where

$$\hat{\psi}(\vec{s}-\vec{s}\,', u) \equiv \langle \hat{\psi}(\vec{s}-\vec{s}\,', u, W)\rangle, \tag{23}$$

$$\hat{\Phi}(\vec{s}, u) \equiv \langle \hat{\Phi}(\vec{s}, u, W)\rangle. \tag{24}$$

Equation (22) is identical to the result of our CTRW procedure.[4] Since $\hat{\psi}(\vec{s}-\vec{s}\,', u)$ is the Laplace transform of $\psi(\vec{s}-\vec{s}\,', t)$, the latter is defined by

$$\psi(\vec{s}-\vec{s}\,', t) \equiv \langle W_{\vec{s},\vec{s}\,'} \exp(-\Gamma_{\vec{s}\,'} t)\rangle. \tag{25}$$

The definition, Eq. (25), was in fact used by SL to calculate $\psi(\vec{s}-\vec{s}\,', t)$.

The factorization procedure is the approximation which makes the resulting problem tractable. A treatment including the effects of correlations between jumps would be desirable, but it too must treat the first jump on an equal footing with the others. The main point of the above discussion is that as a consequence of the general validity of statistical mechanics the occupancy $f(\vec{s}_0)$ associated with the starting point factors out, and all jumps, including the first, are to be treated alike.

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[4] For a more complete discussion, the reader is referred to Appendix B of Ref. 1, where a comparison is made between the procedure outlined here and the ensemble average of a random walk on a random media.

Limits on the Radiative Decay of Neutrinos

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Astronomical observations at the x-ray, optical, and radio frequencies are used to show that the lifetime of neutrinos for radiative decay divided by the rest mass, τ_0/m_ν, exceeds 10^{17} sec/eV. If one makes the further assumption that $m_\nu > 10^{-3}$ eV, then $\tau_0 \gtrsim 10^{19}$ sec. If there are other competing decays of neutrinos, it is then shown that $\Gamma_{\nu_e}(\nu_e \to x+\gamma)/\Gamma_{\nu_e}(\text{total}) \le 10^{-15}$ and $\Gamma_{\nu_\mu}(\nu_\mu \to x+\gamma)/\Gamma_{\nu_\mu}(\text{total}) \le 3\times 10^{-6}$.

In this paper I discuss the limits that can be placed on the radiative instability of the neutrinos from various kinds of observations. In this regard I am motivated by several recent papers[1-5] which have considered the possibility that neutrinos could have finite rest mass and could therefore decay. One particular set of these[3,4] considers the mixing of ν_e and ν_μ and predicts observable widths for the lepton-number-nonconserving decays, such as $\mu \to e+\gamma$ and $\nu_\mu \to \nu_e+\gamma$. Independent of such theoretical considerations it is worthwhile to study the observational limits on such processes.

The astrophysical environment provides excellent possibilities for such a study of very weak processes: Path lengths of $\sim 10^{28}$ cm are available for the decay process to take place, huge in comparison with the $\sim 10^2$ cm available in most laboratory studies. Also there are regions, such as the cores of very hot stars, where the weak processes dominate, as the products of the competing electromagnetic channels are suppressed completely because of the enormous time scales needed for the diffusion of photons to the stellar surface.[6]

Recognizing that both neutral and charged currents are of comparable strength in weak interactions, one finds that there are several locations in nature where copious generation of ν_e, ν_μ, etc., takes place. In this Letter I consider at first that the neutrino decays solely through the channel

$$\nu \to x+\gamma, \tag{1}$$

where x is any particle with a mass smaller than m_ν, and consider later the effects due to competing channels. If the m_ν-m_x mass difference is comparable to m_ν, then the decay photon will carry away roughly half the energy of the neutrino in the decay; then one can make use of observational limits on the photon intensities to place limits on the lifetime for the radiative instability of the neutrino. I briefly derive these limits below, considering sequentially the cases where progressively longer times are available for the decay of the neutrinos.

