# Mixing of States Containing Heavy Quarks

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I discuss the mixing of states containing heavy quarks with new quantum numbers. The mixing of the new states can be substantial in gauge theories with right-handed currents, even though direct neutral couplings conserve all flavors. Some experimental implications of the results are discussed.

During the past year there has been a good deal of interest in the existence of charm-changing neutral currents. Recent experimental evidence on the mixing of the $D^0$-$\bar{D}^0$ states settles this issue by providing a restrictive bound on the strength of direct charm-changing neutral couplings. The bound is comparable to the corresponding bound on strangeness-changing transitions.

The new evidence comes from electron-positron experiments[1] which search for the mixing of the produced $D^0$-$\bar{D}^0$ system. If there are $D^0 \leftrightarrow \bar{D}^0$ transitions, then in addition to the normal decays into $(K^+K^-h_1)$ there would also be decays into $(K^+K^+h_2)$ and $(K^-K^-h_3)$, where $h_1$, $h_2$, and $h_3$ are final states with total strangeness zero. Let us denote the number of decays into each system by $N^{+-}+N^{-+}$, $N^{++}$ and $N^{--}$, respectively. The experimental indications[1] are that

$$\rho = \frac{N^{++}+N^{--}}{N^{+-}+N^{-+}+N^{++}+N^{--}} \leqslant 15\% \tag{1}$$

at the 90% confidence level. When this is compared with an earlier result[2] on the mixing of the $D^0$-$\bar{D}^0$ states over a time interval long in comparison to the lifetimes of the mass eigenstates I obtain

$$\Delta m \leqslant \left(\frac{2\rho}{1-\rho}\right)^{1/2}(\Gamma_L+\Gamma_S), \tag{2}$$

where $\Delta m$ is the mass difference of $D_L$ and $D_S$ and $\Gamma_L$, $\Gamma_S$ their respective widths. Assuming an effective charm-changing interaction of the form

$$\mathcal{L}_{\rm eff} = \tfrac{1}{2}\sqrt{2}\, G\bar{c}\gamma^\mu(h_V+h_A\gamma_5)p\bar{c}\gamma^\mu(h_V+h_A\gamma_5)p, \tag{3}$$

I obtain[3]

$$h_V \text{ or } h_A \leqslant \left(\frac{2\rho}{1-\rho}\right)^{1/4}\times 10^{-3} = 6.8\times 10^{-4}. \tag{4}$$

This is to be compared with the corresponding bound in strangeness-changing transitions which is smaller only by a factor of 5.

The above results are encouraging enough to consider models with neutral currents diagonal in all flavor quantum[4,5] numbers. It is known that in such models $K^0$-$\bar{K}^0$ states mix maximally, while the $D^0$-$\bar{D}^0$ mixing is suppressed[6] by a factor of $\tan^4\theta_C$, $\theta_C$ being the Cabibbo angle. It is now of interest to ask whether states containing heavy quarks could mix to any appreciable extent over their lifetimes due to transitions induced by higher-order corrections. In theories with only left-handed currents and generalized Cabibbo universality the mixing is generally, but not always, small. This can be seen by considering, for example, a bound state of a heavy $b$ quark and a $d$ antiquark. Figure 1 shows a typical one-loop diagram, where the intermediate state $x$ is either another heavy quark, $t$, or a $u$ quark. In the case of the heavy quark the $\bar{t}d$ coupling is proportional to a small Cabibbo-like angle, $\varphi$, since universality is already saturated mostly by the $\bar{u}d\cos\theta_C$ coupling. Thus this case is similar to the $D^0$-$\bar{D}^0$ system with

$$\frac{\Delta m}{(\Gamma_L+\Gamma_S)} \approx \tan^2\varphi. \tag{5}$$

If on the other hand the intermediate state is a $u$ quark then the $\bar{u}b$ coupling is proportional to a small angle and in addition there is a suppression arising from the small mass of the intermediate state. Under the above assumptions there is still a rare circumstance resulting in substantial mixing. This would be the case if the total widths of

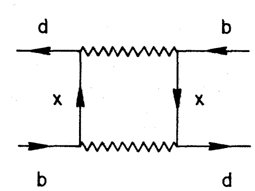


FIG. 1. A typical single-loop diagram for theories with left-handed currents.

the ($b\bar{d}$) states are also suppressed by $\sin^2\varphi$, as happens to be the case when all quarks with significant couplings to the $b$ quark are heavier.

