Nonlinear Dynamics of Runaway Electrons and Their Interaction with Tokamak Liners

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A nonlinear theory of the collective processes in runaway tokamak discharges is presented. The conditions under which runaway electrons can be trapped in toroidal field ripples, leading to liner damage, are derived and compared with the experimental evidence.

In toroidal discharges where the runaway-electron production rate due to the relatively large value of the ratio of the inductive toroidal electric field E to the Dreicer field[1] $E_0 \equiv (4\pi ne^3/T_e) \times \ln\Lambda$ is significant ($\simeq 0.1$), the runaway electrons carry a significant fraction of the energy and momentum of the plasma and can thus become the major factor determining the macroscopic behavior of the discharge. There is ample experimental evidence[2-6] that anomalous macroscopic behavior of such low-density discharges can be attributed to such high-energy runaways. The most damaging of such events was the destruction of the liners in the TFR[3] and Alcator tokamaks.[4] The purpose of this Letter is to present a comprehensive model of the nonlinear runaway dynamics leading to such events.

We present below a simple description of the nonlinear processes leading to liner destruction. The appearance of runaway-electron tails with velocities $v > v_c \equiv (E_0/E)^{1/2} v_e$, where $v_e = (2T_e/m)^{1/2}$ is the electron thermal speed, in the presence of a toroidal electric field E is a well-known phenomenon.[7] During the initial collision-dominated stages of the discharge, E/E_0 is substantial and most of the runaways are produced. They are then freely accelerated by the toroidal electric field and their energy parallel to the magnetic field is limited only by their confinement time. As the runaway confinement time τ appears to be comparable to the bulk energy confinement time, of the order of a few milliseconds in most experiments,[5] their parallel energy $T_\parallel$ can reach a value of $T_e(E/E_0)^2(\nu\tau)^2$, where T_e is the bulk electron temperature and ν is the Coulomb collision frequency. With $E/E_0 \approx 0.1$, typical at the center of low-density discharges, the runaway energy can be of the order of 200 keV, which is consistent with the observed energy range. During this collisional initial stage their perpendicular energy distribution is determined by Coulomb collisions and is found from the Fokker-Planck equation as $T_\perp \approx 2(E_0/E)T_e \ll T_\parallel$ (i.e., typically $T_\perp \approx 20$ keV). On the basis of this temperature anisotropy several authors[7-11] examined the possibility that obliquely propagating plasma waves with $\omega_k = (k_\parallel/k)\omega_p$ can be driven unstable, via the anomalous cyclotron resonance[7] $\Omega + \omega_k = k_\parallel v_\parallel$, where $\Omega = eB/mc$, provided the Landau damping of the wave by particles with $v_\parallel = \omega_k/k_\parallel$ can be overcome. We call this mode the "slow mode" because its phase velocity is much smaller than the velocity of the runaways. A simple instability criterion can be obtained by requiring that the parallel phase velocity of the unstable modes, $v_p = \omega_k/k_\parallel$, must exceed the critical velocity v_c, so that the Landau damping becomes negligible as the distribution function $f(v_\parallel)$ is flat in the runaway region. The condition $v_p > v_c$ is equivalent to $E/E_0 > 2k^2\lambda_D^2$, and since $k\lambda_D \approx 0.3$ for the most unstable mode,[8] we have the approximate instability condition

$$E/E_0 > 0.1. \tag{1}$$

This condition appears to be in good agreement with the experimental observations.[6] As shown in Refs. 8-11 and Haber *et al.*,[12] the unstable modes primarily scatter the resonant runaways in pitch angle, increasing their perpendicular energy at the expense of their parallel. This isotropization of the distribution has been proposed[8] to explain the observed enhanced synchrotron emission. An important consequence of the isotropization is the formation of a bump on the runaway tail and a nonlinear instability, as was first pointed out by Papadopoulos[13] and subsequently studied in detail in Refs. 9, 11, and 12. The formation of the bump in tail can be understood simply as follows. Since the minimum phase velocity of the unstable modes is given by $\omega_k/k_\parallel \gtrsim v_c$, only runaways with parallel velocity

$$v_\parallel > v_M = \left(\frac{\Omega+\omega_k}{k_\parallel}\right)_{\min} \approx \frac{\Omega}{\omega_p}\left(\frac{E_0}{E}\right)^{1/2} v_e \qquad (2)$$

