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BUMPY TORUS*

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Magnetic field configurations resulting from various coil arrangements are being used for thermonuclear studies. In the absence of particle interactions, charged particles which occupy certain regions of velocity space are contained indefinitely provided the particles behave adiabatically. However, there are regions in velocity space in which particles are not contained; e.g., in the mirror machine if the velocity vector of a charged particle makes a sufficiently small angle with the magnetic field the particle escapes through a mirror.[1] Hence, the mean containment time is limited to approximately the time, τ_S, for small-angle scattering to cumulate to a large angle insofar as multiple scattering is the dominant loss mechanism.

A uniformly wound torus would eliminate the end losses; however, another difficulty is introduced. Because of the drift of the charged particles resulting from the gradient of the magnetic field, the particles rapidly impinge on the walls of the device.

A simple case to illustrate the drift motion both in a magnetic mirror machine and in the torus is one where the velocity, v, of the particle is normal to the magnetic field, H, and the longitudinal gradient of the field is zero. The magnetic moment, μ, of the particle motion is an adiabatic invariant[2]; therefore $\mu = W_\perp/H = \frac{1}{2}mv_\perp^2/H = \text{constant}$, and if the particle's energy, W, is a constant the particle's guiding center moves along a path defined by $H = \text{constant}$. The particle drifts are illustrated in Fig. 1.

A rotational transform[3] tends to reduce the loss due to drifts in the torus. However, the rotational transform may be ineffective if the particle motion along the field lines is slow (relative to the drift velocity), with the result that the containment time would again be limited to approximately the scattering time τ_S. The electric field and other phenomena associated with a dense plasma would also affect the containment time.

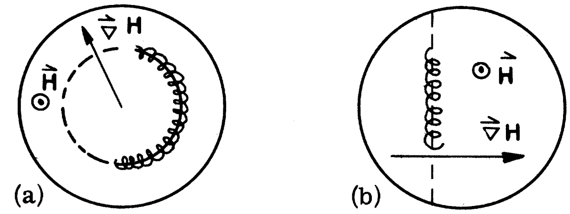

FIG. 1. Particle drift paths. (a) Mirror machine cross section, median plane. (b) Uniformly wound torus cross section.

In this paper the single-particle motion in the magnetic field created by a circular array of circular current loops is investigated (see Fig. 2). The extent to which this geometry would have the advantage of closed precessional sur-

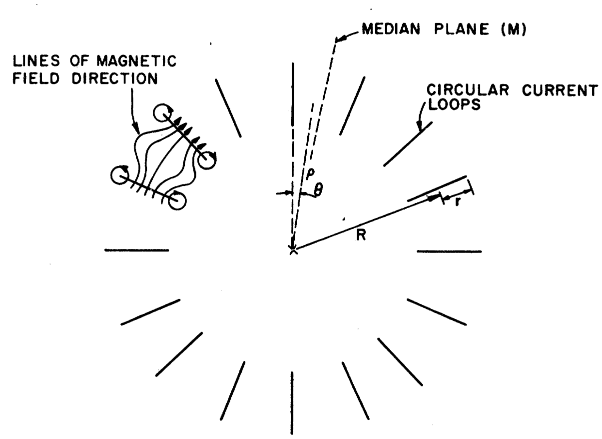

FIG. 2. The bumpy torus geometry and field magnitudes. The symmetry plane, S, is the plane of the sheet of paper.

faces which would not intersect the walls [see Fig. 1(a)], and would also eliminate end effects,[4] is investigated. Because of the complexity of a 3-dimensional calculation, the results in this paper are limited to an investigation of the intersection of the precessional surfaces with the symmetry plane containing the torus. The single-particle model is of special interest during the plasma buildup stage in the high-energy injection schemes.[5,6] The stability of a plasma has not been investigated theoretically for this geometry; of course if a plasma would build up, the cooperative effects could be observed experimentally.

It is assumed that the following quantities are constants of the motion: (1) the kinetic energy, W; (2) the magnetic moment, $\mu = W_\perp/H$; and (3) the action integral, $J = \int p_\parallel dl$, evaluated over a period (as defined later) of the longitudinal motion, where $p_\parallel$ is the component of the momentum parallel to the field and dl is the element of path length parallel to the field.[7]

The IBM 650 is being used to calculate (1) the magnitude of the magnetic field, (2) the direction of the magnetic field, and (3) the action integral for particular values of the magnetic moment and starting coordinates.

The code is set up to calculate these quantities in the symmetry plane S (see Fig. 2) for N equally spaced circular current loops (current I) with a ratio of major radius divided by current loop radius, R/r. Because of the symmetry it is only necessary to investigate the region between a median plane and the plane of an adjacent current loop.

