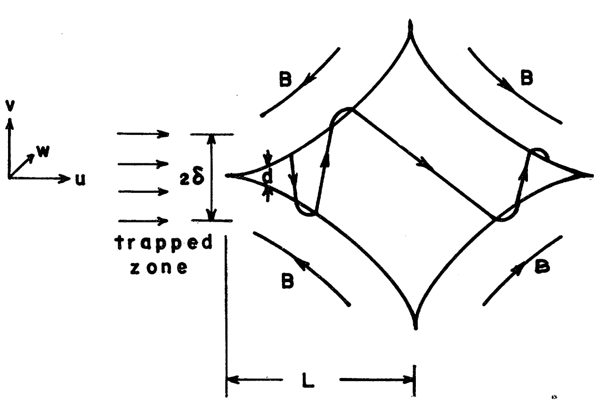

FIG. 1. Two-dimensional cusped geometry.

A simple computation of this persistence effect (based on a calculation of the last encounter made with the original cusp before crossing to the opposite side) shows that all particles are trapped which are injected within a "trapped zone" (see Fig. 1) of width

$$2\delta = 2\lambda\left(1 - \frac{\pi}{16}\frac{\lambda}{L}M^3\right)^{1/2}, \qquad (2)$$

where $\lambda = m(v^2 + w^2)^{1/2}/eB$ is the Larmor radius and $M = u/(v^2 + w^2)^{1/2}$ is the injection "Mach number." To be trapped, the guiding center of an approaching particle should be within a distance δ of the cusp axis. With λ/L fixed, there are no trapped particles when M is sufficiently large. The maximum width of the trapped zone is twice the Larmor radius. For most efficient trapping, the beam width should not exceed the Larmor radius. This formula is an approximation which is valid when either M or L/λ is large.

A similar (but much more intricate) analysis can be performed for the case of axial injection through a point cusp in a three-dimensional geometry (Fig. 2). The maximum size of the trapped zone is in this case a disk of area $\pi\delta^2 \sim L\lambda$; this area has the same order of magnitude as the total trapping area offered by the line-cusp which can be analyzed two-dimensionally as above. An appreciable fraction of those particles which approach the point cusp within the distance δ will be trapped provided that $M = u/(v^2 + w^2)^{1/2}$

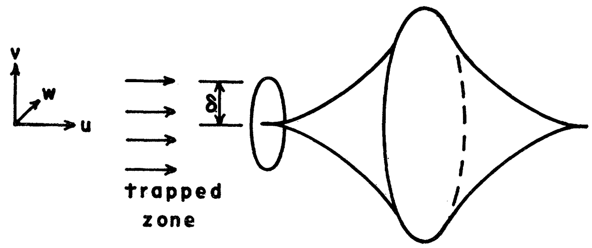

FIG. 2. Three-dimensional cusped geometry.

is smaller than some numerical factor times $(L/\lambda)^{7/6}$.

The problem of computing the trapping of a high-β plasma burst which can alter the magnetic field is much more difficult. A simple fluid-like theory can be developed for the smooth motion of a plasma in a magnetic channel.[9] If we are guided by the similar problem in ordinary fluid dynamics, we can expect complicated phenomena such as shocks (as in a diffuser with supersonic entrance velocity), jet detachment (since the magnetic walls curve away sharply), vortices in the dead space surrounding the jet. Peculiar to the magnetic channel problem, it has been shown that there is no stable transition between subsonic and supersonic regions.[9] Thus the entry of the plasma into the low-field region is a complex matter. At the far end, one cannot conclude (as is done in reference 1), merely from the fact that the magnetic pressure is higher than the energy density in the beam, that the beam will be turned back. For one thing, in the direction parallel to the magnetic field, the Maxwell stress is a tension, not a pressure. It is easy to construct flows of arbitrarily low energy density which can pass through a mirror region, and there is reason to believe that an energetic supersonic jet of this type can be stable.[9]

Theoretical analysis of this entire problem would be a huge task. Fortunately, certain simple experimental expedients should guarantee a large degree of trapping. For example, the cusped configuration can be made slightly asymmetric by misaligning the two coils. The incoming jet will then miss the opposite cusp and be trapped. Or else, two streams can be injected from opposite ends.[10] It should be remarked that a certain amount of trapped transverse magnetic field in the jets is helpful to make them collide instead of passing through one another as has been observed with "plasmoids." Otherwise one might be able to depend on the "two-stream" instability to break up the jets. Again, too much symmetry is undesirable since it might lead to a spurt sideways through the cusped diametral plane, but a slight misalignment (either in direction or in timing) would probably mitigate this effect.

Generally speaking, trapping is apt to be quite efficient in any apparatus with a large approximately field-free region (even in a mirror machine[2]), especially if there is no high degree of symmetry. It is possible that both the high-β and low-β injection techniques could prove to be useful, the first to create a plasma and the second to maintain it against cusp losses for the required time.

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[1]J. L. Tuck, Phys. Rev. Letters 3, 313 (1959), where this is referred to as "entropy trapping."

[2]H. Grad, Atomic Energy Commission Report TID-7520, 1956 (unpublished), p. 148.

[3]A concise summary is given by J. Berkowitz et al., Proceedings of the Second United Nations International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1958 (United Nations, Geneva, 1958), Vol. 31.

[4]Hartland Snyder (private communication, 1955).

[5]For example, see H. Grad, Atomic Energy Commission Report NYO-7969, 1957 (unpublished).

[6]D. Finkelstein, G. A. Sawyer, and T. F. Stratton, Phys. Fluids 1, 188 (1958).

[7]Reference 3, Appendix. The formula quoted there is in error.

[8]H. Grad, Atomic Energy Commission Report TID-7503, 1955 (unpublished), p. 319. See also reference 3.

[9]H. Grad, Bull. Am. Phys. Soc. 3, 288 (1958). This theory can easily be generalized to include plasma and field intermingled.

[10]This proposal, with the two "guns" an integral part of the apparatus rather than external, is being carried out by a group at Stevens Institute.