solving the problem of long-time magnetic confinement of a hot plasma.

We wish to acknowledge many helpful discussions with F. Coensgen, S. Colgate, A. Kaufman, and W. Newcomb, in addition to the past contributions of many others of the Mirror Machine group in the experimental work.

---

*Work done under the auspices of the U. S. Atomic Energy Commission.

[1]I. Bernstein, E. Frieman, M. Kruskal, and R. Kulsrud, Proc. Roy. Soc. (London) A244, 17 (1958).

[2]M. Rosenbluth and C. L. Longmire, Ann. Phys. 1, 120 (1957).

[3]J. Johnson, C. Oberman, R. Kulsrud, and E. Frieman, Phys. Fluids 1, 281 (1958).

[4]J. Berkowitz, H. Grad, and H. Rubin, Proceedings of the Second United Nations International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1958 (United Nations, Geneva, 1959), Vol. 31, p. 177.

[5]R. F. Post, reference 4, Vol. 32, p. 245.

[6]F. Coensgen, F. C. Ford, and R. E. Ellis, reference 4, Vol. 32, p. 266; and reference 5.

[7]A. Garren, R. J. Riddell, L. Smith, G. Bing, L. R. Henrich, T. G. Northrop, and J. E. Roberts, reference 4, Vol. 31, p. 65.

[8]L. Spitzer, Physics of Fully Ionized Gases (Interscience Publishers, New York, 1956).

[9]W. A. Perkins, R. E. Ellis, and R. F. Post, American Physical Society Bulletin for Division of Plasma Physics Meeting, Monterey, Paper B9, p. 8, December, 1959 (unpublished).

[10]E. Gerjuoy and M. Rosenbluth (to be published).

[11]E. Frieman and R. Kulsrud, Advances in Applied Mechanics (Academic Press, New York, 1958), Vol. 5.

[12]M. Kruskal and C. Oberman, reference 4, Vol. 31, p. 137.

[13]M. Rosenbluth and N. Rostoker, reference 4, Vol. 31, p. 144.

[14]W. Newcomb, Ann. Phys. (to be published).

[15]R. F. Post, University of California Radiation Laboratory Report UCRL-5524, July, 1959 (unpublished); Proceedings of the Conference on the Theoretical Aspects of Controlled Fusion Research, Gatlinburg, Tennessee, April 28-29, 1959 [Atomic Energy Commission Report TID-7582 (unpublished)].

[16]E. Harris, Phys. Rev. Letters 2, 34 (1959).

[17]R. F. Post, Annual Review of Nuclear Science (Annual Reviews, Inc., Palo Alto, 1959), Vol. 9, p. 416.

[18]E. Harris, Proceedings of the Conference on the Theoretical Aspects of Controlled Fusion Research, Gatlinburg, Tennessee, April 28-29, 1959 [Atomic Energy Commission Report TID-7582 (unpublished)].

FAR INFRARED ELECTRON-IONIZED DONOR RECOMBINATION RADIATION IN GERMANIUM

S. H. Koenig and R. D. Brown, III

Watson Laboratory, International Business Machines Corporation, Columbia University, New York, New York

(Received January 13, 1960)

Long-wavelength infrared radiation (~100 μ) resulting from the recombination of "hot electrons" in the conduction band of germanium with ionized donor impurities has been detected by observing the photoconductive response of a second sample of germanium. The experimental arrangement is shown in Fig. 1. All measurements were made at 4.2°K. The generator G (the sample producing the radiation) is biased at various points in the breakdown region,[1] so that the carrier density and therefore the photon output will be large. The photodetector D is below and separated from G by a Teflon window (transparent at these wavelengths[2]) so as to minimize convection currents in the helium.

No attempt was made (see Fig. 1) at a focussing geometry since everything in the cavity is either transparent or highly reflecting for 100μ radiation[2] except for the detector which is black at 100 μ and the generator which is "grey." For these conditions, a simple ergodic argument shows focussing to be unnecessary. (An earlier, unsuccessful, search for the recombination radiation, using a focussing geometry, has been reported by Braunstein.[3])

The current-voltage characteristics for a 1-ohm-cm As-doped detector for various values of current through an Sb-doped generator G are shown in Fig. 2. The experiment has been repeated with all possible permutations of Sb-doped (activation energy[4] $\epsilon = 0.0097$ ev) and As-doped ($\epsilon = 0.0127$ ev) germanium as the generator and detector. The magnitude of the photoconductive response of D is relatively insensitive to the doping agent of G, whereas the fractional response of an As-doped receiver is ~50 times greater than that of a similar Sb-doped detector.

