Mode Conversion of the Fast Magnetosonic Wave in a Deuterium-Hydrogen Tokamak Plasma[a]

J. Jacquinot,[b] B. D. McVey, and J. E. Scharer
Electrical and Computer Engineering Department, University of Wisconsin, Madison, Wisconsin 53706
(Received 12 November 1976)

Mode conversion from the fast magnetosonic wave to a slow wave near the two-ion hybrid resonance is shown to explain recent experimental fast-wave damping results. A model for tunneling and mode conversion of the fast wave in the two-ion resonance zone incorporating $k_{\parallel}$ and plasma-density and magnetic-field profiles is used to explain the observations. The strong dependence of the absorption on $k_{\parallel}$ and the species concentration which is obtained has important consequences for major plasma-heating programs which are planned for tokamaks.

Recent experiments on the TFR,[1,2] ATC,[3] TMO1,[4] and T-4[5] tokamaks have measured the damping of the fast wave as the resonance layer for the second ion-cyclotron harmonic ($\omega = 2\omega_{cd}$) is scanned across the plasma cross section. When the second-harmonic resonance for deuterium is in the central plasma region, a strong absorption of the wave is noted. Wave absorption rapidly decreases outside two locations for the cyclotron harmonic resonance zone which are not symmetric with respect to the magnetic axis. A preliminary analysis[2] of the TFR experiment showed that the asymmetry could be explained by the presence of a two-ion hybrid resonance within the plasma due to a hydrogen-impurity component in the deuterium plasma. The experiments on the TMO1[4] and T-4[5] tokamaks found that the presence of even a small amount of hydrogen impurity can greatly increase the wave-absorption process and lower the eigenmode Q. We establish here that the damping rate of the fast wave can be attributed to a mode-conversion process in the two-ion hybrid-resonance layer which is very sensitive to the $k_{\parallel}$ value of the eigenmode. The result of a detailed analysis of the mode-conversion problem is in semiquantitative agreement with the observations.

Local dispersion relation, mode conversion, and its associated damping decrement.—We start with the wave equation

$$\vec{k}\times(\vec{k}\times\vec{E}) + (\omega/c)^2\overleftrightarrow{K}\cdot\vec{E} = 0,$$

where $\overleftrightarrow{K}$ is the hot-plasma dielectric tensor[6] derived from the coupled kinetic Vlasov-Maxwell equations. We further note that for the fast magnetosonic wave the wavelength transverse to the magnetic field is much larger than the ion gyroradius so that $\lambda_j = k_{\perp}^2\rho_j^2/2 \ll 1$. We then expand the Bessel functions which occur in each element of the 3×3 dielectric tensor to second order in their argument λ_j. The resulting tensor is incorporated in the dispersion relation obtained from the wave equation. The resulting matrix is expanded and terms are collected to second order in λ_j. We find from numerical computations that, at spatial locations outside the ion-cyclotron-harmonic resonances ($|\omega - n\omega_{ci}(R)| \gg k_{\parallel}v_i$), the dispersion relation is very well approximated by the following one where only the dominant terms are retained:

$$ak_{\perp}^4 + bk_{\perp}^2 + c = 0, \qquad (1)$$

where

$$a = (c/\omega)^2 S/K_{zz} - K_{xx}^1, \quad b = n_{\parallel}^2 - S,$$

$$c = (\omega/c)^2(n_{\parallel}^2 - L)(n_{\parallel}^2 - R),$$

$$K_{zz} = -(\omega_{pe}/k_{\parallel}v_e)^2 Z'(\omega/k_{\parallel}v_e),$$

$$K_{xx}^1 = \sum_i(\omega_{pi}^2 v_i^2/2\omega_{ci}^2) \times[(\omega^2-\omega_{ci}^2)^{-1} + (\omega^2 - 4\omega_{ci}^2)^{-1}].$$

S, R, and L are defined as in Stix,[6] $n_{\parallel}$ is the parallel index of refraction and Z' is the derivative of the plasma dispersion function.

