Possible Reduction of Laser-Fusion Target Illumination by Enhanced Stimulated Raman Plasmon Scattering

M. Sparks and P. N. Sen
Xonics, Incorporated, Santa Monica, California 90401
(Received 26 May 1977)

When the input irradiance I exceeds a threshold value, two mechanisms cause the stimulated Raman scattering by a plasma to increase as $\exp(g_E I^2 x)$. One mechanism is a nonlinear increase in the plasmon amplitude, and the other is a second-order process of converting two laser photons into a Stokes photon and an anti-Stokes photon. In laser fusion, the enhanced scattering by the residual fusion-chamber gas may greatly reduce the laser irradiance at the target. This theoretical possibility should be investigated experimentally.

Laser fusion has recently spurred interest in nonlinear interactions of laser fields with *pellet plasmas*.[1-4] It will be shown that enhanced stimulated Raman plasmon scattering, a process directly analogous to enhanced stimulated Raman scattering[5] with the optical phonon replaced by a plasmon, *in the ionized residual gas* in the fusion chamber may considerably reduce the laser irradiance at the pellet in currently envisioned commercial laser-fusion power plants. In scientific demonstration experiments, the pressure can be reduced sufficiently to avoid serious scattering problems. The scattering decreases with increasing laser frequency or plasma damping and with decreasing electron density or gain length.

The rate of the Raman process, shown in Fig. 1(a), is given by the second-order perturbation-theory result $2\pi\hbar^{-2}|\mathcal{H}|^2\delta(\sum_i \omega_i)$ and the well-known matrix elements of the creation and annihilation operators $a^\dagger$ and a (see in particular the second paper in Ref. 5):

$$\partial n_S/\partial t \sim (n_S+1)(n_p+1)n_L - n_S n_p(n_L+1) \simeq (n_S+n_p+1)n_L, \qquad (1)$$

where n_L and n_S are the numbers of laser and Stokes photons, and n_p is the number of plasmons. The saturation (pump-depletion) term $n_S n_p$ was dropped. The usual steady-state spontaneous-scattering result $I_S \sim (\bar{n}_p+1)Ix$ is obtained from (1) by neglecting $n_S n_L$, adding the propagation term $c\,\partial n_S/\partial x$, setting $\partial n_S/\partial t = 0$ and $n_p = \bar{n}_p$, where the bar denotes thermal equilibrium, and integrating over x. The Stokes irradiance is $I_S = \hbar\omega_S c_S n_S/V$, where ω_S and c_S are the Stokes frequency and velocity, respectively, and V the interaction volume.

The steady-state (unenhanced) stimulated-Raman-scattering result $n_S = (\bar{n}_p+1)[\exp(gIx)-1] + n_{S0}\exp gIx$, including amplification of noise and zero-point energy as well as the usual Stokes gain, is obtained from (1) by keeping the term $n_S n_L$, but still setting $n_p = \bar{n}_p$ formally. The enhanced result [same n_S, but with gIx replaced by $gIx/(I-I_R)$, where $I_R = \nu/gc_S$ and ν is the plasmon (energy) relaxation frequency] is obtained[5] from (1) by allowing n_p to increase above $\bar{n}_p$, as determined by the equation of motion for n_p. The unenhanced results (including $n_p = n_S I/I_R$) are regained in the limit $I \ll I_R$, and the spontaneous-scattering result is regained in the limit $I \ll I_R$ and $gIx \ll 1$.

The results above are for the high-optical-dispersion case of negligible anti-Stokes generation. In the opposite limiting case of negligible optical dispersion, as in the low-density plasma of the fusion chamber, the corresponding expressions for I_S and the anti-Stokes irradiance I_{AS} in the high-irradiance limit $I \gtrsim I_R$ are[5]

$$I_S \simeq I_{AS} \simeq \tfrac{1}{2}\exp g_E I^2 x; \quad g_E = 4cg^2/\nu. \qquad (2)$$

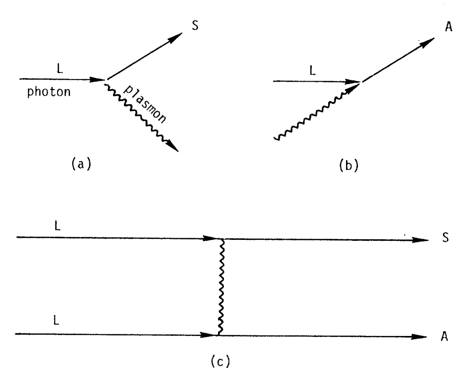

FIG. 1. (a) Stokes and (b) anti-Stokes Raman scattering process and (c) the second-order process converting two laser photons into a Stokes photon and an anti-Stokes photon. Both time orderings of the vertices and both directions of plasmon arrow must be included in (c).

