β Spectra of ^{12}B and ^{12}N Reanalyzed[a]

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Reanalysis of our experimental results on ^{12}B and ^{12}N provides shape factors that yield $(a_-^0 - a_+^0)_{\rm exp} = +(0.86 \pm 0.24)\%$ MeV^{-1}, still in good agreement with the theoretical prediction of $(a_-^0 - a_+^0)_{\rm theo} = +0.86\%$ MeV^{-1}. The experimental evidence for the conserved-vector-current theory as manifested in the mass-12 triad is on no less firm ground than it was at the time of our previous publication.

In a recent article Calaprice and Holstein[1] showed that there is a big difference between the Fermi functions F for positrons of Bhalla and Rose[2] ($F_{\rm B\text{-}R}$) and those of Behrens and Jänecke ($F_{\rm B\text{-}J}$).[3] The authors further argued that since $F_{\rm B\text{-}R}$ was used in Lee, Mo, and Wu's[4] analysis of ^{12}B and ^{12}N spectra and $F_{\rm B\text{-}R}$ had been suggested[5] to be in error, an analysis with the use of the new $F_{\rm B\text{-}J}$ greatly alters the conclusions on the shape factors of the β spectra of ^{12}B and ^{12}N. This conclusion seemed to undermine the confidence in the simple conserved-vector-current (CVC) theory, particularly the existence of the weak-magnetism term.[6]

Using currently improved and revised functions and informations we have reanalyzed our previous experimental data on ^{12}B and ^{12}N. It turns out that while the replacement of the erroneous Fermi functions $F_{\rm B\text{-}R}$ by the $F_{\rm B\text{-}J}$ indeed greatly reduces the slope of the shape factors of ^{12}N and ^{12}B, the presently accepted values of the branching ratios and the integrated F functions, f, the effects of which were considered negligible in Calaprice and Holstein's article, actually affect the slopes of the shape factors considerably but in the opposite direction to that of the $F_{\rm B\text{-}J}$. The final experimental shape-factor slopes for ^{12}B and ^{12}N of $(0.46 \pm 0.10)\%$ and $-(0.50 \pm 0.09)\%$ MeV^{-1} from this analysis are still in good agreement with the prediction of weak magnetism of the CVC theory. This more comprehensive analysis shows that the conclusions[1] based on the sole consideration of F functions and end points are misleading. Hence their implication for the CVC theory seems hardly warranted at the present time.

Shape correction factors.—In our experiments, the β^- or β^+ counting rates were normalized to the counting rates of the recoil protons, corrected for the background, and then divided by the momentum to adjust for acceptance of a magnetic spectrometer. This constitutes our experimentally observed composite spectrum, $S_{\rm exp}$.

The composite theoretical spectrum of allowed shape including the first and second branching transitions could be expressed as

$$S_\Sigma = pEF(\pm Z,p)[C_0(E-E_0)^2R_0 + C_1(E-E_1)^2R_1 + C_2(E-E_2)^2R_2], \qquad (1)$$

where $F(Z,p)$ is the Coulomb correction, $R_i(E, E_i)$ the radiative correction, and E_i the end-point energies. Here C_i is related to the branching ratios b_i and f_i as explained later. The weak-magnetism term in the *isotriplet* conserved-vector-current hypothesis as proposed by Feynman and Gell-Mann introduces an interference term which gives, if a slight curvature is neglected, a shape correction factor $(1 + a_\mp E)$,[6] so that

$$S = S_\Sigma(1 + a_\mp E) = S_{\rm exp}. \qquad (2)$$

(1) *Coulomb correction function* $F(\pm Z, p)$.—A complete table of F for all values of Z and electron momenta was not available at the time of our experiment. In Dzhelepov and Zyrianova's[7] Table II the finite-nuclear-size effects and electron-screening effects were taken into account, but the highest energy was only 10 MeV. We used this table ($F_{\rm D\text{-}Z}$) in our first analysis[4a] for no other reason than that the energy range of $F_{\rm D\text{-}Z}$ was wider. For a later paper[4b] Bhalla prepared the F functions for ^{12}B and ^{12}N taking into account the finite-size correction and the finite de Broglie wavelength.

That the $F_{\rm B\text{-}R}$ functions for positrons were in error was first suggested by Huffaker and Laird.[5] At our recent suggestion, Bhalla confirmed an error in his F calculations and prepared new F tables for ^{12}B and ^{12}N. In the following analysis, we have replaced $F_{\rm B\text{-}R}$ by $F_{\rm B\text{-}J}$ in order to enable us to make direct comparison with Calaprice and Holstein's analysis.