We assume the $1/R$ variation in toroidal field and a parabolic density profile appropriate to tokamak conditions. Figure 1 shows that mode conversion occurs in the immediate vicinity of the position R_S where $n_{\parallel}^2 = S$. The large imaginary part of $k_{\perp}^2$ in the propagation region of the slow wave corresponds to a short spatial-damping length. Points of cutoff for the fast wave are well approximated by the position where $n_{\parallel}^2 = R(R_R)$ and $n_{\parallel}^2 = L(R_L)$. We note that the finite-temperature corrections change considerably the slow branch of the dispersion curve but that R_R, R_L, and R_S are almost identical to the cold-plasma case. Figure 2 shows the location of the R_S, R_R, and R_L surfaces in the plasma cross section. Inside the R_R surface (region 1) the fast wave is

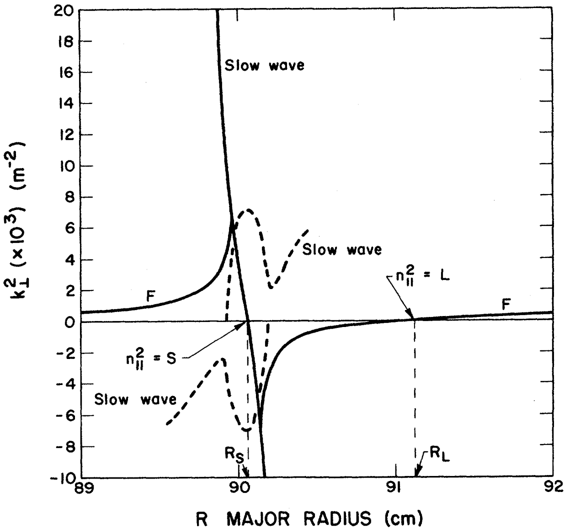

FIG. 1. Two-ion hybrid resonance zone, $n_e^{\text{peak}} = 5 \times 10^{13}\ \text{cm}^{-3}$, $n_h/n_d = 0.25$, $\nu = 61$ MHz, $T_d = T_h = 0.5$ keV, $T_e = 1.5$ keV, $R_c = 100$ cm, $k_{\parallel} = 15\ \text{m}^{-1}$; $\text{Re}(k_{\perp}^2)$, solid line; $\text{Im}(k_{\perp}^2)$, dashed line.

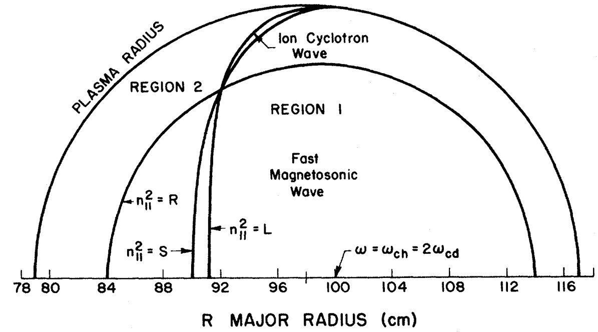

FIG. 2. TFR tokamak cross section (same parameters as Fig. 1).

propagating except in the crescent-shaped layer between R_S and R_L. On the contrary, outside the R_R surface (region 2) the ion cyclotron wave is evanescent everywhere except between R_S and R_L. Region 1 decreases with increasing $k_{\parallel}$; however, for the maximum value of $k_{\parallel}$ ($k_{\parallel\,\max}$) corresponding to the last eigenmode in the TFR torus, region 1 remains larger than region 2.