This value of I_S is much greater than the previous, unenhanced value $I_S \sim \exp gIx = \exp[(I/I_R) \times (\nu/c)x]$. For $I \lesssim I_R$, this previous result is still valid. Note that $g_E I^2 x = 4(I/I_R)^2(\nu/c)x$.

Enhanced stimulated Raman *plasmon* scattering results can be obtained directly from the previous enhanced stimulated Raman scattering results[5] once the quantized Hamiltonian $\mathcal{H} = C a_L a_S^\dagger \times a_q^\dagger + \text{H.c.}$ is known since the Hamiltonians and dispersion relations are formally the same for the two cases. In fact, since the gain coefficient g for stimulated Raman plasmon scattering is known[2,4,6] ($g = \pi e^2 q^2 \omega_p / m^2 c^2 \omega_L^2 \omega_S \nu$), the enhanced stimulated Raman plasmon gain coefficient g_E can be written down directly from (2) as $g_E = 4\pi^2 e^4 q^4 \omega_p^2 / m^4 c^3 \omega_L^4 \omega_S^2 \nu^3$, where ω_p is the plasma frequency and q the plasmon wave vector.

Since the standard plasma kinetic-theory or hydrodynamic methods have not included the second-order process [Fig. 1(c)] in which two laser photons are converted into a Stokes photon and an anti-Stokes photon with the virtual exchange of a plasmon, a direct derivation[7] of the second-order result, as well as of the first-order result, is outlined. The interaction Hamiltonian for the scattering of the electromagnetic radiation, having vector potential $\vec{A}_L$, from the charge-density fluctuations $e\,\delta\rho$ in the plasma is[8,9] $\mathcal{H} = \int d^3r\, e^2 A_L^2 \times \delta\rho/2mc^2$. With the usual[10] quantization of $\vec{A}_L$ and the quantization of $\delta\rho$ from pages 35–36 of Ref. 9, $\mathcal{H}$ becomes $\mathcal{H} = C_1(\omega_S^{-1/2} a_L a_S^\dagger a_q^\dagger + \omega_A^{-1/2} \times a_L^\dagger a_A a_q^\dagger) + \text{H. c.}$, where $C_1 = -i\pi e^2 q (2n_e \hbar^3 / m^3 \omega_q \omega_L)^{1/2}$, n_e is the electron density, ω_A and ω_q are the anti-Stokes and plasmon frequencies, and H.c. denotes the Hermitian conjugate. The wave-vector Kronecker δ function eliminated one sum, and all of the other terms in the remaining two sums that contribute to the scattering are retained. The scalar products $\hat{\epsilon}_L \cdot \hat{\epsilon}_S$ and $\hat{\epsilon}_L \cdot \hat{\epsilon}_A$ of the polarization vector were set equal to unity since the important scattering is at small angles.

The remaining analysis, which is straightforward and directly analogous to that for enhanced stimulated Raman phonon scattering,[5] gives

$$\partial n_S/\partial t + c\,\partial n_S/\partial x = (\nu/n_R)[(n_S + n_p + 1)n_L - n_S n_p], \tag{3}$$

$$\partial n_A/\partial t + c\,\partial n_A/\partial x = (\nu/n_R)[n_L n_p - (n_L + n_p + 1)n_A], \tag{4}$$