The situation with small mixings can easily be reversed in models with right-handed currents and in particular, models with right-handed currents and a high-$y$ anomaly. In order to be specific I consider a prototype model,[7] whose symmetry group is $SU(2)_L \otimes SU(2)_R \otimes U(1)$ and the quark assignments

$$\begin{pmatrix} u \\ d_\theta \end{pmatrix}_L, \begin{pmatrix} c \\ s_\theta \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L, \quad \begin{pmatrix} u \\ b \end{pmatrix}_R, \begin{pmatrix} c \\ s_\varphi \end{pmatrix}_R, \begin{pmatrix} t \\ d_\varphi \end{pmatrix}_R \tag{6}$$

with $s_\varphi = s\cos\varphi - d\sin\varphi$ and $d_\varphi = d\cos\varphi + s\sin\varphi$. The right-handed currents introduce new contributions[8] to the $K^0$-$\bar{K}^0$ problem, which require

$$\sin^2\varphi \leq \left(\frac{m_c}{m_t}\right)^2 \sin^2\theta_C. \tag{7}$$

Thus for all practical purposes I can set $\varphi = 0$. Such a small value for $\varphi$ is also consistent with (i) the observed decays of charmed mesons and (ii) theoretical studies[9] of the $\Delta I = \frac{1}{2}$ rule. The quark assignments in (6) do not bring significant new terms into the $D^0$-$\bar{D}^0$ problem. The situation is quite different when we consider the lightest state containing a $b$ quark, that is the $b\bar{d}$ bound state to be denoted by $B^0$. With the strong interactions switched off there are two diagrams contributing to off-diagonal mass matrix elements, shown in Fig. 2. The internal line can be either a $u$ or a $t$ quark. In either case one of the vertices is left-handed and the other is right-handed. Consequently each internal fermion line involves one mass insertion.

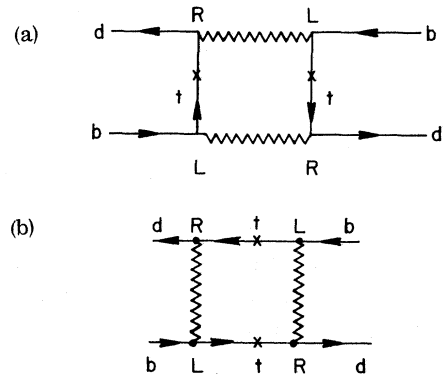


FIG. 2. One mass insertion diagrams in theories with both left- and right-handed currents.

In the approximation of zero external momenta, the effective interaction arising from diagram 2(a) is

$$\mathcal{L}_{\rm eff}{}^{\rm I} = (G^2/8\pi^2)[\ln(M_W/M_b) - 1]m_t{}^2\{\bar{d}(q_-)(1+\gamma_5)\gamma_\mu\gamma_\nu b(q_+)\bar{d}(k_+)(1+\gamma_5)\gamma^\nu\gamma^\mu b(k_-)\} + \text{H.c.}, \tag{8}$$

and a similar interaction arises from diagram 2(b)

$$\mathcal{L}_{\rm eff}{}^{\rm II} = (G^2/8\pi^2)[\ln(M_W/M_b) - 1]m_t{}^2\{\bar{d}(q_-)(1+\gamma_5)\gamma_\mu\gamma_\nu b(k_-)\bar{d}(k_+)(1+\gamma_5)\gamma^\nu\gamma^\mu b(q_+)\} + \text{H.c.} \tag{9}$$

