Molecular-Orbital Interpretation of the $\Delta l = 1\hbar$ Transfer Anomaly in Heavy-Ion Reactions

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The effect of the operator causing one-nucleon transfer in heavy-ion reactions in the molecular-orbital (MO) approach is contrasted to that of the usual distorted-wave Born approximation in a simple diffraction model calculation. For the case of $\Delta l = 1\hbar$ transitions, it is shown that the MO approach predicts an angular distribution exactly out of phase with that of the distorted-wave Born approximation. The MO approach therefore offers a simple explanation of the "anomalous" angular distributions recently observed.

The distorted-wave Born approximation (DWBA) has been used successfully to analyze the angular distribution of many heavy-ion–induced one-nucleon-transfer reactions. At sufficiently high bombarding energy, these angular distributions have a pronounced diffractive behavior.[1-3] For some particularly simple reactions (where the angular momentum transferred in the reaction is $1\hbar$), however, the DWBA fails to reproduce the data, yielding maxima where the data have minima and minima where the data have maxima.[4-6]

The purpose of this Letter is to present the first results of a model of the transfer process in which the interaction of the transferred nucleon with both cores is included explicitly during the entire process, thus allowing the wave function to respond to the presence of the target potential in more than first-order perturbation theory. We do not attempt here to give a full, detailed explanation of the molecular-orbital (MO) approach, which has an extensive literature in atomic physics,[7,8] but rather wish to point out the striking differences which can result from the application of that approach to one-nucleon transfer induced by heavy ions, compared to the usual treatment. Even in nuclear physics, a two-center approach has been considered for some time.[9-11] That such an approach might be required is indicated by the energies at which this maxima-minima anomaly often appears. For example, in the experiment[5] on ^{40}Ca(^{13}C, ^{14}N)^{39}K the velocity of the carbon at the point of transfer is the same as that of a 3-MeV nucleon, while the kinetic energy of the nucleon to be transferred from the top of the Fermi sea in the projectile is about 35 MeV. The result of the simple model calculation presented in this Letter will be that the angular distribution for the unique l transfer of $1\hbar$ in the MO approach is indeed *out of phase* with the angular distribution as predicted by the DWBA. Other l transfers are not so affected. The model, therefore, offers some hope of explaining the "anomaly" rather simply.

To compare the two approaches effectively, the origin of the DWBA diffractive structure in the cross section must be understood. It is explained in this paragraph using a no-recoil approximation to the full amplitude. The combined effect of the optical potential and the short range of the strong interaction is to confine the reaction to the annular region $(R, \Theta = \pi/2, \varphi)$ about the equator of the target if the beam is approaching the south pole (z axis in beam direction). The transition amplitude $T_{f\leftarrow i}$ to a final state involving angular momentum transfer l and projection m becomes (if $k_i \cong k_f$)[3]

$$T_{f\leftarrow i} \propto P_l{}^m(\epsilon = \tfrac{1}{2}\pi) \int d\varphi\, e^{im\varphi} e^{iqR\cos\varphi};$$

the properties of the associated Legendre polynomials then imply that $T_{f\leftarrow i} = 0$ unless $l+m$ is even. The integral is recognizable as a representation of the Bessel function of the first kind, $J_m(qr) = J_m(k_i R \sin\Theta_{\rm scat})$. From this follows the phase rule that if l is even, only even J_m contribute to the angular distribution $(d\sigma/d\Omega) \propto |T|^2$, while for odd l (as in a $p \rightarrow s$ transition), only odd J_m contribute. This is the reason for the dominance of the $\Delta m = 1$ transition in the DWBA amplitudes, which is the root of its disagreement with experiment. That this is a reasonable representation of the angular distribution for small enough angles is noted in Ref. 5 (see p. 302). Here, however, the difference in the results of model calculations is to be stressed and not a fit to any particular data.

The model proposed below is similar to the calculation of the transfer of an electron in an ion-atom collision and hence may be referred to as an MO approach. The Hamiltonian can be written in a no-recoil approximation as in the usual (DWBA)

treatment:

$$H = T_R + U_{\text{opt}} + T_n + V_1 + V_2,$$

where T_R is the relative kinetic energy of target and projectile; U_{opt} is the optical potential for elastic scattering, containing a real and imaginary part; T_n is the kinetic energy of the transferred nucleon; and V_1 (V_2) is the potential due to core 1 (2) which is assumed to be inert. However, rather than treating either V_1 or V_2 as the perturbation, the internal wave function of the transferred nucleon is obtained by considering R (the core-core distance) fixed and solving

$$H_{\text{int}}\Psi = (T_n + V_1 + V_2)_{\text{int}}\Psi = \epsilon(R)\Psi$$

for each of the states of relevance in the problem as a function of R. This corresponds to the intuitive notion that, in the region of transfer, the strong interaction of the transferred nucleon with each core should be included explicitly.