(a) *Studies at particle accelerators.*—Here the neutrino fluxes are generated by the decay of mesons and muons produced by a high-energy proton beam interacting with a target of nuclei. The neutrino fluxes have a mean energy of ~1 GeV and their decay would give rise to γ rays of similar energy. These γ rays will be easily detected in the spark chambers. The decay length available is at least several meters (~10^2 cm). One can safely assert that the number of γ rays that would be detected arriving in the beam direction is less than the total number of neutral-current events which have a similar signature. Equating the expected number of decays in a 10^2-cm path with the number of ν-induced events (σ ~10^{-39} cm^2) in a target of thickness $N_T \approx 10^{25}$ nuclei/cm^2, we have

$$\tau_\nu = \frac{F_\nu}{F_\gamma}\Delta t > \frac{1}{N_T\sigma}\frac{l}{c} \text{ or } \tau_\nu > 3\times10^6 \text{ sec.}$$

Since $\tau_\nu = \tau_0 E_\nu/m_\nu$,

$$\tau_0/m_\nu > 3\times10^{-3} \text{ sec/eV}, \tag{2}$$

with $E_\nu = 10^9$ eV for the neutrinos at the accelerators.

(b) *Atmospheric neutrinos.*—Cosmic rays induce[7,8] similar reactions as considered in case (a) in Earth's atmosphere and the events induced by these are observed by experiments deep underground.[9] The mean energy of the neutrinos is ~1 GeV and the flux is ~10^{-1}/cm^2 sec sr. The total event rate, $\mathcal{R}$, is 10^{-13}/cm^2 sec sr. The decay length available at the apparatus is again several meters. We can therefore write

$$\tau_\nu = \frac{F_\nu}{\mathcal{R}}\frac{l}{c} > 10^4 \text{ sec or } \frac{\tau_0}{m_\nu} > 10^{-5} \text{ sec/eV}. \tag{3}$$

(c) *Solar neutrinos.*—If we make the eminently reasonable assumption that the energy generation in the sun is due to the synthesis of protons to helium nuclei, then at Earth we expect a neutrino flux of ~10^{11}/cm^2 sec irrespective of any specific solar models or the details of the nuclear reaction chains.[6] These neutrinos have a mean energy of ~200 keV and their decay during the ~500-sec flight from the sun to Earth would result in an intense x-ray flux. The observations[10] indicate that the x-ray flux from the quiet sun is below the level of detectability of ~10^{-4}/cm^2 sec; this yields

$$\tau_\nu > \frac{10^{11}\times500}{10^{-4}} = 5\times10^{17} \text{ sec or } \frac{\tau_0}{m_\nu} > 2\times10^{12}\,\frac{\text{sec}}{\text{eV}}. \tag{4}$$

(d) *White dwarfs and central stars of planetary nebulae.*—It is well known[11,12] that these stars cool rapidly by neutrino emission. An order-of-magnitude estimate of the neutrino fluxes emitted by such objects can be obtained by equating the gravitational energy released in the formation of these objects, GM^2/R, with the energy carried away by the neutrinos. With the assumptions $M = M_\odot = 2\times10^{33}$ g and $R \approx 10^{8.5}$ cm, a total flux of ~10^{58} neutrinos of $E_\nu \approx 100$ keV is radiated into the universe during the formation of such an object. The rate of formation of white dwarfs is ~1/yr in a galaxy. With $M_{\text{gal}} \approx 10^{44}$ g and the mean density of the universe due to galaxies $\rho_U \approx 10^{-31}$ g/cm^3, one will expect an x-ray flux from space of

$$F_\gamma \approx \frac{GM^2}{RE_\nu}\,\frac{\rho_U\tau_U R_U}{3\times10^7\times10^{44}\tau_\nu}, \tag{5}$$

where $\tau_U = 10^{18}$ sec is the "age" of the universe and $R_U \approx 3\times10^{28}$ cm is its "radius." The observations of x-ray astronomers[13] yield a flux limit of $F_{\text{x ray}} < 10^{-1}$/cm^2 sec sr which yields the limit

$$\tau_\nu > 10^{22} \text{ sec or } \tau_0/m_\nu > 10^{17} \text{ sec/eV}. \tag{6}$$

(e) *Supernovae.*—These occur in the galaxy once once in a hundred years and radiate energies about a few hundred times larger than in the formation of a white dwarf. The mean energy of the radiated neutrinos is $E_\nu \approx 10$ MeV, and with the use of γ-ray[14] flux limits of ~10^{-3}/cm^2 sec sr, one obtains

$$\tau_\nu > 3\times10^{23} \text{ sec or } \tau_0/m_\nu > 3\times10^{16} \text{ sec/eV}. \tag{7}$$