can be in cyclotron resonance with these slow modes and participate in the turbulent pitch-angle diffusion. Electrons with $v_\parallel < v_M$ do not suffer any pitch-angle scattering since the phase velocity of any mode in cyclotron resonance with them must be below v_c and is thus stable. Therefore, as schematically shown in Fig. 1, the line $v_\parallel = v_M$ becomes a boundary between rapid ($v_\parallel > v_M$) and negligible ($v_\parallel < v_M$) pitch-angle diffusion. The resulting decrease in parallel energy for $v_\parallel > v_M$, i.e., the receding of the tail of the distribution with $v_\parallel > v_M$, causes backward diffusion (corresponding to a smaller parallel temperature) while leaving the distribution with $v_\parallel \leqslant v_M$ unchanged. This phenomenon makes particles pile up near v_M and leads to the formation of a positive slope in the distribution (Fig. 1). The formation of the bump destabilizes parallel-propagating plasma waves with phase velocity near the bump, to which we refer below as the fast mode. The growth rate of the fast mode can be estimated as follows. Let the mean parallel velocity of the runaways be $v_0(t)$, with $v_0(0) = V_0$, and the perpendicular velocity be $v_1(t)$, changing in time because of pitch-angle scattering. Then the bump around $v_\parallel \approx [v_0 + v_M]/2$ is formed with width Δ of the order of $[v_0(t) - v_M]/2$. By conservation of the number of resonant runaways, the fraction of the runaways going into the bump at time t is $[1 - v_0(t)/v_0(0)]$. The minimum $v_0(t)$ is attained when the slow mode stabilizes (at $t=\tau$). At such a moment, particles with $v_\parallel > v_M$ are distributed more or less isotropically with average velocity $v_0(\tau) = v_b$. v_b can be calculated from energy conservation since pitch-angle scatterings preserve the particle energy. We find that $v_b/v_M = 1.15$ if $v_M/v_0(0) = \frac{3}{4}$. The number of runaways in the bump at τ is $n_b = (1 - v_b/v_0)\Delta n$, where the first factor indicates the fraction of the runaways depleted from the tail and forming a bump. The growth rate of the fast mode (parallel-propagating plas-

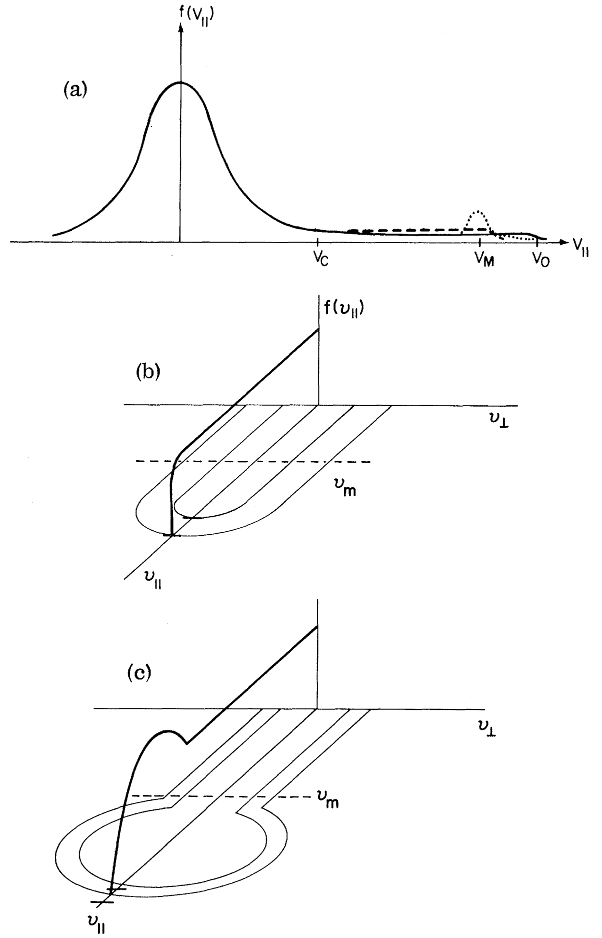

FIG. 1. (a) The solid curve denotes the initial distribution. The dotted curve denotes the formation of a bump by pitch-angle scatterings. The dashed curve denotes the flattened bump by parallel-propagating plasma waves. Notice the effective reduction of the average parallel velocity. (b), (c) Qualitative contour map of the distribution. The heavier curve is the distribution function integrated over $v_\perp$.