The details and limitations of the MO approach are discussed in the references; but what we wish to stress here is the effect of the perturbation on the angular distribution. The perturbation- (L_{ab}) inducing transitions between states is the relative motion of the cores and, for the transition between states of different angular momentum, is the result of the rotation of the internuclear axis with respect to the laboratory frame. The perturbation then has the same form as that of the coupling of a single particle to a rotating deformed core and, in fact, is given by the Coriolis coupling term $\vec{L}\cdot\vec{l}$, where $\vec{l}$ is the angular momentum of the transferred nucleon about the symmetry axis, namely the core-core radius vector $\vec{R}$, and $\vec{L}$ is the core-core angular momentum. The Coriolis coupling term can be written as $L_- l_+ + L_+ l_-$, where L_+ (L_-) and l_+ (l_-) are the raising (lowering) operators for $\vec{L}$ and $\vec{l}$, respectively.

Of critical importance to carrying out the evaluation of the transition amplitude in an approximation similar to that used for the DWBA (recall paragraph three) is the demonstration that the transfer is in fact localized near the nuclear surface. This is done in the two-state approximation to the time-dependent scattering problem[7,8] in an impact-parameter formulation. One substitutes into the Hamiltonian the following wave function:

$$\Psi(\vec{R}, t) = C_a\chi_a \exp(-iE_i t/\hbar) + C_b\chi_b \exp(-iE_f t/\hbar),$$

where $\chi_{a(b)}$ are the molecular states, which asymptotically look like a nucleon attached to one core or the other,

$$E = E_i + \epsilon_a(\infty) = E_f + \epsilon_b(\infty),$$

with the initial conditions $C_a(t=-\infty)=1$, and $C_b(-\infty)=0$. The substitution yields the coupled equations

$$i\dot{C}_a - (V_{aa}/\hbar)C_a = \hbar^{-1}L_{ab}C_b e^{-i\omega t},$$
$$i\dot{C}_b - (V_{bb}/\hbar)C_b = \hbar^{-1}L_{ba}C_a e^{i\omega t},$$

where $\omega = (E_i - E_f)/\hbar$ and L_{ab} is the coupling term previously described. The probability of a transition is then given by $|C_b(+\infty)|^2$ which, if it is small, can be evaluated as

$$|C_b(+\infty)|^2 = \hbar^{-2}\left|\int L_{ba}(t)\cos\Omega(t)\,dt\right|^2, \quad (1)$$

where $\Omega(t)$ is the difference of the energy of the molecular-orbital states at each instant in time. $\Omega(R)$ for the ^{40}Ca(^{13}C, ^{14}N)^{39}K($\frac{1}{2}^+$) system is shown in Fig. 1. If $\Omega(R)$ equalled zero at any point (an energy level crossing), Kelvin's stationary-phase argument would imply that the major contribution comes from that point. Since $\Omega(R)$ is not zero, the maximum contribution comes from where $\Omega'(R)$ has a maximum (the radius R indicated on Fig. 1). Again the region inside this surface is not probed because of absorption.

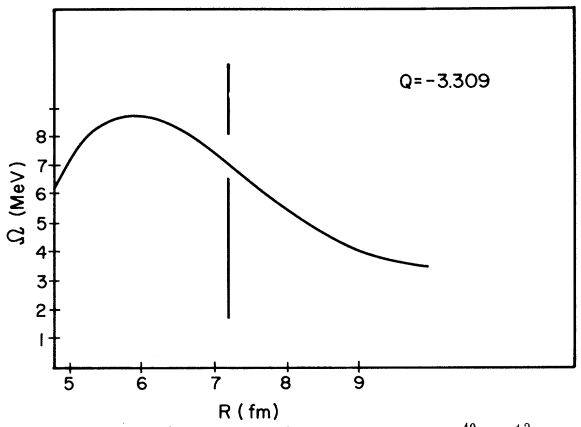

FIG. 1. Plot of $|\Omega|$ vs R for the reaction ^{40}Ca(^{13}C, ^{14}N)^{39}K (2.53 MeV).