(f) *Neutrinos in "big-bang" cosmology.*—Noting that the neutrinos of energy ~100 keV live longer than 10^{22} sec, one realizes that the neutrinos generated during the condensed phase of the universe will have survived till the present epoch. I had placed[15] a rather stringent limit of $m_\nu < 8$ eV, considering the gravitational effects due to such neutrinos. If indeed $m_\nu \geqslant 10^{-3}$ eV, the neutrinos would have behaved as a nonrelativistic gas during the expansion of the universe at red shifts Z

<1 (i.e., during the relatively recent past of $\sim 3\times10^{10}$ yr). This expansion would effectively have brought the neutrinos to rest, and the hypothetical decay of the neutrinos will yield photons of energy comparable to their rest mass. The number density[15] of any one type of neutrinos and antineutrinos is 600 cm^{-3} and would yield a photon flux of

$$F_\gamma = n_\nu R_U/\tau_0 \approx 2\times10^{31}/\tau_0 \tag{8}$$

(assuming $\tau_0 > \tau_U \approx 10^{18}$ sec). The background photon flux has a maximum at the peak of the blackbody curve at 2.7°K, where the relic microwave radiation[16] is observed. Asserting that the decay of the neutrinos cannot contribute substantially to the microwave background (~ 400 photons/cm^3) yields

$$\tau_0 > 10^{19}\ \text{sec if}\ m_\nu \approx 10^{-3}\ \text{eV}. \tag{9}$$

On the other hand, if $m_\nu \approx 1$ eV, their decay would yield a flux of optical photons. The observational limit on the background starlight flux is $\sim 3\times10^8/\text{cm}^2$ sec. This yields

$$\tau_0 > 10^{23}\ \text{sec if}\ m_\nu \approx 1\ \text{eV}. \tag{10}$$

These limits which have been derived with generous overestimates for the allowed decay rates are summarized in Table I. The limits based on accelerators and atmospheric neutrinos are necessary to preclude the possibility that the neutrinos will decay inside the stellar sources themselves. The limits (c)–(e) allow us to consider the neutrinos of cosmological origin and show that these neutrinos would have survived decay until the recent past, i.e., at least until the era where radiation and matter have decoupled.[17]

The limits presented here are sufficiently general, not based on any specific details of the processes considered, and therefore provide a reliable constraint on the models of weak interactions. For example in the model of Eliezer and Ross[3] the mere application of the mass limit[14] $m_\nu < 8$ eV yields

$$\frac{\Gamma(\mu\to e\gamma)}{\Gamma(\mu\to e\nu\bar{\nu})} < 10^{-30}\ \text{and}\ \frac{\tau_0}{m_\nu} > 10^2\ \text{sec/eV} \tag{11}$$

for the radiative instability of the muon and the neutrino, respectively. Furthermore, if m_ν is indeed roughly ~ 8 eV, the limits on the radiative decay derived in this Letter indicate that the mixing angle θ is less than $10^{-7.5}$. Conversely, if one maintains maximal mixing, we are restricted to $m_\nu < 10^{-6}$ eV.

Let us now consider briefly the consequences of the existence of competing processes to the radiative decay of the neutrino. For example, the very existence of the vertex $\nu\to x+\gamma$ implies a corresponding electromagnetic interaction of the ν with the Coulomb fields of nuclei: $\nu + Z \rightleftharpoons Z + x$.

TABLE I. Lower limits on the lifetime of neutrinos derived from various observations.