In the case of $m_t < m_b$ an absorptive term also arises from diagram 2(b), whose presence does not affect the qualitative features of the conclusions. The two terms quadrilinear to the quark fields are related by the Fierz rearrangement theorem. Both terms (8) and (9) contribute to off-diagonal elements of the mass matrix. In estimating their contributions I use the bag model[10] where

$$\langle B^0|\bar{d}(1+\gamma_5)\gamma_\mu\gamma_\nu b\bar{d}\gamma^\nu\gamma^\mu b|\bar{B}^0\rangle = -8\langle B^0|\bar{d}(1+\gamma_5)\gamma_\mu b\bar{d}(1+\gamma_5)\gamma^\mu b|\bar{B}^0\rangle = 16\langle B^0|\bar{d}(1+\gamma_5)b\bar{d}(1+\gamma_5)b|\bar{B}^0\rangle. \tag{10}$$

Following Lee, Primack, and Treiman[11] I evaluate the matrix elements by inserting the vacuum intermediate state between the $\Delta B = 1$ vector currents

$$\langle B^0|[\bar{b}\gamma_\mu(1-\gamma_5)d]^2|\bar{B}^0\rangle \approx 4\,\frac{(f_B m_B)^2}{2m_B} \tag{11}$$

and equating the decay coupling constants of different flavors

$$f_\pi \approx f_K \approx f_D \approx f_B, \tag{12}$$

as would be the case in the limit of unbroken hadronic symmetry. Several of the above steps are model dependent, expecially the step of keeping only the contribution of the vacuum state. The corrections to this analysis are difficult to estimate in the absence of a more complete theory, but the successes of similar estimates in the $K^0$-$\bar{K}^0$ and $D^0$-$\bar{D}^0$ systems are encouraging. The above assumptions lead to the following off-diagonal mass matrix element

$$M_{12} = (G^2/\pi^2)m_t{}^2[\ln(m_W/M_b) - 1]2f_\pi{}^2 m_B. \tag{13}$$

In the quark model the main contribution to the nonleptonic decay of the $B^0$ ($b\bar{d}$) state is given by the elementary process $b \to u + d + \bar{u}$. Combining Eq. (4) with the estimate of the total width I obtain

$$\rho \geq 0.31 \qquad (14)$$

for $m_b \lesssim m_t$, $m_b = 5\ \mathrm{GeV}/c^2$, $m_W = 70\ \mathrm{GeV}/c^2$, and $f_\pi = 0.9 m_\pi$. Similarly, if one considers the time development of a single $B^0$ state, then in addition to the normal decays into ($\mu^- \bar{\nu}$ hadrons) there are also decays into ($\mu^+ \nu$ hadrons). Denoting by $N^-$ and $N^+$ the number of decays into ($\mu^- \ldots$) and ($\mu^+ \ldots$), respectively, I find

$$\rho_1 = \frac{N^-}{N^- + N^+} = \frac{1}{2}\,\frac{4(\Delta m)^2 + (\Gamma_L - \Gamma_S)^2}{4(\Delta m)^2 + (\Gamma_L + \Gamma_S)^2} = \rho. \qquad (15)$$

The essential point in the previous considerations is the presence of a heavy quark with substantial couplings to the quarks which form the bound state. The same considerations hold for a ($t\bar{p}$) bound state, since the $b$ quark has substantial couplings to both of them. These features are in general present in gauge models with right-handed currents and special arrangements must be made in order to avoid them. Thus, in view of these considerations, I expect right-handed currents to be accompanied by new mixing phenomena.

As an application consider the high-$y$ anomaly. If the anomaly is genuine and it arises from the production of a new quark $b$, then in addition to the decay chain

$$\bar{\nu}N \to \mu^+ + B + \text{hadrons}, \quad B \to \mu^- + \bar{\nu}$$

I expect the occurrence of events,

$$\bar{\nu}N \to \mu^+ \mu^+ \nu + \text{hadrons},$$

arising from the development of a $\bar{B}^0$ component. Thus among the events attributable to the high-$y$ anomaly one expects to observe dimuon events of the opposite sign as well as dimuons of the same sign in the approximate ratio of 6:1. This provides an additional test for the presence or (most likely) absence of a high-$y$ anomaly.[12] Furthermore, associated production of $B^0$-$\bar{B}^0$ states by neutrinos leads to trimuon events with the charges $\mu^-\mu^+\mu^-$, $\mu^-\mu^-\mu^-$, and $\mu^-\mu^+\mu^+$ in the ratios of $\approx 4:1:1$. The overall rate is hard to estimate since the rate for associated production is unknown. Finally, associated production of $B^0$-$\bar{B}^0$ states in electron-positron collisions will produce phenomena analogous to those discussed at the beginning of this Letter.