The fact that an Sb-doped generator (with its lower value of ϵ) can induce photoconductivity in an As-doped detector indicates that the radia-

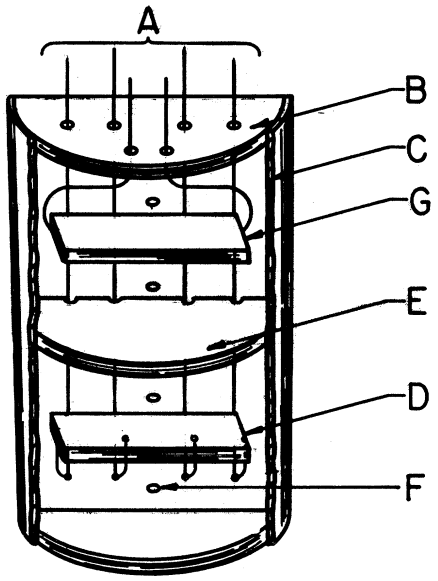

FIG. 1. Sketch of sample holder. A, Teflon-insulated copper leadwires; B, silver-slated copper chamber; C, cutaway of silver-plated cover; D, detector; E, Teflon window; F, inlet for helium; G, generator. The entire holder is inserted inside another light-tight container.

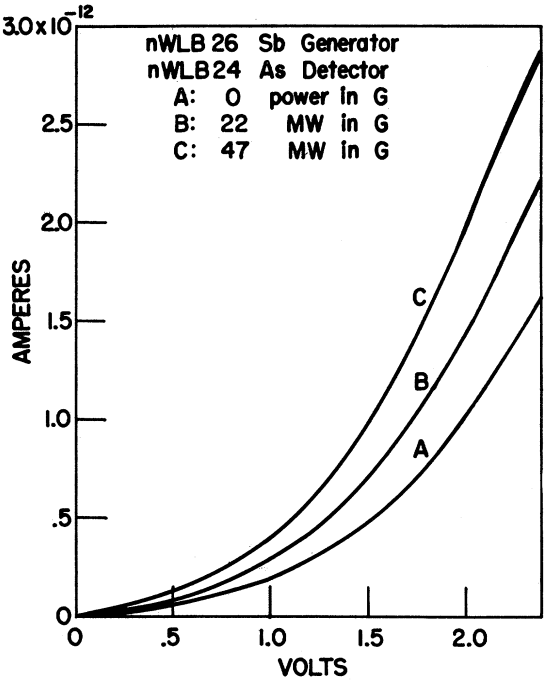

FIG. 2. A tracing of a recorder plot of the current-voltage characteristics of an As-doped detector for several values of power (in milliwatts) in an Sb-doped generator. The thickness of the tracing represents the peak to peak noise. Further data on these samples are given in reference 1.

tion must be due to recombination between reasonably "hot" electrons ($\sim 10kT$) and the donor ground state(s) (or else between unreasonably energetic electrons and an excited donor state).

There is the obvious implication here that by studying the relative response of Sb- and As-doped detectors as the electron "temperature" of G (Sb- or As-doped) is varied, it should be possible to obtain information relating to the shape of the electron distribution function in G. However, varying the bias point in the "breakdown" region of G (which may at first seem the obvious thing to do) only changes the carrier density without significantly altering the distribution (since the electric field varies very little in this range). Biasing G beyond breakdown to obtain a greater electron temperature, on the other hand, requires a prohibitively large power dissipation. It is possible, though, by using samples with differing amounts of compensating impurities, to have the breakdown occur at different electron "temperatures."[5] These experiments have yet to be done.

The sensitivity of the detector as a function of electric field E can be computed by using Price's[6] kinetic equations. If A_O is the optical generation rate of carriers in D per donor, A_T the thermal generation rate, A_I the impact ionization rate, and B_T the single electron-ionized donor recombination rate, the expression for n, the carrier density, valid for the range of Fig. 2 is

$$(A_O+A_T)(N_D-N_A)+nA_I(N_D-N_A)-nB_TN_A=0; \quad (1)$$

$$\Delta n/n_0 \equiv (n-n_0)/n_0 = A_O/A_T,$$

where n_0 is the "dark" value of n, and N_D and N_A are the donor and acceptor concentrations, respectively. Equation (1) assumes that A_O is uniform throughout the detector, which will of course be true only in the limit $\alpha t \to 0$, where α (in cm^{-1}) is the absorption coefficient of D (proportional to $N_D - N_A$) and t the thin dimension of D. The data of Fig. 2 were taken with a photodetector which was lapped down from an initial value of $t \gg \alpha$ to one such that $t(\sim 0.3$ mm) $\lesssim \alpha$, so that Eq. (1) is appropriate for this case. The value of α estimated from the measurement

of the efficiency of the infrared detection for various values of t agrees with the values reported by Fisher and Fan.[7]