(2) *Radiative corrections.*—Calculations by Ki-

noshita and Sirlin[8] based on local $V-A$ theory and the method of regularization were used. The new value for E_i gives slight changes in the R's and in the slope of the shape factor for ^{12}N by $\Delta a_+ = -0.01\%$ MeV^{-1}.

(3) *End-point energy* E_0.—E_0 is related to the nuclear mass difference $\Delta = M_i - M_f$ and $M_{ave} = (M_i + M_f)/2$ by

$$E_0 = \frac{\Delta(1 + m_e^2/2\Delta M_{ave})}{1 + \Delta/2M_{ave}}. \qquad (3)$$

The atomic mass differences between ^{12}N and ^{12}C and between ^{12}B and ^{12}C of (17.344 MeV) $- 2mc^2$ and 13.370 MeV, respectively,[9] were used to calculate Δ. The recoil formula shown above was used to calculate E_0. With the use of 4.4391 and 7.6552 MeV for the levels in ^{12}C, E_i's for branching transitions are tabulated in Table I.

(4) $C_i = b_i/f_i$.—The branching ratios, b_i, are given by the probability of β decays per second,

$$b_i = C_i \int_{mc^2}^{E_0} F(Z,E) R(E,E_0) pE(E-E_0)^2 dE \equiv C_i f_i,$$

where f_i is given by the usual definition, the integral of the F function over the spectrum. The accurate determination of branching ratios b_i is rather difficult, since electron detection efficiency depends on energy, particularly near the low discrimination levels of the detector. The b_1 for ^{12}N was originally reported to be 2.4/94[10] but revised to 2.10/94.45 in recent years.[9] The value $b_1 = 2.10 \times 10^{-2}$ was indirectly derived from four experimental values of $R = [I(\beta^+)_{4.4}/I(\beta^+)_0]_{^{12}N}/[I(\beta^-)_{4.4}/I(\beta^-)_0]_{^{12}B}$. The weighted average of all four data points is $R_{ave} = 1.65 \pm 0.04$.[11] Using $b_1 = 1.29 \times 10^{-2}$ for ^{12}B yields $b_1 = 2.13 \times 10^{-2}$ for ^{12}N. The presently adopted value of b_1 for ^{12}N is $(2.10 \pm 0.16) \times 10^{-2}$ with a rather large uncertainty.[11]

TABLE I. Changes in slope ($\Delta a_\pm$) of the shape correction factors in ^{12}N and ^{12}B due to replacement of each term separately and all terms at once. Only narrow-slit data are used.

	^{12}N			^{12}B		
	New	Old	Δa_+ (% MeV^{-1})	New	Old	Δa_- (% MeV^{-1})
F	F_{B-J}	F_{B-R}	+0.20	F_{B-J}	F_{B-R}	−0.16
E_0/mc^2	32.918	32.037	+0.06	27.147	27.162	+0.08
E_1/mc^2	24.242	24.50	−0.03	18.470	18.493	
E_2/mc^2	17.954	18.17	−0.03			
b_0	0.94	0.94	0	0.97	0.97	
b_1	0.021	0.024	−0.07	0.0129	0.0129	
b_2	0.027	0.030	0.01			
f_0	1132 700	1200 000	−0.08	561 130	560 000	Small
f_1	244 520	224 000	−0.04	81 739	83 500	
f_2	53 200	50 000				
$a_\mp$	$a_+ = -0.52$	$a_+ = -0.52$	$\sum \Delta a_+ \sim 0$	$a_- = -0.48$	$a_- = +0.55$	$\sum \Delta a_- \sim 0.08$

In order to examine how sensitively the shape factors are affected by the presently adopted ratio, we calculated the slopes a_+ by using different regions of the ^{12}N spectrum. When only twelve points above 7.4 MeV and ten points above 8.4 MeV were used, the slopes a_+ obtained were -0.52% MeV^{-1} and -0.49% MeV^{-1}, respectively. These are to be compared with the slope -0.64% MeV^{-1} found by using all fifteen experimental points above 6 MeV. It can be ssen that above 7.4 MeV the contribution due to the transition to the second excited state has already dropped to near 1% and that due to the first excited state has also diminished to <3%, as shown in Fig. 1(a). Any uncertainty introduced by the errors in the branching ratios can be improved by excluding the low-energy points $E < 7.4$ MeV. For this reason, only the twelve points above 7.4 MeV were taken into account in the least-squares fitting in the new analysis of the ^{12}N spectrum. The new f_i values were taken from the paper of McDonald *et*