ma wave) with phase velocity $v_f = \omega_p/k_\parallel$ due to the positive slope of the bump is thus approximately given by

$$\gamma^f \approx \omega_p \frac{n_b}{n_0}\frac{V_f^{\,2}}{\Delta^2} \approx \frac{\Delta n}{n}\,\omega_p. \qquad (3)$$

Because $\Delta n/n$ is typically a few percent, the growth rate of the fast modes is of the order $10^{-2}\omega_p$, which can be an order of magnitude faster than that of the slow modes. This is possible because the quasilinear relaxation time of the bump, given by

$$\tau_{\mathrm{QL}} \approx 4\pi^2 \gamma_f^{-1} \ln\left[\left(1 - \frac{v_b}{v_0}\right)\Delta n \lambda_{\mathrm{D}}^{3}\frac{v_M}{v_e}\right],$$

is still much longer than the relaxation time due to the slow modes, as they have already attained high amplitude. During the quasilinear relaxation of the bump in tail, the runaways lose approximately $\frac{2}{3}$ of their parallel energy to plasma waves, while their perpendicular energy remains unchanged.

With the onset of the nonlinear instability of the fast modes and their quasilinear flattening of the resonant runaway distribution of the bump, the final distribution of these resonant runaways, between v_c and v_b, after a time $\tau_{\rm QL}$, is therefore a flat distribution in $v_\parallel$ between v_c and v_b with a perpendicular temperature given by v_b as the isotropy is reached for $v_\parallel > v_M$ at about the same time as the bump is formed. It is given by[14,15]

$$f_R = \frac{1}{\pi}\left(1 - \frac{v_0(\tau)}{V_0}\right)\frac{\Delta n}{n}\frac{1}{(v_M - v_c)v_b^2}\exp\left[-\left(\frac{v_\perp^2}{v_b^2}\right)\right], \tag{4}$$

where $V_0 = v_0(0)$, $v_b = 1.15\,v_M$, for $v_b > v_\parallel \geq v_c$. The final perpendicular average velocity is v_b as isotropy is approached. Because of the flattening of the bump, there are now energetic electrons with very large pitch angles that can be trapped by the ripples of the toroidal magnetic field of strength δB:

$$v_\parallel / v_\perp < (2\delta B/B)^{1/2}. \tag{5}$$

From Eq. (4), we find the density ratio of the particles trapped by the field ripples to the bulk plasma to be

$$\begin{aligned}\left(\frac{\delta n}{n}\right)_{\rm tr} &\approx \left(1 - \frac{v_b}{v_0}\right)\frac{\Delta n}{n}\frac{1}{v_b - v_c}\int_{v_c}^{v_b} dv_\parallel \exp\left(-\frac{B}{2\delta B}\frac{v_\parallel^2}{v_b^2}\right)\\ &\approx \frac{\sqrt{\pi}}{2}\frac{\Delta n}{n}\left(1 - \frac{v_b}{v_0}\right)\left(\frac{v_b}{v_b - v_c}\right)\left(\frac{2\delta B}{B}\right)^{1/2}\operatorname{erfc}\left[\left(\frac{B}{2\delta B}\right)^{1/2}\left(\frac{v_c}{v_b}\right)\right],\end{aligned} \tag{6}$$

where erfc is the complementary error function. Once trapped, the electrons drift vertically with the velocity $v_D = \frac{1}{2}mcv_\perp^2/eBR$. We note that the energy above which the trapped particle can convect vertically down the minor radius without suffering a detrapping collision is also about 50 keV. The energy flux due to the vertical drift of these trapped electrons can then be calculated using the distribution given by Eq. (4)

$$\begin{aligned}F &= \pi\int_{v_c}^{v_M} dv_\parallel \int_{v_\parallel (B/2\delta B)^{1/2}}^{\infty} dv_\perp^2 f_R v_D \tfrac{1}{2}mv_\perp^2\\ &\simeq \epsilon_b \bar{v}_D \Delta n\left(1 - \frac{v_b}{v_0}\right)\frac{v_b}{v_b - v_c}\left(\frac{2\delta B}{B}\right)^{1/2}\int_{x_c}^{(B/2\delta B)^{1/2}} dx\, e^{-x^2}(x^4 + 2x^2 + 2),\end{aligned} \tag{7}$$

