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CONFINEMENT OF CHARGED PARTICLES BY PLANE ELECTROMAGNETIC WAVES IN FREE SPACE*

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A number of papers[1-9] have been written recently on the confinement of charged particles by oscillating electromagnetic fields within a cavity or waveguide. There appear to be two cases under which confinement can occur. Conditions for one case occur when ∇E^2 is normal to $\vec{E}$ and the magnetic field $\vec{B}$ is 90° out of phase with $\vec{E}$ so that $\vec{B}$ is in phase with the velocity of the particle.[3,8,9] The confinement then derives from the $\vec{v}\times\vec{B}$ term in the Lorentz formula. Conditions for the other case occur when ∇E^2 is parallel to $\vec{E}$, which results in confining forces associated with the nonuniformity of the electric field and bear a close relationship to strong-focusing forces.[3,7] Both conditions can be obtained in the region where two plane waves in free space intersect.

For the first case, let the waves intersect symmetrically about the y axis as shown in Fig. 1, except that the electric vectors are normal to the paper. Using the mks system, the electric vectors are given by

$$\vec{E}_j = \hat{k}E_0 \exp[i(\vec{k}_j\cdot\vec{r} - \omega_j t_j)], \tag{1}$$

where $\vec{k}_j = (\omega_j/u_j^2)\vec{u}_j$ and $\hat{k}$ is a unit vector in the z direction. Take $\omega_1 = \omega_2$ and $u_1 = u_2 = c$ and superimpose the electric vectors to obtain

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \hat{k}E_0 \exp[i(\beta_y - \omega t)] \times\{\exp(i\beta_x) + \exp[-i(\beta_x + \alpha)]\}, \tag{2}$$

where $\beta_x = (\omega/c^2)u_x x$ and $\beta_y = (\omega/c^2)u_y y$. We have taken $t_1 = t$ and $t_2 = t + (\alpha/\omega)$, where α is the phase difference between the two waves.

Similarly we superimpose the magnetic vectors to obtain

$$\vec{B} = (\vec{k}_1/\omega)\times\vec{E}_1 + (\vec{k}_2/\omega)\times\vec{E}_2,$$

$$\vec{B} = (E_0/u^2)\exp[i(\beta_y - \omega t)]\{(u_y\hat{i} - u_x\hat{j})\exp(i\beta_x) + (u_y\hat{i} + u_x\hat{j})\exp[-i(\beta_x + \alpha)]\}. \tag{3}$$

For the special case where $\alpha = \pi/2$, we find for $\vec{E}$ and $\vec{B}$ at $x = y = 0$,

$$\vec{E} = \hat{k}\sqrt{2}\,E_0[\cos(\omega t + \tfrac{1}{4}\pi) - i\cos(\omega t - \tfrac{1}{4}\pi)], \tag{4}$$

$$\vec{B} = \sqrt{2}\,(E_0/u)\{\hat{i}\cos\theta[\cos(\omega t + \tfrac{1}{4}\pi) - i\cos(\omega t - \tfrac{1}{4}\pi)] - \hat{j}\sin\theta[\cos(\omega t - \tfrac{1}{4}\pi) + i\cos(\omega t + \tfrac{1}{4}\pi)]\}. \tag{5}$$

The $\vec{B}$ vector is elliptically polarized while the $\vec{E}$ vector remains plane polarized. When the real part of $\vec{E}$ is zero, the real part of $\vec{B}$ has a

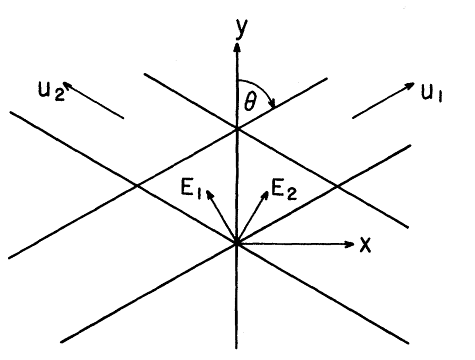

FIG. 1. Intersecting plane waves with electric vectors in plane of paper.

component in the minus y direction. From Eq. (2) it is evident that ∇E^2 is normal to $\vec{E}$. Thus the confinement conditions for the first case appear to exist.

For the second case, let the waves intersect symmetrically about the y axis as shown in Fig. 1, and let the electric vectors be given in the mks system as

$$\vec{E}_1 = (-x_0\hat{i}+y_0\hat{j})E_0 \exp[i(\vec{k}_1\cdot\vec{r} - \omega t)], \tag{6}$$

$$\vec{E}_2 = (x_0\hat{i}+y_0\hat{j})E_0 \exp[i(\vec{k}_2\cdot\vec{r} - \omega t - \alpha)], \tag{7}$$

where $x_0 = \cos\theta$ and $y_0 = \sin\theta$.

As before, we find

$$\vec{E} = E_0 \exp[i(\beta_y - \omega t)]\{(y_0\hat{j} - x_0\hat{i}) \exp(i\beta_x) + (y_0\hat{j}+x_0\hat{i}) \exp[-i(\beta_x+\alpha)]\}, \tag{8}$$

$$\vec{B} = \hat{k}(E_0/u) \exp[i(\beta_y - \omega t)]\{\exp(i\beta_x) + \exp[-i(\beta_x+\alpha)]\}. \tag{9}$$

The electric vector is now elliptically polarized, while $\vec{B}$ remains plane polarized.

We can now form $\nabla(\vec{E}\cdot\vec{E})$ and examine the result to see if it is parallel to $\vec{E}$. Because of the complexity of ∇E^2, we present here the result for the special case when $\theta = \frac{1}{4}\pi$. In this case

$$\nabla E^2 = \sqrt{2}\, E_0^{\,2}(\omega/u)i \exp[2i(\beta_y - \omega t)] \times\{\hat{i}[\exp(2i\beta_x) - \exp(-2i(\beta_x+\alpha))] + \hat{j}[\exp(2i\beta_x) + \exp(-2i(\beta_x+\alpha))]\}. \tag{10}$$

Taking the vector cross product of ∇E^2 with $\vec{E}$ and setting the result equal to zero, we find that the condition for ∇E^2 to remain parallel to $\vec{E}$ is

$$2\beta_x + \alpha = \tfrac{2}{3}n\pi, \tag{11}$$

where n is an integer with values 0, 1, 2...

Hence one would expect regions to exist within the intersecting volume where strong focusing occurs.

Detailed calculations of individual particle trajectories must still be made. It may be necessary to use multiple plane waves to block open ends which may exist in the volume wherein only two waves intersect.

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DURATION OF NUCLEOSYNTHESIS*

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In a recent Letter on a determination of the age of the elements, Reynolds[1] reported the important discovery of isotopically anomalous xenon in the stony meteorite Richardton. The isotopes which appear to occur in significant excess over atmospheric xenon are Xe^{128}, Xe^{129}, Xe^{130}, and Xe^{131}, with the Xe^{129} dominant by an order of magnitude. At present it does not appear possible to explain all of these data by any single mechanism. Because of the existence of