for the first-order processes. Here $n_R \equiv \nu V / c^2 g \hbar \omega_L$, and $I_R = \hbar \omega_L c n_R / V$. For the second-order process,[5] the term $B \equiv (\nu/n_R^2)[n_L(n_L - 1)(n_S + 1) \times (n_A + 1) - n_A n_S (n_L + 1)(n_L + 2)]$ is added to the right-hand sides of both (3) and (4). This second-order process is of particular significance since there is no time delay caused by the buildup in the value of n_p as there is in the first-order enhanced process. In particular, the solution to (3) and (4) with the right-hand sides both replaced by B (that is, considering only the effect of the second-order process) is, for zero initial and boundary conditions and $n_L = n_{L0}\,\theta(t - x/c)$, where θ is the unit step function,

$$n_S = n_A = \tfrac{1}{2}[\exp(\tfrac{1}{2} g_E I^2 x) - 1]\theta(t - x/c), \tag{5}$$

which shows an instantaneous buildup of the Stokes irradiance to its steady-state value at the instant the laser pulse and the previously generated Stokes irradiance arrive at any given point x. By contrast, the solution to the first-order equations (3) and (4) and the corresponding equation for n_p exhibit a complex time dependence, the steady-state being approached only after the time[7] $t \gg g_E I^2 x / 2\nu$. With both the first- and second-order processes included and $I > I_R$ satisfied, this steady-state value is given by (5) with $g_E/2$ replaced by g_E and the step function in time deleted. Thus, the instantaneously attained value (5) is the same as the steady-state value (2), but with half the gain exponent. In either case, for large I this striking I^2 dependence causes the conversion of the incident radiation to Stokes and anti-Stokes radiation in a very short gain distance. For $I < I_R$, n_S is equal to the ordinary unenhanced value $n_S \sim \exp(gIx)$, and $n_A \ll n_S$.

Next, it is shown that the enhanced stimulated Raman plasmon scattering is expected to considerably reduce the laser irradiance at the pellet in a currently envisioned commercial laser-fusion plant. The fusion chamber contains a considerable amount of mass from the pellet and the chamber walls after a firing, and the pressure can be reduced to only ~0.1 Torr between firings.[11] This pressure corresponds to a density of atoms $N = 10^{15}$ cm^{-3} at 10^3 K. Values of other relevant parameters are as follows[11]: 1.06 μm wavelength; 10^{-9} s pulse duration (t_p); 10^4 cm^2 first-mirror area; ~3 J cm^{-2} per pulse of laser-energy density on the first mirror; 10 m from the pellet to the first mirror; and 500 μm pellet diameter.

The ambient gas is ionized by multiphoton and tunneling processes.[12] Within a distance $z \leq 22$ cm of the geometrical focus, greater than 90% of the atoms are ionized and the enhanced-gain result (5) applies since $I > I_R$ is satisfied and the steady state is not attained.[7] For the exemplary case of $z = 1$ cm, the irradiance is $I = 3\times 10^{15}$ W/cm^2, the number of electrons ionized per atom[12] is $Z = 4$, and the electron temperature from inverse bremsstrahlung heating[13] is $k_B T_e = 160$ eV. The corresponding plasma frequency is $\omega_p = (4\pi NZe^2/m)^{1/2} = 3.57\times 10^{12}$ s^{-1}, Debye wave vector is $k_D = 6.7\times 10^3$ cm^{-1}, plasmon relaxation frequency[14] is $\nu = 3.57\times 10^9$ s^{-1}, threshold field is $I_R = 3.84\times 10^{13}$ W/cm^2, and gain exponent is $\frac{1}{2}g_E I^2 x = 1.46\times 10^3 x$ (for $q = k_D/5$). The gain coefficient g_E increases with increasing q until $q \simeq k_D/5$. Further increase in q causes g_E to decrease precipitously from increased Landau damping.[13] Thus, $q \simeq k_D/5$ effectively, and the Stokes and anti-Stokes fields are peaked in a cone of angle $\theta_s = q/k_L = k_D/5k_L$. Notice that the enhanced gain *exponent* $\frac{1}{2}g_E I^2 x = 2(I/I_R)^2(\nu/c)x$ is a factor of ~ 200 greater than the unenhanced value $gIx = (I/I_R)\times(\nu/c)x$. With a total number of incident photons $n_L = \exp(53)$, there is total conversion into the scattered Stokes beam when $1.46\times 10^3 x = 53$, or $x > 3.6\times 10^{-2}$ cm, which is well satisfied in the present case.[7]

It remains to be shown that the scattered radiation misses the pellet. For $k_D = 6.7\times 10^3$ cm^{-1} and $k_L = 5.9\times 10^4$ cm^{-1}, the scattering angle is $\theta_s = k_D/5k_L = 2.3\times 10^{-2}$. Thus the scattered radiation misses the geometrical focus by a distance $\theta_s z = 230$ μm, which is approximately equal to the pellet radius. This is, in fact, why $z = 1$ cm was chosen for the example. Scattering from the region 1 cm $< z \leq 22$ cm, including repeated scattering, will greatly increase the net displacement of the beam.