Since the transfer is localized, the calculation of the resultant angular distribution from this model can be done in the same spirit as that of the DWBA previously discussed.[12] The interaction region will again be taken as the ($\Theta = \pi/2$, φ) plane at the surface (R), which will be as good an

approximation as for the DWBA and the models of Refs. 1–3, and the wave function outside that region will therefore be simplified to be the final and initial asymptotic states centered on the target and projectile, respectively. The evaluation of the transition amplitude corresponding to the coupling matrix element $(\vec{L}\cdot\vec{I})$ requires the wave function expressed about the center of mass [in momentum space $\exp(i\vec{k}\cdot\alpha\vec{R})\varphi(\vec{k})$, which can further be expressed by the expansion of the shift operator $\exp(i\vec{k}\cdot\alpha\vec{R})$ in spherical harmonics; noting $\vec{R}$ to be at $\Theta=\pi/2$ restricts the sum to those terms with $l+m$ even.

The transition amplitude then becomes, for the case of a $\Delta l=1\hbar$ transfer (e.g., from a p state, Y_{11}, to an s state, Y_{00}),

$$\int d\varphi \exp(-i\vec{k}_f\cdot\vec{R})\langle e^{i\vec{k}\cdot\alpha\vec{R}}\varphi_s(k)|L_+l_-+L_-l_+|e^{i\vec{k}\cdot\beta\vec{R}}\varphi_p(k)Y_{11}(\hat{k})\rangle_{k\ space}\exp(i\vec{k}_i\cdot\vec{R})$$

which may be shown to be zero by the following argument:

$$e^{i\vec{k}\cdot\beta\vec{R}}\varphi_p(k)Y_{11}(\hat{k})=4\pi\sum_{\substack{l,m\\ l+m=even}} i^l j_l(\beta kR)Y_{lm}^*(\hat{R})Y_{lm}(\hat{k})\varphi_p(k)Y_{11}(\hat{k})$$

but

$$Y_{lm}(\hat{k})Y_{11}(\hat{k})=\sum_{\mathcal{L}}\cdots C\begin{pmatrix} l & 1 & \mathcal{L}\\ 0 & 0 & 0\end{pmatrix}C\begin{pmatrix} l & 1 & \mathcal{L}\\ m & 1 & m+1\end{pmatrix}Y_{\mathcal{L}m+1}(\hat{k}),\quad \mathcal{L}+l+1=\text{even},$$

the possible terms of which are displayed in Table I. The evaluation of the overlap of the final $\langle Y_{l'm'}|\times l_\pm|Y_{\mathcal{L}m+1}\rangle$ will be zero; since $l'=\mathcal{L}$ each side will have the same evenness or oddness, but from the table one sees that $m+1$ and m' then also have the same evenness or oddness. Such states are not connected by $l_\pm$ which changes m by only 1. These matrix elements being zero in this model destroys the dominance of the $\Delta m=1$ transfer amplitudes. (Since $L_\pm$ does not strictly commute with the shift operator $e^{i\vec{k}\cdot\beta\vec{R}}$, a nonvanishing matrix element can be obtained. The result is of order β and a suppression of the $\Delta m=1$ amplitude remains.)

It can now be shown that the transition from a state described by a $Y_{1,0}$ to a state with $Y_{0,0}$ produces the $J_0(qR)$ behavior suggested by the data. It is easier to calculate the overlap by first expressing the wave function quantized along the beam direction in terms of wave functions quantized along the $\vec{R}$ axis (the double-prime coordinate system). The rotation required to transform between these two coordinate systems is described by the Euler angles $(\varphi, \pi/2, 0)$, or in Table II, and the initial $Y_{0,0}$ state is rewritten as $(1/\sqrt{2})(Y_{1,1}''-Y_{1,-1}'')$ along the $\vec{R}$ axis. This wave function in the coordinate system with origin at the center of mass does not produce any m substates other than $m=\pm1$ because the translation [via $\exp(i\vec{k}\cdot\alpha\vec{R})$] has $\Theta=0$, so only the Y_{lm}'s with $m=0$ contribute. Then

$$T_{f\leftarrow i}\propto\int d\varphi\exp(-i\vec{k}_f\cdot\vec{R})\cdots\langle Y_{l0}''|L_+''l_-''+L_-''l_+''|\{Y_{l,+1}-Y_{l,-1}\}\rangle_{k\ space}\exp(i\vec{k}_i\cdot\vec{R})$$

and application of $l_\pm$ yields

$$T_{f\leftarrow i}\propto\int d\varphi\exp(-i\vec{k}_f\cdot\vec{R})(L_+''-L_-'')\exp(i\vec{k}_i\cdot\vec{R})$$

$$\propto\int d\varphi\exp(-i\vec{k}_f\cdot\vec{R})\left(Z''\frac{d}{dX''}-X''\frac{d}{dZ''}\right)\exp(i\vec{k}_i\cdot\vec{R})\propto k_z\int d\varphi\, e^{iqR\cos\varphi}\propto J_0(qR),$$

where the relation of Table II and the fact that scattering is coming from the $Z=0$ plane have been used.