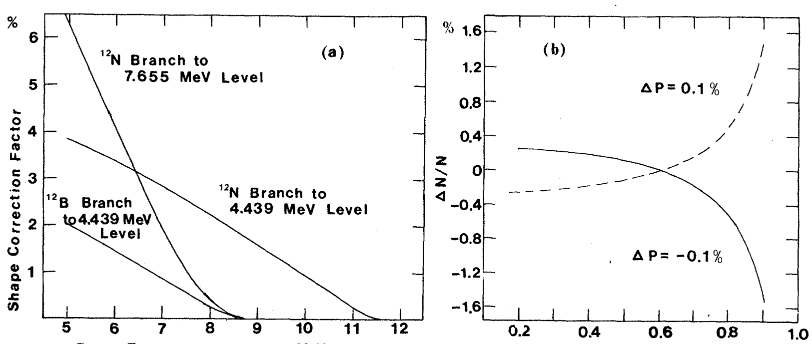

FIG. 1. (a) Correction to shape factors due to the branching transitions in ^{12}N and ^{12}B. (b) The change in the β spectrum due to a change in momentum calibration by ±0.1%. Here p is the β momentum, p_0 is the maximum β momentum, and N is the β spectrum.

al.[11] and also checked by numerical calculations using a recent parametrization by Wilkinson and Macefield.[12]

We obtained both the spectral shape factors (S_{exp}/S_{Σ}) for ^{12}N and ^{12}B and the slopes $a_\pm$ of the linear plots by least-squares fitting methods using $F_{B\text{-}J}$ and currently accepted E_i, b_i, and f_i values as shown in Table I. A systematic study of $a_\pm$ was made by varying only one parameter at a time between the new and old values. The Δa_+ thus obtained are listed in the fourth column of Table I. It is interesting to see that the great reduction in slope of the ^{12}N spectrum due to the replacement of the erroneous $F_{B\text{-}R}$ by the $F_{B\text{-}J}$ as reported by Calaprice and Holstein is completely compensated for by the sum of the increases in slope due to the changes in all other parameters. Similar behavior also occurred in the case of ^{12}B to a lesser extent.

Although the $F_{B\text{-}J}$ function includes the finite-nuclear-size correction ($\delta_{f.s.}$) the electron wave function is evaluated at the *nuclear center.* To take into account properly a distributed charge, the electron wave function must be averaged over the nuclear volume (δ_{ave}). Furthermore, the corrections due to the "*small*" Coulomb solutions f_{-1}, g_1, etc., must be included (δ_{small}). So the modified F should include at least three correction terms:

$$S \sim R(E,E_0)pE(E-E_0)^2F_0\{(1+a_\mp{}^{w.m.}E)(1\mp\delta_{f.s.}\mp\delta_{ave}\mp\delta_{small})\}, \tag{4}$$

or

$$S \sim RpE(E-E_0)^2F_{B\text{-}J}\{(1+a_\mp{}^{w.m.}E)(1\mp\delta_{ave}\mp\delta_{small})\} \tag{5}$$

if one ignores branching transitions for the sake of clarity of argument, and uses the relation $F_{B\text{-}J}=F_0(1\mp\delta_{f.s.})$. Here $a_\mp{}^{w.m.}$ represents the correction due to the weak-magnetism term only. The shape correction factors $1+a_\mp E$ obtained from our experimental spectrum shape factors represent the quantity in the curly brackets in Eq. (5), and are shown in Fig. 2.

Recently, Armstrong and Kim[13] and also Huffuker and Laird[5] and Wilkinson[14] have calculated these small additional Coulomb corrections to be

$$\delta_{f.s.}=0.053\% \text{ and } \delta_{ave}+\delta_{small}=0.045\% .$$

Therefore, our experimental results based on $F_{B\text{-}J}$ give $a_-=0.46\%$ and $a_+=-0.50\%$ MeV^{-1} after averaging the narrow- and wide-slit data. All earlier theoretical calculations of the shape correction factors used the point-charge Fermi function F_0 and denoted the quantity in the curly brackets in Eq. (4) as $1+a_\mp{}^0E$. The experimental values and theoretical predictions of $a_-{}^0-a_+{}^0$ are summarized in Table II.

