EFFECT OF MASS SPLITTINGS ON THE CONSERVED VECTOR CURRENT*

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In order to explain the apparent lack of renormalization of the vector β-decay coupling constant, Feynman and Gell-Mann and Gerstein and Zeldovich have suggested the idea of a conserved vector current.[1] The proposed current is strictly conserved only in the approximation in which one neglects small deviations from isotopic spin conservation such as those originating from multiplet mass splittings and from the usual electromagnetic corrections. At present, the effect of these deviations is of considerable interest in view of the existence of an apparent small discrepancy between the beta and muon decay coupling constants.[2] It is the aim of this note to prove some theorems, valid to all orders in the strong coupling, concerning the effect of these mass splittings on the effective vector β-decay matrix element.

These corrections can most easily be calculated by writing an "effective" Lagrangian which consists of the usual free and strongly interacting charge-independent boson-fermion Lagrangian, L_0, the weak decay interaction L^w, plus terms involving operators which represent the multiplet mass splittings (and which, when taken between physical states, give the experimentally observed mass differences). These mass operators may be interpreted as arising from interactions which violate charge independence, such as the electromagnetic coupling, and represent the clothing by such a field of the fermion and boson propagators.[3] The total Lagrangian may be written

$$L = L_0 + L^w + L_1 + L_2,$$

$$L_1 = \tfrac{1}{2}\delta m_{n,p}\bar{N}\tau_3 N + \tfrac{1}{2}\delta m_{\Xi^0,\Xi^-}\bar{\Xi}\tau_3\Xi + \tfrac{1}{2}(\delta m^2)_{K^0,K^+}\bar{K}\tau_3 K + \tfrac{1}{2}\delta m_{\Sigma^-,\Sigma^+}\bar{\Sigma}T_3\Sigma,$$

$$L_2 = (\delta m^2)_{\pi^0,\pi^\pm}\pi^\dagger(T_3^{\,2} - \tfrac{1}{3}T^2)\pi + \tfrac{1}{2}(\delta m_{\Sigma^0,\Sigma^+} + \delta m_{\Sigma^0,\Sigma^-})\bar{\Sigma}(T_3^{\,2} - \tfrac{1}{3}T^2)\Sigma, \qquad (1)$$

where $\delta m_{ab} = m_a - m_b$, $(\delta m^2)_{ab} = m_a^{\,2} - m_b^{\,2}$. It should be noted that in isotopic spin space, L_0, L_1, and L_2 behave as spherical harmonics $Y_0^{\,0}$, $Y_1^{\,0}$, and $Y_2^{\,0}$, respectively.

The weak-interaction Lagrangian will be expressed in the form $L^w = gj_\mu^\dagger j^\mu$, where j^μ is the strangeness-conserving part of a conserved charged vector current. For the following theorems, we will assume that the part of j^μ constructed from the strongly interacting fields satisfies the condition[4]

$$\mathcal{G} j^\mu \mathcal{G}^{-1} = j^\mu, \qquad (2)$$

where $\mathcal{G} = \mathcal{C}e^{i\pi T_2}$. It is not necessary to specify further the form of j_μ. In momentum space, the most general vector matrix element for β decay arising from L is of the form

$$M = [\bar{u}(p)\Gamma^\mu(q, \delta m_1, \delta m_2)u(p')][\bar{u}_e\gamma_\mu(1+\gamma_5)v_\nu], \qquad (3)$$

where Γ^μ is the vertex operator

$$\Gamma^\mu = (a\gamma^\mu + bq^\mu + c\sigma^{\mu\nu}q_\nu)\tau_+,$$

and p' and p are the four-momenta of the neutron and proton, $q = p' - p$ is the four-momentum transfer, and δm_1 and δm_2 denote the various mass differences which appear in L_1 and L_2, respectively. The quantities a, b, and c are invariant functions of q^2 and of δm_1 and δm_2. The first theorem states that a and c must be even functions and b an odd function of δm_1. In order to prove this statement let us first observe that under the transformation $\mathcal{G}$, L is changed only in that the sign in front of L_1 is reversed.

From this, it follows that under this transformation

$$\Gamma^{\mu}(q, \delta m_1, \delta m_2) \rightarrow \Gamma^{\mu}(q, -\delta m_1, \delta m_2). \tag{4}$$

On the other hand, it is possible to show by direct evaluation that under $\mathcal{G}$

$$\Gamma^{\mu} \rightarrow G\Gamma_{\mu}^{\ T} G^{-1}, \tag{5}$$

where $G = Ce^{i\frac{1}{2}\pi\tau_2}$. Comparing Eqs. (4) and (5) and remembering Eq. (3), we find that a and c are even functions and b is an odd function of δm_1.

The second theorem states that the contributions to M of first order in any of the δm_2 (but of zero order in the δm_1 and for $q_{\mu} = 0$) is zero to all orders in the strong coupling. In order to prove this statement we first note that the above contributions to the matrix element may be written in the Heisenberg representation in the form

$$M^{(1)} \sim \int d^4x d^4y \langle p | T\{j^{\mu}(x) L_2(y)\} | n\rangle \times e^{iq\cdot x}[\bar{u}_e \gamma_{\mu}(1+\gamma_5)v_{\nu}], \tag{6}$$

where $|n\rangle$ and $|p\rangle$ are eigenstates of the total charge-independent Hamiltonian H_0 derived from L_0.

Now we observe that $U = \int d^3x\, j^0(x)$ is a constant of the motion because j^{μ} is conserved and therefore commutes with H_0. Therefore, U acting on either $|p\rangle$ or $|n\rangle$ generates a linear combination of eigenstates of H_0 corresponding to the nucleon energy.[5] (We assume that the only possible eigenstates of this energy correspond to $i = \frac{1}{2}$.) Now, setting $q = 0$ and going to the nucleon rest frame, we notice that only the zero component of j^{μ} gives a nonvanishing contribution and $\int j^0 e^{iq\cdot x} d^3x$ reduces to U. Remembering the above-mentioned property of U and the fact that L_2, behaving as $Y_2^{\ 0}$, cannot connect two isotopic spin $\frac{1}{2}$ states, we see that $M^{(1)} = 0$.

From the two previous theorems, we conclude that the corrections to the effective vector β-decay matrix element due to the various multiplet mass splittings may be expected to be typically of the order $(\delta m_{np}/m_N)^2 \sim 10^{-6}$. Although the corresponding corrections in O^{14} may be perhaps larger by a factor of ten because of the greater energy difference between initial and final nucleon energies, the previous results indicate that these effects may be completely neglected in determining the ratio of the muon- and β-decay coupling constants.

In a spirit of conjecture, it is perhaps interesting to observe that if it is feasible to construct appropriate strangeness-nonconserving currents which are conserved in the limit of neglecting certain baryon mass differences, then the possible existence of theorems similar to those above would indicate that the renormalization effects would be of higher order in the mass splittings than might be originally expected.

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[3] R. E. Behrends and L. Landovitz, Phys. Rev. (to be published).

[4] For a discussion of possible generalizations of this condition, see S. Weinberg, Phys. Rev. 112, 1375 (1958).

[5] See S. Okubo, Nuovo cimento 13, 292 (1959).