The amount of energy coupled to the slow wave can be calculated by extending the analysis of Swanson[7] to include a finite $k_{\parallel}$. The coefficients in Eq. (1) are expanded in a Taylor series about the mode-conversion point, R_s. For plasma parameters which vary slowly in space, a is approximately constant, and b and c can be approximated as linear functions of position. Inverse Fourier transformation yields a fourth-order differential equation for which an integral solution can be obtained by Laplace's method. Asymptotic analysis of the integral solution determines tunneling and absorption coefficients for the fast wave. Our results[8] can be cast in a form similar to Swanson's with his parameter $\beta \equiv 1$. For TFR, we find that the mode-conversion zone is spatially distant from the ion-cyclotron-harmonic resonance. In this case, Swanson's and our results reduce to the Budden form of Whittaker's differential equation[9]:

$$d^2E(x)/dx^2 + k_{\perp f}^{\infty 2}(1 - w/x)E(x) = 0, \qquad (2)$$

where w is the width of the evanescent zone, $w = |R_L - R_S|$, $k_{\perp f}^{\infty}$ is the asymptotic wave number, and x a position variable in the direction of increasing major radius. In terms of the Taylor-series expansion of the dielectric elements, $k_{\perp f}^{\infty}$ and w in our case are approximated by

$$k_{\perp f}^{\infty 2} = \frac{(d/dx)[(L - n_{\parallel}^2)(R - n_{\parallel}^2)]}{(d/dx)(S - n_{\parallel}^2)}$$

and

$$w = \frac{-(L - n_{\parallel}^2)(R - n_{\parallel}^2)}{(d/dx)[(L - n_{\parallel}^2)(R - n_{\parallel}^2)]},$$

where S, R, and L are all evaluated at R_s. The fraction of energy lost by the fast wave in passing through the mode-conversion zone is $A_R = \exp(-\pi\eta) - \exp(-2\pi\eta)$ for a wave from the low-field side and $A_L = 1 - \exp(-\pi\eta)$ for a wave from the high-field side, where $\eta = |k_{\perp f}^{\infty} w|$.

The above results do not take into account the boundary properties of the tokamak configuration. For the TFR parameters, $\pi\eta \lesssim 1$ and most of the fast-wave energy incident from either side tunnels through the mode-conversion and evanescent zones, establishing a standing wave. These effects are included to lowest order by considering averaged quantities: $\langle w \rangle = \frac{1}{2}w$(measured at the midplane), $\langle A \rangle = \frac{1}{2}(A_R + A_L)$, and $\langle k_{\perp f}^{\infty} \rangle = \frac{1}{2}(k_{\perp r} + k_{\perp l})$, where $k_{\perp r}$ and $k_{\perp l}$ are calculated in the propagation zones of the fast wave which are adjacent to the mode-conversion region. When $\pi\eta \gtrsim 1$, wave energy incident from the high-field side is strongly absorbed and that from the low-field side is largely reflected. The averaged quantities do not apply in this case and a different analysis is required.

Now let α be the damping decrement of the field amplitude of the fast wave as it propagates along the confining magnetic field in an infinite cylinder. Then $\alpha = P/2S$, where P is the fast-wave power absorbed via mode conversion and S the flux of the Poynting vector along the static magnetic field. Assuming a uniform wave intensity over the cross section, we obtain

$$\alpha = \langle A\rangle\langle k_{\perp f}\rangle / [4 k_{\parallel} a (1 + \omega^2/\omega_{cd}^2)], \qquad (3)$$

where a is the plasma radius. In a toroidal configuration, the wave field E_t is obtained from the calculation of the self-interference pattern of the wave as it decays exponentially during the rotations around the torus. For toroidal eigenmode conditions ($k_{\parallel}R_0$ = integer, where R_0 is the major radius of the magnetic axis) and field measurements at a port a major diameter away from the antenna, we find

$$E_t/E_0 = \sqrt{2}\,[\cosh(2\pi \alpha_t R_0) - 1]^{-1/2}, \qquad (4)$$

where E_0 is the field under the antenna in the infinite-cylindrical case and α_t is the sum of the damping decrements due to mode conversion (α) and other absorption mechanisms (α_0).