In order to compare our experimental analysis which is given by the quantity in the curly brackets of Eq. (5) with the theoretical predictions in Eq. (4), we must add $\mp\delta_{f.s.}=\mp 0.053\%$ MeV^{-1} to our $a_\mp$ values to undo the correction for finite size in $F_{B\text{-}J}$. Our results based on F_0 are then

$$a_-{}^0=+(0.41\pm0.10)\% \text{ MeV}^{-1},$$
$$a_+{}^0=-(0.45\pm0.09)\% \text{ MeV}^{-1},$$

and

$$a_-{}^0-a_+{}^0=(0.86\pm0.24)\% \text{ MeV}^{-1}.$$

So the agreement between experimental and theoretical results is excellent. The uncertainties

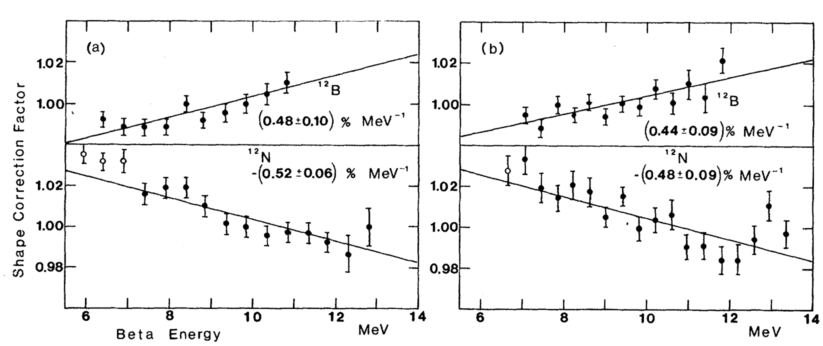

FIG. 2. Shape correction factors for ^{12}B and ^{12}N. $S_{exp}/S=1+a_\mp E$ measured with (a) the narrow ($\frac{3}{16}$ in.) and (b) the wide ($\frac{3}{8}$ in.) annular slits. The open circles for ^{12}N are not used for fitting (see the text). The points are normalized to the value near the middle of each spectrum.

TABLE II. Experimental and theoretical values of the difference between a_-^0 (^{12}B) and a_+^0 (^{12}N).

Experimental	$0.41+0.45=0.86\pm 0.24$
Theoretical[a]	
Gell-Mann and Berman, Ref. 15	0.86 ± 0.14
Morita, Ref. 16	0.86 ± 0.07
Huffaker and Laird, Ref. 5	0.90 ± 0.07
Bohr and Mottelson, Ref. 17	$0.42+0.49=0.91$
Calaprice and Holstein, Ref. 1	$0.37+0.47=0.84$

[a] $a_-^0 - a_+^0$ in Ref. 15 was originally $+1.33\%$ MeV^{-1} based on $\Gamma_{M1}=53$ eV. The Coulomb effect is -0.25% MeV^{-1}. After correction using $\Gamma_{M1}=37$ eV, the total effect is $1.33\times(37/53)^{1/2}-0.25=0.86\%$ MeV^{-1}. Calaprice and Holstein's value is corrected for $\delta_{\mathrm{f.s.}}$.

quoted for each individual slope are calculated directly from the least-squares fittings and are <0.1 MeV^{-1}. However, the uncertainty assigned to $a_-^0 - a_+^0$ includes a conservative estimate of systematic errors. The major source of systematic error could be the momentum calibration of the spectrometer. Although the linearity of the spectrometer was shown to be better than one part in 2000, the absolute calibration is known to be only better than one part in 1000. Now if one uses a rather severe criterion such as in Fig. 1(b), the maximum systematic error due to momentum calibration is 0.13% MeV^{-1}. The error in branching ratios adds 0.03% MeV^{-1} to the uncertainty for the case of ^{12}N and less for the case of ^{12}B. When these uncertainties are added in quadrature, the uncertainty becomes 0.17% MeV^{-1} for individual $a_\mp$ and $\pm 0.24\%$ MeV^{-1} for $a_- - a_+$ including all systematic errors which is the number given in our previous publications.[4]

The above analysis indicates that a thorough reexamination of our previous experimental data provides a shape factor still in good agreement with the prediction of weak magnetism based on the isotriplet CVC theory. Thus the experimental evidence for the CVC theory as manifested in the mass-12 triad is on no less firm ground than it was at the time of our previous publication.

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