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*Note added*—After submittal of this article I received a paper [J. Ellis *et al.*, CERN Report No. 2346, 1977 (to be published)] where mixing of $B^0$ and $\bar{B}^0$ is discussed in models with left-handed currents.

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# Measurement of $\varphi$ Production in Proton-Nucleus Collisions at 400 GeV/$c$

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We have measured the invariant cross section for inclusive $\varphi$ production in proton-nucleus collisions at 400 GeV/$c$ near Feynman $x = 0$. For transverse momenta in the range between 0.8 and 3.5 GeV/$c$ the ratio of $\varphi$ to $\pi^-$ rises from 1 to 7%. We also report on correlations with particles opposite the $\varphi$ in the center-of-mass system as they relate to the Okubo-Zweig-Iizuka rule.

We have observed the $\varphi$ in its $K^+K^-$ decay mode in 400-GeV/$c$ proton-nucleus collisions at the Fermi National Accelerator Laboratory. An accurate measurement of the production of a vector meson in high-energy hadronic interactions is possible with the $\varphi$ because its narrow width allows a clear separation from the background. This measurement is of interest for two reasons. First, measurement of the $\varphi$, a meson containing "hidden" strangeness, allows a test of the Okubo-Zweig-Iizuka (OZI) rule which in this case states that the production of the $\varphi$ should be suppressed unless accompanied by strange particles. Second, the level of prompt muons contributed by the decay $\varphi \to \mu^+\mu^-$ can be established. Measurements by others of $\varphi$ production have been described previously.[1,2]

In our experiment the $\varphi$ was detected when both decay kaons traversed the same arm of a two-arm spectrometer. This spectrometer has been described previously.[3] Briefly, it consisted of two identical arms each positioned at 100 mrad with respect to a diffracted proton beam. Each arm included a momentum-analyzing magnet, sixteen drift-chamber planes, trigger scintillation counters, three threshold Cherenkov counters, and, at the downstream end, muon identifiers. In most of the data collected subsequent to that reported in Ref. 3, the Cherenkov counter in each arm closest to the target was replaced by steel shielding with apertures for the beam and spectrometers. The function of the displaced counters was taken over by new Cherenkov counters positioned behind each analyzing magnet. Most of the data for this Letter were collected in the latter configuration. Data were collected for measurement of inclusive $\varphi$ production and for correlations with particles in the spectrometer arm opposite that in which the $\varphi$ was observed.

For the inclusive measurement, the effective-mass spectrum of neutral pairs within a single arm is displayed in Fig. 1 for those events whose

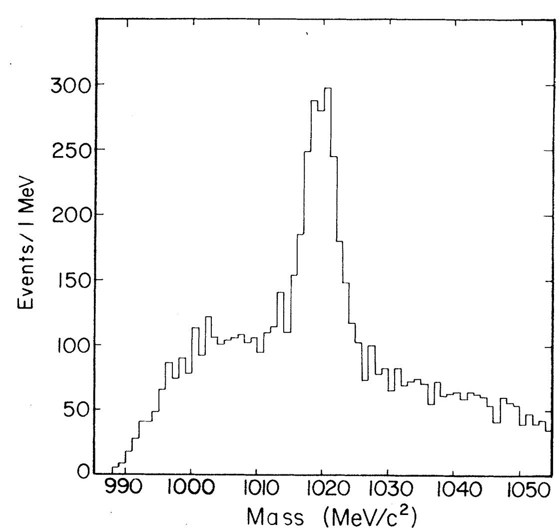


FIG. 1. Uncorrected effective-mass spectrum of $K^+K^-$ candidates in a single spectrometer arm obtained in the inclusive measurement.