Comparison to TFR measurements in a deuterium plasma.—When the location of the harmonic cyclotron resonance (R_c) for deuterium is out of the plasma the damping decrement as measured from the width of the eigenmode was reported to be[1] $\alpha_t = \alpha_0 \simeq 9\times10^{-3}$ m^{-1}. α_t increased by more than an order of magnitude when $-0.2 \leqslant (R_c - R_0)/a \leqslant 0.9$. With use of Eq. (4), a similar increase of α_t can also be independently deduced from the observed variation of the wave amplitude versus R_c (Fig. 4). The striking features are the asymmetry of the curve with respect to the magnetic axis and the fact that α_t is an order of magnitude greater than the value expected from either second-harmonic damping or fundamental cyclotron absorption by a minority hydrogen component. It was also noted that the enhanced absorption profile was present over a large density range and that the phenomenon disappears at the first cyclotron harmonic in a hydrogen plasma. Finally, in the deuterium plasma, mass-spectrum analysis of gas composition immediately after each shot revealed an important hydrogen-impurity content. The hydrogen- to deuterium-density ratio was $n_h/n_d \simeq 0.2$.

In order to compare these results with the mode-conversion model, we first point out that the width of the evanescent zone w in region 1 depends strongly on the values of R_c and $k_{\parallel}$ (Fig. 3). The width vanishes for $k_{\parallel} \geqslant 23$ m^{-1}. Modes with $k_{\parallel} \simeq 15$ to 20 m^{-1} are mode converted in the central plasma region with a strong asymmetry with respect to the magnetic axis. The width drops much more sharply on the low-field side than on the high-field side because of the change of sign of the density gradient relative to the magnetic-field gradient. Modes with small values of $k_{\parallel}$ are heavily mode converted even when R_c is outside the plasma on the low-field side. The amplitude of the wave field deduced from w and Eqs. (3) and (4) is represented in Fig. 4. It is clear that the curves corresponding to $k_{\parallel} \simeq 15$ m^{-1} agree very well with the observed asymmetry of the wave-amplitude curve and yield the correct value of the enhanced damping.

Measurements of the $k_{\parallel}$ values of the eigenmodes generated in TFR have not been reported but $k_{\parallel}$ has been calculated by solving the boundary-

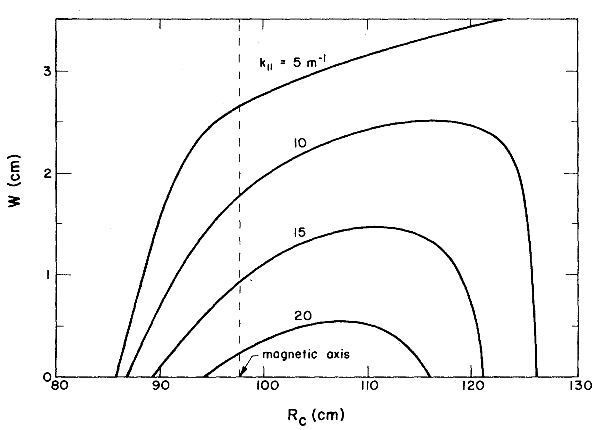

FIG. 3. Width of the evanescent zone $w = R_L - R_S$ in the midplane vs the position of $\omega = 2\omega_{cd}$. $n_e = 5\times10^{13}$ cm^{-3}, $n_h/n_d = 0.21$, $\nu = 60$ MHz.

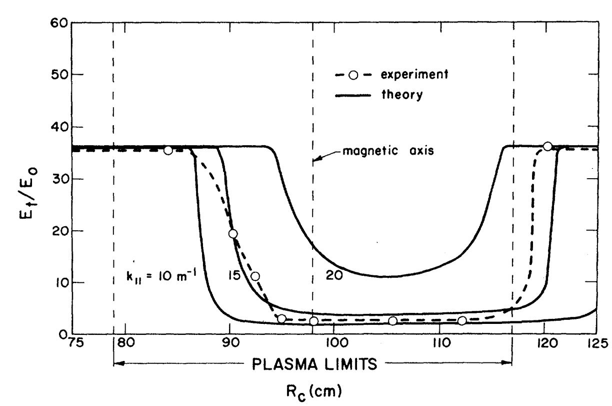

FIG. 4. Comparison of the theoretical and experimental wave amplitude for TFR. The sharp decrease in the central region is due to mode-conversion damping. Parameters are identical to those fro Fig. 3. $\alpha_0 = 9 \times 10^{-3}$ m^{-1}.

value problem for a fast wave propagating in an inhomogeneous plasma confined inside a cylindrical conducting wall.[2] For the conditions of Fig. 4, this model shows that the $m = 1$ mode has the largest possible value of $k_{\parallel}$ ($k_{\parallel\max}$), which is found to be $\simeq 18\ \mathrm{m}^{-1}$. Therefore, in agreement with the theoretical results, the $m = 1$ mode was the last one to survive mode conversion in TFR.