In experiments at KMS Fusion,[15] increasing the helium pressure in the chamber to ~ 20 Torr caused the radiation to miss the pellet (even for an irradiance, $I \simeq 10^{11}$ W/cm^2 at $z = 1$ cm, that is considerably less than, and for an effective f number, $f/0.3$, that is considerably greater than, those of proposed commercial systems). Simple estimates indicate that absorption and defocusing by the electron plasma are not sufficiently strong to explain this result, whereas the present enhanced scattering is sufficiently strong. Further experiments to measure the Stokes and anti-Stokes irradiances, while monitoring such parameters as temperature and plasma quality factor, should be performed. We are currently investigating methods of avoiding deleterious effects of Raman scattering.

Useful discussions with Dr. J. Maniscalco, Dr. R. Johnson, and Dr. K. Brueckner are gratefully acknowledged. This research was partially supported by the Defense Advanced Research Projects Agency, by the U. S. Energy Research and Development Administration, and by the Lawrence Livermore Laboratory (under the auspices of the U. S. Energy Research and Development Administration, Contract No. W-7405-ENG-48).

Note added.—The question of the validity of the use of second-order perturbation theory (Fermi Golden Rule) to calculate the occupation numbers and the relation between the resulting gain and the gain of the mode amplitudes has been raised by D. Eimerl. His analysis and our response will be the subject of forthcoming publications.

[1]K. Brueckner and S. Jorna, Rev. Mod. Phys. 46, 325 (1974).

[2]*Advances in Plasma Physics*, edited by A. Simon and W. B. Thompson (Interscience, New York, 1976), Vol. 6.

[3]V. N. Tsytovitch, *Nonlinear Effects in Plasma* (Plenum, New York, 1970).

[4]R. E. Kidder, in *Physics of High Energy Density, International School of Physics, "Enrico Fermi," Course XLIII*, edited by P. Caldirola and H. Knoepfel (Academic, New York, 1971).

[5]M. Sparks, Phys. Rev. Lett. 32, 450 (1974), and Phys. Rev. A 11, 595 (1975), and J. Appl. Phys. 46, 2134 (1975); M. Sparks and J. H. Wilson, Phys. Rev. B 12, 4493 (1975); M. Sparks and H. C. Chow, Phys. Rev. B 10, 1699 (1974).

[6]The gain coefficient g is smaller than that given in Ref. 4 by a factor of 2 but agrees with that given in Ref. 2.

[7]P. N. Sen and M. Sparks, unpublished.

[8]C. Kittel, *Quantum Theory of Solids* (Wiley, New York, 1963).

[9]D. Pines, *Elementary Excitations in Solids* (Benjamin, New York, 1964). The direct quantization of plasmons is not necessary. The fluctuation-dissipation theorem gives $(n_p + 1)$ from the structure factor $\sim\langle\delta\rho\,\delta\rho\rangle$.

[10]W. Heitler, *The Quantum Theory of Radiation* (Oxford Univ. Press, London, 1954), 3rd ed.

[11]J. Maniscalo, private communication.

[12]M. Sparks, unpublished.

[13]L. Spitzer, *Physics of Fully Ionized Gases* (Interscience, New York, 1962).

[14]A conservative value of the plasmon quality factor $Q = \omega_p/\nu = 10^3$ is used. If instead we used the electron-ion collision frequency $\nu_c = 8.8\times 10^7$ s^{-1} (or $Q = 4.1\times 10^4$), the gain *exponent* $\frac{1}{2}g_E I^2 x$ would be *greater* by a factor of $(4.1\times 10^4/10^3)^3 = 6.9\times 10^4$.

[15]R. Johnson and K. Brueckner, private communication.