Splitting of the Giant Quadrupole Resonance in Light Deformed Nuclei

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The coupled monopole and β and γ quadrupole $T=0$ vibrations in sd-shell nuclei are studied within the generator-coordinate method, together with a scaling assumption. In contrast to heavier deformed nuclei, the splitting due to the nuclear deformation appears as the dominant mechanism in the spreading of the $E2$ strength. The available experimental data support our analysis.

It has now been clearly established[1] that the giant quadrupole resonance (GQR) in nuclei with mass $A>28$ lies around $63A^{-1/3}$-MeV excitation energy and exhausts most ($\gtrsim 60\%$) of the isoscalar energy-weighted sum rule (EWSR). In lighter systems, where the situation was more confused, recent α-scattering experiments have revealed the existence of a noticeable $E2$ strength (30–60%) between 12 and 32 MeV. In contrast with heavier nuclei, this GQR strength is not generally concentrated in a single peak but is spread over several states or clusters of states.

From a theoretical point of view, studies toward $E2$ strength location have been for the most part restricted to spherical nuclei (for a review see Ref. 1). Considerably less work has been done in deformed nuclei where a splitting of the GQR is expected in analogy with the dipole resonance. However in a well-deformed heavy nucleus, the quadrupole splitting, estimated to be nearly 2 MeV, will be masked by the damping width.[2-4] The data are indeed consistent with just a small broadening of the GQR due to the nuclear deformation.[1] As we shall see below things are different in a light deformed nucleus.

On the other hand, while the monopole and quadrupole vibrations are almost decoupled in spherical nuclei, this is no longer the case in the deformed ones. As a matter of fact it has been shown by two of us[5] that there exists a strong coupling between monopole and β-quadrupole modes in a light deformed nucleus such as ^{20}Ne. However the γ nonaxial mode which carries a significant fraction of the EWSR was not considered there.

The aim of this paper is precisely to study the coupling of all three vibrations (monopole and β and γ quadrupole) in sd-shell nuclei. Although interesting predictions can be made using just Inglis's approximation,[6] a complete study requires the use of the microscopic generator-coordinate method (GCM) treatment. This method indeed provides a convenient framework where anharmonicities of the potential-energy surface as well as nonadiabatic effects are taken into account.

In the generator-coordinate approach[7] the many-body wave function Ψ_n is described as a superposition of antisymmetrized generating functions $\Phi(x;\eta)$:

$$\Psi_n(x)=\int\Phi(x;\eta)f_n(\eta)d\eta,$$

where x stands for all variables of the nucleons and η represents a set of parameters. The amplitude $f_n(\eta)$ is determined from the Hill-Wheeler equation

$$\int[H(\eta',\eta)-E_nI(\eta',\eta)]f_n(\eta)\,d\eta=0$$

with the Hamiltonian and overlap kernels defined as $\langle\Phi(x;\eta')|H|\Phi(x;\eta)\rangle$ and $\langle\Phi(x;\eta')|\Phi(x;\eta)\rangle$.

The basic functions $\Phi_{JMK,\vec{P}=0}(x;\alpha,\beta,\gamma)$ used here to generate the vibrational states have been obtained by projection,[8] on both angular momentum J and center-of-mass momentum $\vec{P}=0$, from Slater determinants built by filling a deformed harmonic well with lengths b_x, b_y, and b_z. The volume parameter $\alpha=(b_xb_yb_z)^{1/3}$ generates the monopole mode, while $\beta=b_z/(b_xb_y)^{1/2}$ and $\gamma=b_y/b_z$

generate, respectively, the axial and nonaxial volume-conserving quadrupole oscillations. In this picture the nuclear wave functions are thus expressed as

$$\Psi_{JM,\vec{P}=0}{}^{n}(x)=\int d\eta \sum_K f_K{}^{n}(\eta)\Phi_{JMK,\vec{P}=0}(x;\eta).$$

The Hill-Wheeler equation has been solved, for each J value separately, by discretization allowing $\eta\equiv(\alpha,\beta,\gamma)$ to change in finite steps rather than continuously. The number of mesh points is chosen large enough to ensure a good convergence of the observables under consideration (energies, transition rates). As in Ref. 5, where more details can be found, all results presented here have been obtained using the Brink-Boeker $B1$ effective interaction.

It appears that the equilibrium shape of the nucleus (prolate, oblate, or triaxial) plays a major role in determining the pattern of the vibrational spectrum.

As shown in Fig. 1 for the axial nuclei ^{20}Ne and ^{28}Si, the quadrupole splitting mechanism in light nuclei is characterized by the following: (i) The large energy separation, nearly 8 MeV, between the β and γ modes. The sign of the shift is such that the β oscillation preserving axial symmetry is the lowest for prolate shape and the highest for oblate shape (such a result has already been obtained on other grounds by Bohr and Mottelson[9]).

(ii) The coupling between monopole and β vibrations which remains significant even when the energy shift between the uncoupled modes is large, as in ^{28}Si. In contrast, the coupling of the γ vibrations with the two other modes is less important, at least as far as the energies are concerned. However, a correct treatment of the γ mode is necessary, both for the $E2$ strength carried by such vibrations (respectively 34% and 57% of the EWSR in ^{20}Ne and ^{28}Si) and for the coupling which affects substantially the strength distribution, especially in a triaxial nucleus like ^{24}Mg.