For a lower plasma density, the minimum value of $k_{\parallel}$ for elimination of mode conversion decreases. However, $k_{\parallel\max}$ also decreases accordingly so that the damping of the $m = 1$ mode remains unchanged, in agreement with the observations. The calculated damping has a broad maximum for $0.1 \lesssim n_h/n_d \lesssim 0.2$ and becomes comparable to the cyclotron damping only when $n_h/n_d \simeq \simeq 0.01$. Such a qualitative behavior has been observed in the TMO1 experiment.[4]

It has been demonstrated that the mode-conversion process near $n_{\parallel}^2 = S$ dominates the fast-wave absorption in present tokamaks at $\omega \simeq 2\omega_{cd}$ in a deuterium plasma with a minority hydrogen component. The predicted absorption agrees well with the experimental observation as the cyclotron harmonic resonance is scanned across the plasma cross section. The mode-conversion process is weakest for the mode with the highest possible $k_{\parallel}$ value consistent with eigenmode excitation. Strong damping from mode conversion has also been calculated for larger tokamaks.

The occurrence of mode conversion has the disadvantage of considerably reducing the antenna loading resistance ($Q \approx 50$ for TFR). On the other hand, mode conversion could offer the advantage of a well-defined absorption process dominating both cyclotron damping and recent experimental observation of other "parasitic" absorption mechanisms.[1,2] However, the evaluation of such a heating scheme requires detailed examination of the fate of the energy coupled to the slow wave.[10]

One of us (J.J.) would like to thank Dr. J. Adam for suggestions and encouragement during the course of this work.

[(a)]This work was supported in part by National Science Foundation Grant No. ENG 75-19259.

[(b)]Permanent address: Commissariat à l'Energie Atomique, Fontenay-aux-Roses, France.

[1]TFR Group, in *Proceedings of the Third International Conference on Theoretical and Experimental Aspects of Heating Toroidal Plasmas, Grenoble, France, 1976* (Commissariat à l'Energie Atomique, Grenoble, France, 1976), Vol. 1, p. 87.

[2]TFR Group, in Proceedings of the Sixth International Conference on Plasma Physics and Controlled Nuclear Fusion Research, Berchtesgaden, West Germany, 1976 (to be published), Paper IAEA CN35/G8.

[3]P. Studier, H. Hsuan, R. R. Smith, and H. Takahashi, Bull. Am. Phys. Soc. 21, 1157 (1976).

[4]V. L. Vdovin, N. V. Shapothovskii, and V. D. Rusanov, in *Proceedings of the Third International Conference on Theoretical and Experimental Aspects of Heating Toroidal Plasmas, Grenoble, France, 1976* (Commissariat à l'Energie Atomique, Grenoble, France 1976), Vol. 2, p. 349.

[5]V. V. Buzankin *et al.*, in Proceedings of the Sixth International Conference on Plasma Physics and Controlled Nuclear Fusion Research, Berchtesgaden, West Germany, 1976 (to be published), Paper IAEA CN35/G10.

[6]T. H. Stix, *The Theory of Plasma Waves* (McGraw-Hill, New York, 1962).

[7]D. G. Swanson, Phys. Rev. Lett. 36, 316 (1976).

[8]B. D. McVey, J. E. Scharer, and J. Jacquinot, Bull. Am. Phys. Soc. 21, 1158 (1976).

[9]K. G. Budden, *Radio Waves in the Ionosphere* (Cambridge Univ. Press, London, 1961), Chap. 17.

[10]F. W. Perkins, to be published.