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Absence of Inertial Induction in General Relativity

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I review arguments indicating that there is no real, physically detectable, local inertial-induction effect in general relativity, contrary to recent comments by Tittle.

In a recent Letter Tittle[1] has brought up an old suggestion of Einstein's that there is some sort of inertial-induction effect in his standard general-relativistic theory of gravitation. In his book Einstein[2] devoted about ten pages to a discussion of this point, particularly in reference to the role of Mach's principle in his theory. Over the years, many and varied expressions of Mach's principle have been proposed, making it one of the most elusive concepts in physics.[3] However, it seems clear that Einstein intended to show that locally measured inertial-mass values are gravitationally coupled to the mass distribution in the universe in his theory. For convenience I repeat the first-order geodesic equations given by Einstein to support his argument:

$$(d/dl)[(l+\bar{\sigma})\vec{v}] = \nabla\bar{\sigma} + \partial\vec{A}/\partial l + \nabla\times(\vec{A}\times\vec{v}),$$

$$\bar{\sigma} = (\kappa/8\pi)\int(\sigma/r)\,dV_0,$$

$$\vec{A} = (\kappa/2\pi)\int(\sigma d\vec{x}/dl)\,r^{-1}\,dV_0.$$

Here σ is the source-mass density while l is *coordinate* time and $\vec{v}$ is *coordinate* velocity of a test particle. Einstein's claim is that "The inertial mass is proportional to $l+\bar{\sigma}$, and therefore increases when ponderable masses approach the test body."[2] This Letter is meant to call attention to arguments which indicate that this conclusion is not consistent with the usual interpretation of general relativity.

In the 1950's, R. H. Dicke at Princeton University was stressing the importance of Berkeley's and Mach's ideas that if space is to be regarded as a subject for physical theory, then its physical characteristics ought to be determined by the mass distribution within it. In studying this problem, Dicke and I were not satisfied that general relativity met this criterion. In fact, we came to the conclusion that Einstein's claim of inertial induction was a purely coordinate effect and thus could have no physically detectable consequences. The basic reason is that Einstein's theory is generally covariant, with gravitational effects carried by a tensor field alone whose effects are transformed away approximately in any local inertial reference frame. We neglect, of course, tidal forces which have no significant effect in a cosmological context. Because of the central importance of this problem, I have given a careful and thorough treatment of it.[4,5] Since Tittle, and perhaps others, do not seem to be aware of this work, a review of the main points of the argument will be given here.

First, let us recall the importance of giving operational definitions for our terms, as stressed by Einstein, above all. Thus the concept of inertia must be tied to some, at least ideally possible, measurement. Of course, mass is a dimensional quantity, so we must pick some standard unit. Since there does not seem to be any direct connection (pending development of a complete unified field theory) between small electrical and other atomic and nuclear fields and gravitational and inertial forces, we choose atomic units for standards—for example, the charge of the electron, together with some atomic length or time unit and the precription that the velocity of light be 1.

Next we assume that all laws of physics are to be valid in the same form in every local inertial reference frame when measurements are referred to such standards. This is, of course, one version of the general principle of relativity which forms an important part of the operational interpretation of the Einstein theory. We are now left with the task of specifically defining inertial mass. The obvious notion, intuitively, is that of the ratio of force to acceleration, where each is measured in some standard way. In Ref. 5, a specific choice for inertial mass was made by assuming that inertial mass times covariant acceleration should equal the force four-vector, as given, for example, by the Lorentz electromagnetic force. Of course, other choices could be made, but it seems evident that any definition for inertial mass consistent with the above ideas of general covariance would lead to the same conclusions. A sketch of such a general argument is given in Ref. 5. The result, as would be expected, is that *global, i.e., nontidal, gravitational fields are completely invisible in such local standard measurements of inertial mass,* contrary to Einstein's claim. In fact, in terms of the analysis above, Einstein considered only accelerations as measured in noninertial reference frames. This cannot be physically significant, however, since any result can be obtained by an arbitrary coordinate choice. Einstein ought to have normalized his local space-time measurements to inertial frames, in which the metric has been transformed approximately to the standard Minkowski values, and for which distant-matter contributions are not present. Equivalently, only proper-time and proper-distance measurements should have been considered.

Because of our failure to find any influence of distant matter on local, inertial reference frame physics in general relativity, Dicke and I were led to consider an additional mass coupling through a scalar field, whose effects could not be transformed away. The result was a scalar-tensor theory of gravity.[6] The scalar field turned out to be operationally related to the reciprocal of the locally measured Newtonian gravitational "constant," which becomes a function of the mass distribution of the universe.

For completeness I should also mention another statement of Mach's principle supported most prominently by Wheeler and his associates.[7] This version points out that the geometry of space is determined, up to conformal transformations, by the distribution of mass in the universe, assuming that the spacelike sections are compact. While this is true, it does not seem to contradict the above comments that local, *internal* physics done in inertial reference frames is unaffected by global cosmological distributions of mass in the universe. Certainly, the notion of the inertial mass of a particle would have to fall in this category.

Finally, it should be pointed out that recent astronomical studies[8] have cast serious doubt on the assumption that we do indeed live in a universe with compact spacelike (complete) sections. This would disturb the basic premise of the above argument. In fact, the possibility of an infinite universe should cause us to reconsider seriously the entire question of the significance of theories based on constraint equations of an elliptic nature. While these equations are thoroughly understood, and have quite satisfying uniqueness properties on compact spaces, they become physically worthless on noncompact spaces without the introduction of boundary conditions. How can such boundary conditions be seriously considered, without again putting man at the center of the universe providing an origin for a coordinate r, with fields required to assume trivial values as $r \to \infty$?

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