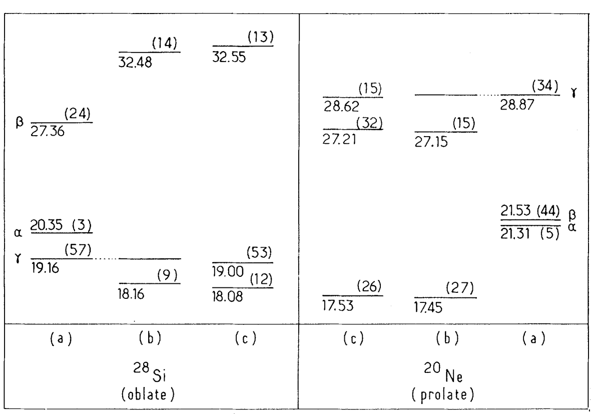

FIG. 1. Excitation energies (in MeV) and isoscalar $E2$ strengths (shown in parentheses in percent of the EWSR) of the lowest 2^+ vibrational states deduced from GCM calculations: (a) all three vibrations uncoupled; (b) monopole and β-quadrupole coupled, γ uncoupled; (c) all three modes coupled.

The $E2$ strengths deduced with all three vibrations coupled are shown in Fig. 2 (those of the low-lying rotational states which reproduce the data rather well are not reported).

Experimentally, the GQR in ^{40}Ca is found[1] at 18.0 ± 0.5 MeV, while GCM calculations with $B1$ predict it at 21.3 MeV. On the other hand, the GQR energy is known to be strongly dependent on the effective interaction.[17] For these reasons the theoretical energies in Fig. 2 have been shifted by 3.3 MeV downward to fit the ^{40}Ca data. Such a procedure, which also gives a good account of the GQR centroid in ^{16}O, will permit a more reliable comparison with experiment in open sd-shell nuclei.

As seen in Fig. 2, one observes striking varia-

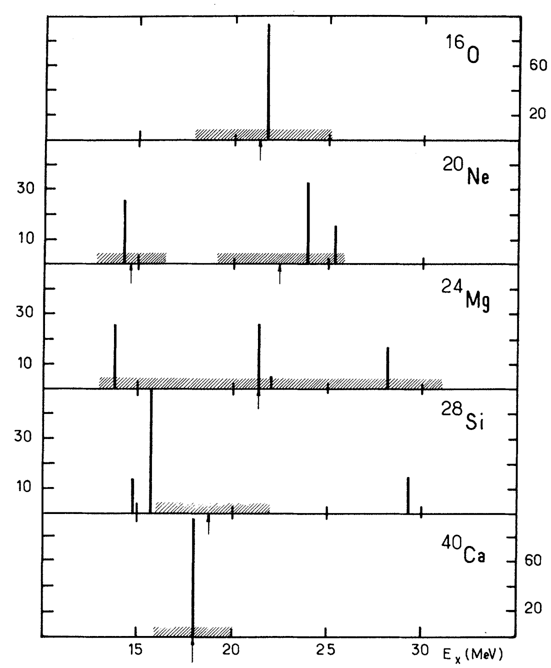

FIG. 2. Distribution of the isoscalar $E2$ strength (in percent of the EWSR) as a function of the excitation energy deduced from the GCM calculation. The arrows and hatched areas represent, respectively, the observed GQR centroids and the regions where noticeable strength is found experimentally (Refs. 1, 10–16). The calculated GCM energies have been shifted to fit the ^{40}Ca data.

tions in the strength distribution. Almost concentrated in a single collective state in spherical nuclei (^{16}O and ^{40}Ca), the GCM strength is clearly fragmented in deformed nuclei and the splitting, as discussed above, depends on the nature of the deformation.

The bulk properties of the observed GQR shapes in ^{20}Ne, ^{24}Mg, and ^{28}Si are consistent with the predicted behavior, and support the dominant influence of the deformation in the spreading of the strength. Of course, one must not forget that, in a collective picture like the GCM, the resulting states represent only centroids of the strength distribution[18] (mixing with continuum and discrete particle excitations will give decay width and additional fine structure).

In good agreement with our results, recent experiments[12] have revealed the splitting of the $E2$ distribution in ^{20}Ne into two distinct components: a high-energy component centered at 22.4 MeV ($\Gamma \sim 5.5$ MeV) and a low-energy component near 14.5 MeV ($\Gamma \sim 3$ MeV). These two components exhaust, respectively, 35% and 20% of the EWSR, while the GCM predicts 47% and 26%. Somewhat similar conclusions have been reached in an excited-core model.[19]

As expected from Fig. 2, the spreading of the GQR in ^{24}Mg observed in α-scattering experiments[13-15] is considerably larger than in other sd-shell nuclei. Indeed, (50–70)% of the EWSR has been located, widely distributed among several states between 12 and 31 MeV, with some clustering near 13, 18, and 25 MeV excitation energy. In comparison, the GQR strength is much more concentrated in ^{28}Si (essentially between 16 and 22 MeV[12,15,16]).

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