where $x_c = (v_c/v_b)(B/2\delta B)^{1/2}$, $\epsilon_b = \frac{1}{2}mv_b^2$ and $\bar{v}_D = \frac{1}{2}mv_b^2c/eBR$. The energy flux as a function of $\delta B/B$ for typical tokamak parameters is plotted in Fig. 2. For $E/E_0 \approx 0.1$ and $\omega_p/\Omega \sim 0.3$, the perpendicular energy of the runaways is approximately $\frac{1}{2}mv_b^2 \approx (\Omega/\omega_p)^2(E_0/E)T_e$ and of the order of 50 keV, for $T_e \approx 0.5$ keV, precisely the range observed in the experiments.[3] Thus with $v_M \approx \frac{3}{4}v_0$ and $\Delta n/n$ a few percent, ϵ_M of 50 keV, $\delta B/B \approx 0.5\%$, the flux F is about 10 W cm^{-2}. This is sufficient to burn a hole in the stainless steel of 0.5 mm thickness because of an additional focusing effect in the toroidal direction.[16]

Finally, we may remark on the various time scales of the processes considered. The growth rate of the slow mode is typically $10^{-3}\omega_p$ while that of the fast mode (nonlinear instability) is $10^{-2}\omega_p$. The quasilinear flattening of the bump in tail occurs on the time scale $\tau_{\rm QL} \approx 10^3\gamma_f^{-1} \approx 1$ μsec. In a collision period of the order of a fraction of a millisecond, the plasma turbulence will be damped out by collisions. In a runaway-production time, about 10 msec, the runaways will be reaccelerated and the above-mentioned processes will repeat again. Many cycles of bursting instability are therefore expected during the discharge. The details of those processes will be discussed separately.

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Note added.—After this paper had been submitted for publication, there appeared a paper by Molvig *et al.*[17] considering the steady-state run-

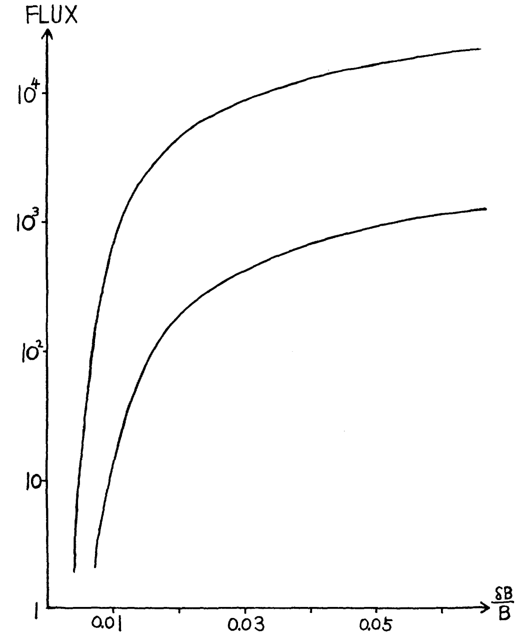

FIG. 2. The energy flux in units of watts per square centimeter. The numerical values are taken to be $B = 40$ kG, major radius = 100 cm. For the upper curve $\frac{1}{2}mv_0^2 = 300$ keV, $v_M/v_0 = \frac{3}{4}$, $v_C/v_M = \frac{1}{3}$, $\Delta n = 9\times 10^{11}/\text{cm}^3$ at the center. For the lower curve $\frac{1}{2}mv_0^2 = 180$ keV, $v_M/v_0 = \frac{3}{4}$, $v_C/v_M = 0.38$, $\Delta n = 5.5\times 10^{11}/\text{cm}^3$ at the center.

away distribution. By integrating the quasilinear equation over $v_\perp$, they obtained a "dynamic friction" F, proportional to $v_\parallel$, from the obliquely propagating modes, which brakes the acceleration, leading to a bump in tail. A simple check shows that F, in fact, is inversely proportional to $v_\parallel^2$, and therefore cannot stop the runaways. Thus without taking into account the minimum phase velocity of the unstable modes and the consequent backward diffusion as is done in this paper, there can be no bump formation. Furthermore, in the present work, the time evolution of the nonlinear processes is thoroughly discussed.

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