From Eq. (1), it is seen that the sensitivity $(\Delta n/n_0)$ varies inversely as A_T, and therefore as $\exp(-\epsilon/kT)$. The relative sensitivities of similar Sb- and As-doped detectors at 4.2°K should then be in the ratio $\exp[(\epsilon_{Sb}-\epsilon_{As})/kT] \sim 600$, assuming A_O the same for both. This factor of 600 is larger than the observed relative response (see above) of similar Sb- and As-doped detectors. It is felt that the difference is due to the fact that the As detector is responding to a smaller integrated portion of the emitted spectrum (because of its greater value of ϵ) than the Sb detector. Equation (1), in addition, predicts that $\Delta n/n_0$ should be independent of electric field (on the assumption that optically generated carriers quickly come into equilibrium with the rest of the distribution). The data of Fig. 2 are in agreement with this.

The detected power is exceedingly small. From the measured resistivity of the detector of Fig. 2 (2.5×10^{10} ohm cm), the estimated compensation and mobility[1] ($\sim10^5$ cm^2/volt sec), and from the known value of B_T,[8] A_T is estimated to be 1.5×10^{-4} (donor cm^3 sec)$^{-1}$. Knowing the detector volume and the value of $(N_D - N_A)$ (from room temperature resistivity), one estimates that $\sim10^9$ photons/sec or $\sim1.5\times10^{-12}$ watt[9] (with a net uncertainty of a factor ~2) are being detected. A few percent of this power, judging from Fig. 2, would be the limit of detectability for the present arrangement.

It is somewhat more difficult to estimate the generated power. The optical recombination cross section has been calculated by Lampert[10] using a hydrogenic model, and discussed independently by Burstein, Picus, and Sclar.[11] Though both estimate a somewhat different cross section for the (energy-dependent) capture process, a reasonable compromise appropriate to the present situation is $\sigma\sim10^{-22}$ cm^2. This corresponds to one recombination in 10^9 being radiative.[12] For the highest power curve of Fig. 2, one estimates, from the known values of N_A (number of ionized donors), conductivity, and volume, that $\sim2\times10^9$ photons/sec or $\sim3\times10^{-12}$ watt are being generated. The value of αt for the generator is ~0.1 so that the majority of the photons generated should escape (even allowing for several internal reflections). One may say, then, from the numbers, that most of the photons generated in the "grey" generator are being detected by the "black" photodetector. Consistent with this, we find that if a fraction of the cavity wall is painted with Aquadag, the detector response decreases.

A final point should be mentioned. Since the detected power is so small, an argument must be made to eliminate heating of G as the major source of the effect. For example, were G at 5°K, the integrated blackbody radiation sufficiently energetic to produce a photoconductive response in D would be $\sim10^{-13}$ watt/cm^2 (the area of G is ~1 cm^2). Were D at 4.5°K, this number (which decreases exponentially with decreasing temperature in this range) would drop to $\sim10^{-15}$ watt/cm^2. Experimentally, the response of D as a function of P, the power input to G, is linear (rather than exponential). For G biased in the breakdown region, the carrier density is proportional to P since the mobility and electric field are essentially independent of the bias current. Therefore, the photon output of G and the photoresponse of D should be linear in P.

From the published values of heat flow across an interface to liquid helium,[13] G at the maximum power input of Fig. 2 should rise to less than 0.1° above the bath ambient of 4.2°K. As a further experimental check to assure that heating of G was not contributory to the response of D, G was replaced by a 0.1-watt carbon resistor, H, of the same area and volume. H is made of a central resistive core covered by an insulating (both thermally and electrically) plastic jacket, which is presumably transparent in the far infrared. When 50 mw is dissipated in H, the core heats to ~6°K (determined from the known temperature dependence of its resistance), and there is an $\sim6\%$ response of D. When the surface of H is painted with Aquadag to absorb any internally generated radiation, so that the only radiation from H will be from its surface, there is no detectable response of D. Since the germanium generator cannot support any significant thermal gradient internally, the above experiment on H conclusively demonstrates that heating of G is not responsible for the radiation reported here.

---

[1]See, for example, S. H. Koenig and G. R. Gunther-Mohr, J. Phys. Chem. Solids 2, 268 (1957).

[2]See, for example, R. A. Smith, F. E. Jones, and R. P. Chasmar, The Detection and Measurement of Infrared Radiation (Oxford University Press, New York, 1957).

[3]R. Braunstein, Signal Corps Report (unpublished).