Taking account of the constancy of the kinetic energy and of the magnetic moment, the action integral may be written as

$$J = mv\int\left(1 - \frac{H}{H_0}\sin^2\delta\right)^{1/2} dl,$$

where mv is the momentum, H_0 is the initial field magnitude, and δ is the initial angle that the velocity makes with the field direction. For a given value of μ, this integral is calculated as a function of the radial position of the starting point in the median plane, M. The integrals are stopped either at the turning point where $v_\parallel$ has decreased to zero or at the plane of the adjacent coil if there is no turning point.[8]

Figure 3 shows the results of the action integral calculations for seven values of μ and also some of the field magnitude calculations that

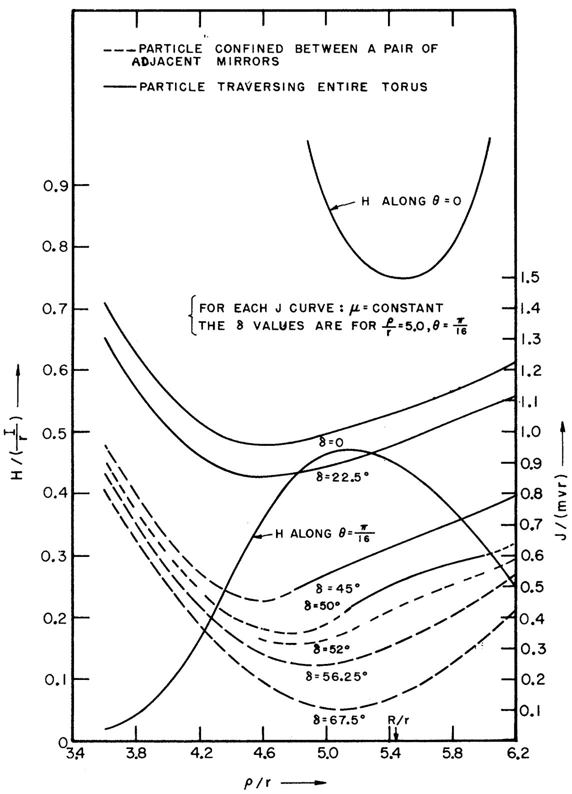

FIG. 3. Adiabatic invariants and field magnitudes for a bumpy torus with $N = 16$ and $R/r = 5.44$.

have been obtained for the case of $N = 16$ and $R/r = 5.44$. All of the action integral curves have a minimum (for $J \neq 0$). Choosing a particular curve of μ = constant, then for a chosen value of J the two values of ρ (inner and outer), where the precessional surface intercepts the plane of symmetry S, are uniquely determined. It is most unlikely that these surfaces are not closed outside of the S plane; however, a 3-dimensional calculation is required for a rigorous proof.

For the particular case of precession in the median plane M, $J = 0$ and the intercepts of the precessional surface are not defined. However, as shown above, the precession for this special case is constrained to a path along which H = constant. The plot of H vs ρ/r in the plane S has a maximum (this must be a maximum for displacements out of the plane S also). Thus, inner and outer intercepts for the precessional curve (which must be closed) with the plane of symmetry S may be found from the curve of H vs ρ in the median plane. In the limit as μ approaches the value characteristic of a particle trapped in the median plane (holding the kinetic energy constant), the intercepts predicted by the J curves appear to approach those predicted by the median-plane H curve as they should.

In case $v_{\parallel}$ is nearly zero at a mirror (transition from dashed to solid lines in Fig. 3), J does not necessarily remain constant in time (see footnote[8]). Such a particle has $v_{\perp} \simeq v = \text{constant}$ and so follows a closed precessional curve for which $H = \text{constant}$ in the plane of the coil (i.e., at $\theta = 0$).

Another special case is for $\mu = 0$. In this instance J is proportional to the length of the field line (over the path of integration), and since J is shown to have a minimum value in the S plane this is certainly a minimum with respect to displacements out of the S plane also. Hence, these precessional surfaces are closed.

Scattering can change the value of μ suddenly so that a vertical transition occurs in Fig. 3 from one of the $\mu = \text{constant}$ curves to another at a nearly constant ρ. After the transition the particle is constrained to the new precessional surface associated with the new values of μ and J until another scattering occurs. Since the J vs ρ curves do not all have the same shape and since their minima do not all occur at a single value of ρ, scattering can result in a new type of diffusion, that is, the particles can diffuse onto larger and larger diameter precessional surfaces. (This diffusion is independent of the orbit diameter.) It is planned to investigate different values of N and R/r. Increasing N and R/r proportionally should enable one to approach the degenerate case of a linear series of mirror machines and hence reduce the effect of this new type of diffusion. Initial results with $N = 4\times 16$ or 64 and $R/r = 4\times 5.44$ or 21.76 indicate that the minimum number of 90° changes in pitch angle of the velocity that can change the diameter of the precessional surfaces by an amount equal to the coil radius is approximately three for particle positions such that the distance from the axis is $\geqslant r$. Hence it is expected that the mean time for this change is about $3^2\tau_S$, or $9\tau_S$.

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[8]The justification for this assumption is that it makes J approach the same value in two limiting cases (holding the kinetic energy and the magnetic moment constant): (1) for a particle that is trapped between two adjacent mirrors as the turning point approaches the mirror, and (2) for an untrapped particle as $v_{\parallel}$ at the mirror approaches zero. A more detailed analysis of this case should be made because the longitudinal period becomes very long as the above limit is approached, and the action integral theorem is not proven for this case.