In conclusion, the essential features of the DWBA approach (which assumes one of the core potentials to be a perturbation) has been contrasted with an MO approach for the case of p- to s-state transitions. Within the same scheme of approximation, inert cores and diffractive scattering from an annulus in the

TABLE I. Signs of vector coupling coefficients and angular momentum coupling scheme.

$$i^lY_{lm}Y_{11}=\Sigma_{\mathcal{L}}\cdots i^lC\begin{pmatrix} l & 1 & \mathcal{L}\\ m & 1 & m+1\end{pmatrix}C\begin{pmatrix} l & 1 & \mathcal{L}\\ 0 & 0 & 0\end{pmatrix}Y_{\mathcal{L},m+1}$$

l	$\mathcal{L}$	m	$C\begin{pmatrix} l & 1 & \mathcal{L}\\ 0 & 0 & 0\end{pmatrix}$	$C\begin{pmatrix} l & 1 & \mathcal{L}\\ m & 1 & m+1\end{pmatrix}$
even	$l+1$	odd	+	+
	l		0	
	$l-1$	odd	−	+
odd	$l+1$	even	+	+
	l		0	
	$l-1$	even	−	+

TABLE II. Transformation between laboratory-fixed (unprimed) and rotating (double primed) coordinate systems.

$$X = Z''\cos\varphi - Y''\sin\varphi$$
$$Y = Z''\sin\varphi + Y''\cos\varphi$$
$$Z = X''$$

$(\Theta = \pi/2, \varphi)$ plane, the two theories predict rather different cross sections. The DWBA yields $(d\sigma/d\Omega) \propto |J_1(qR)|^2$, while the MO approach leads to a dominant $|J_0(qR)|^2$. Experiment seems to suggest that the latter function better represents the data. Other angular-momentum transfers ($\Delta l = 0$ and $\Delta l = 2\hbar$) do not have this same striking discrepancy between the DWBA and MO approach.

It should be stressed that the approximations used in deriving these results severely limit their generality. It is a sensitive function of energy and particular system considered that allows the angular distribution to have diffractive features while occurring slowly enough compared to internal motion to insure the validity of the MO approximation. In general, a full coupled-channels calculation including a treatment of the relative motion with nuclear and Coulomb distortions and the effect of both core potentials exactly is required. Such calculations are presently under investigation.

Since the conclusions reached here are based primarily on the *geometrical* character of the MO perturbation, we expect that for the $\Delta l = 1\hbar$ transitions the MO approach will predict an angular distribution out of phase with that of the DWBA even in more complete calculations. Still, it is interesting to see that nature may have provided a few examples where these approximations are valid, and these analytic results hold.

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[12]This localization minimizes difficulties associated with the incorrect asymptotic behavior (spurious rotation) of the molecular-orbital states since an expansion into molecular orbitals is required only within the region of transfer. This feature is particular to the nuclear case compared to atomic physics calculations.

Photon Antibunching in Resonance Fluorescence

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The phenomenon of antibunching of photoelectric counts has been observed in resonance fluorescence experiments in which sodium atoms are continuously excited by a dye-laser beam. It is pointed out that, unlike photoelectric bunching, which can be given a semiclassical interpretation, antibunching is understandable only in terms of a quantized electromagnetic field. The measurement also provides rather direct evidence for an atom undergoing a quantum jump.

The tendency of photons in a light beam emitted by a thermal equilibrium source to arrive in bunches, rather than strictly at random, has been well known since the classic experiments of Hanbury Brown and Twiss.[1] The bunching phenomenon was studied more explicitly in time-resolved correlation experiments,[2] and it was confirmed that the joint probability density of photodetection $P_2(t, t+\tau)$ by a phototube at two times t and $t+\tau$ is greatest when τ is near zero, and falls to a constant lower value once τ appreciably exceeds the coherence time. It is possible to look on the bunching phenomenon as a characteristic quantum feature of thermal bosons. If the wave