[4]T. H. Geballe and F. J. Morin, Phys. Rev. 95, 1085 (1954).
[5]S. H. Koenig, J. Phys. Chem. Solids 8, 227 (1959).
[6]P. J. Price (unpublished).
[7]P. Fisher and H. Y. Fan, Bull. Am. Phys. Soc. 3, 128 (1958).
[8]S. H. Koenig, Phys. Rev. 110, 988 (1958).
[9]See W. S. Boyle and K. F. Rodgers, Jr., J. Opt. Soc. Am. 49, 66 (1959).
[10]M. A. Lampert, Signal Corps Report (unpublished).
[11]E. Burstein, J. S. Picus, and N. Sclar, Proceedings of the Conference on Photoconductivity, Atlantic City, November 4-6, 1954, edited by R. G. Breckenridge et al. (John Wiley and Sons, Inc., New York, 1956), p. 353.
[12]S. H. Koenig, Phys. Rev. 110, 986 (1958).
[13]A. Karagounis, Suppl. Bull. inst. intern. froid, Annexe 1956-2, p. 195.

INTERNAL IMPURITY LEVELS IN SEMICONDUCTORS: EXPERIMENTS IN p-TYPE SILICON*

Solomon Zwerdling, Kenneth J. Button, Benjamin Lax, and Laura M. Roth
Lincoln Laboratory, Massachusetts Institute of Technology, Lexington, Massachusetts
(Received December 29, 1959)

The concept of the existence of multiple sets of localized impurity levels, each associated with the various band extrema pertinent to its donor or acceptor character, has been verified by the observation of newly-found transitions to acceptor excited states in silicon lying near the split-off $p_{1/2}$ valence band maximum, within the continuum of the $p_{3/2}$ bands as shown in Fig. 1. The detection of such discrete internal impurity levels should provide information about the location of higher bands in energy-momentum space as well as about their characteristics and band parameters.

In the case of the impurity levels of the valence bands in silicon, transitions are possible between the ground state, which is primarily associated with the $p_{3/2}$ band (band a), and the excited states of the $p_{1/2}$ band (band b), as shown in Fig. 1. If we treat the coupling between the $p_{3/2}$ and $p_{1/2}$ bands as a perturbation, the transition probability from level 1 of band a to level 2 of band b has the form

$$W \propto \left[\vec{\Pi}'_{a(1)b(2)} + \frac{\mathcal{H}'_{a(1)b(1)}}{\mathcal{E}_{a(1)b(1)}}\vec{\Pi}_{b(1)b(2)} - \frac{\mathcal{H}'_{a(2)b(2)}}{\mathcal{E}_{a(2)b(2)}}\vec{\Pi}_{a(1)a(2)}\right]^2 \quad (1)$$

Here $\vec{\Pi}$ is a momentum operator between ground and excited states,[1] $\mathcal{H}'_{ab}$ is a matrix element of the effective-mass Hamiltonian connecting the two bands, and $\mathcal{E}_{ab}$ is the energy separation of the levels. Transitions are also allowed between the ground state of band b to an excited state of band a permitting, in principle, population inversion between state 2 and state 1 of band b or between state 1 of band b and state 2 of band a.

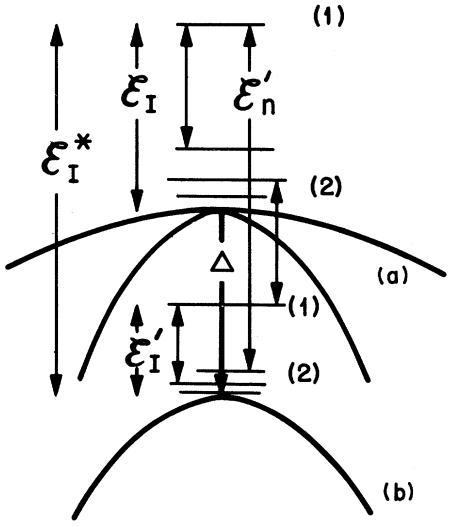

FIG. 1. Valence energy bands of silicon showing the acceptor ground state (1) and excited states (2) for both the degenerate $p_{3/2}$ bands (a) and the split-off $p_{1/2}$ band (b). The "internal" impurity levels are those nearest band b. The unlabeled arrows are also allowed transitions.

Optically-induced transitions of holes from the ground state in the forbidden gap to excited states associated with the split-off $p_{1/2}$ valence band were observed at 4°K in the wavelength range 10-15 microns using both boron- and aluminum-doped silicon. The spectra for boron-doped silicon in Fig. 2 show clearly-defined absorption maxima for transitions to the first few quantum levels, and the energies appeared to fit a Rydberg series for both boron and alumi-