Source of Neutrinos	Neutrino produing process	Neutrino Density	E_ν	Observational Evidence	Lower Limit on $\frac{\tau_0}{m_\nu}$
a Accelerators	$p+N\to K$'s, πs $K, \pi\to(e,\mu)+\nu$	–	~ 1 GeV	Total number of induced events	3×10^{-3} sec/eV
b Cosmic-ray induced atmospheric neutrions	"	0.1 cm^2 sec sr	~ 1 GeV	$F_\gamma \approx 10^{-13}\ \text{cm}^{-2}\ \text{sec}^{-1}$	10^{-5} sec/eV
c. Sun	$4p+2e\to He+2\nu_e$	$10^{11}/\text{cm}^2$ sec	~ 200 keV	$F_\gamma \approx 10^{-4}\ \text{cm}^{-2}\ \text{sec}^{-1}$	$2\ 10^{12}$ sec/eV
d White dwarfs and central stars of planetary nebulae	$e^+ + e^- \to \nu\bar{\nu}$ $e^- + z \to e^- + z + \nu + \bar{\nu}$ $e^- + \gamma \to e^- + \nu + \bar{\nu}$ $\gamma_{\text{plasmon}} \to \nu + \bar{\nu}$	10^{58}/W.D.	~ 100 keV	$F_\gamma \approx 10^{-1}\ \text{cm}^{-2}\ \text{sec}^{-1}\ \text{sr}^{-1}$	10^{17} sec/eV
e. Supernovae	as in white dwarfs and $p+e\to n+\nu$	4×10^{58}/S.N.	~ 10 MeV	$F_\gamma \approx 10^{-3}\ \text{cm}^{-2}\ \text{sec}^{-1}\ \text{sr}^{-1}$	3×10^{16} sec/eV
f. Cosmos	Thermodynamic equilibrium	600/cm^3	$\sim 6\times10^{-4}$ eV	$F_\gamma \approx 3\times10^8\ \text{cm}^{-2}\ \text{sec}^{-1}$ at ~ 1 eV	$\tau_0 > 10^{23}$ sec[a]
				$F_\gamma \approx 10^{13}\ \text{cm}^{-1}$ at $\sim 10^{-3}$ eV	$\tau_0 > 10^{19}$ sec[h]

[a] If $m_\nu \approx 1$ eV.

[b] If $m_\nu \approx 10^{-3}$ eV.

This interaction can possibly generate a comparitively rapid "stimulated-decay" chain $\nu \to x \to \nu \to x \ldots$ during the transit of the neutrino from the stellar interiors to the surface. Since the stars do evolve by rapid loss of energy in some form [see section (d)], one can assert that the particles ν and x do escape from the star, still retaining a good fraction of their energy. Under these circumstances the limits derived above are not affected substantially.

There is also the possibility, though theoretically unlikely, that other relatively rapid modes of decay besides $\nu \to x + \gamma$ exist, such as

$$\nu \to x + y + z + \ldots. \tag{12}$$

In this hypothetical decay, the sum of the masses of x, y, z, ... should be less than m_ν, and none of the known particles can satisfy this condition other than the neutrinos and photons, of course. And yet, if any of x, y, z, ... is easily observable, we may try to estimate the decay rate from the flux of the decay product and discuss its importance relative to the radiative decay. If, however, the decay products x, y, z, ... have weak interactions alone and therefore are not easily observable, one may present the following arguments. Using the studies of ν-induced reactions at nuclear reactors and deep underground one can set $T_0^{\nu_e}/m_\nu \geq 3\times10^{-13}$ sec/eV and $T_0^{\nu_\mu}/m_\nu > 3\times10^{-11}$ sec/eV, where T_0 is the lifetime due to all the decay modes. These limits are not very restrictive, and if $T_0/m_\nu < 10^{-5}$ sec/eV the neutrinos will decay inside the stars and the limits derived earlier will then apply to the radiative instability of the decay products x, y, z, ... On the other hand, on general considerations, the expected lifetime for the decay of neutrinos into only weakly interacting particles is rather long. Here I wish to consider two cases: (I) When the lifetime T for the competing process is longer than or comparable to the typical times involved in the various discussions (c)–(f), the limits remain essentially unaffected [i.e., $T \geq 500$ sec for the solar neutrinos and $T \geq 10^{18}$ sec for cases (d)–(f) above]. (II) If 10^{18} sec $> T > 1$ sec, then the neutrinos will escape the stars ($T_0/m_\nu > 10^{-5}$ sec/eV) and their radiative decay will become observable, depending on the relative competition between radiative and other decay modes. Arguments presented in (c)–(e) imply the following: (c′)

$$\frac{\tau_0^{\nu_e}}{T_0^{\nu_e}} > 10^{15} \text{ or } \frac{\Gamma(\nu_e \to x+\gamma)}{\Gamma(\text{total})} < 10^{-15}; \tag{4′}$$

(d′)

$$\frac{\tau_0}{T_0} > 10^{4} \text{ or } \frac{\Gamma(\nu \to x+\gamma)}{\Gamma(\text{total})} < 10^{-4}; \tag{6′}$$

(e′)

$$\frac{\tau_0}{T_0} > 3\times10^{5} \text{ or } \frac{\Gamma(\nu \to x+\gamma)}{\Gamma(\text{total})} < 3\times10^{-6}. \tag{7′}$$

In summarizing I remark that the present considerations indicate that either the rate for $\nu \to x + \gamma$ is negligible or it is an extremely small part of the total decay rate of the neutrinos.

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