Two-Body Decays of Vector and Pseudoscalar Mesons in a Broken SU(4) Scheme[(a)]

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The two-body decays of the "old" and the "new" mesons are studied in a broken SU(4) scheme. The scheme succeeds in suppressing $\Gamma(\psi\to\eta_c\gamma)$. Sum rules among amplitudes are derived to support $\Gamma(\rho\to\pi\gamma)\simeq 75$ keV.

There are some outstanding problems in the radiative decay of vector mesons. The first problem is posed by the experimental[1] rate $\Gamma(\rho\to\pi\gamma) = 35\pm 10$ keV. This is not understood easily.[2] In a recent Letter Bohm and Teese[3] claim to fit $\Gamma(\rho\to\pi\gamma)$ in a SU(4) scheme. One of the purposes of this Letter is to demonstrate that the work of Ref. 3 is in error and that an understanding of the experimental value of $\Gamma(\rho\to\pi\gamma)$ is still to be found. Secondly, the rate[4] $\Gamma(\psi\to\eta_c\gamma)<3.5$ keV is difficult to understand since both ψ and η_c are largely $c\bar{c}$ states and one should expect a rate of a few hundred keV. A possible mechanism for the suppression of this rate could be SU(4) multiplet mixing.[5] In this Letter we offer a different explanation based on SU(4) symmetry breaking. The third problem is that the rates[4] $\Gamma(\psi\to\eta\gamma) = 55\pm 12$ eV and $\Gamma(\psi\to\eta'\gamma) = 152\pm 117$ eV are much larger than $\Gamma(\psi\to\pi\gamma) = 5\pm 3.2$ eV.[4] Explanation for this puzzle can be given in terms of pseudoscalar mixing angles.[6] We show, in this Letter, that one can understand this puzzle also in terms of SU(4) symmetry breaking. We also mention the work of Kazi, Kramer, and Schiller[7] done in the context of a broken SU(4) scheme before many of the rates in the charm sector were available.

We assume that the decays of the kind $V\to P\gamma$ and $P\to V\gamma$ proceed through vector-meson dominance (VMD). The hadronic VVP vertex is parametrized in terms of symmetry-breaking terms transforming like the eighth and the fifteenth component of a mixed second-rank SU(4) tensor. The general structure of the $V^m\to P^iV^n$ vertex (we shall suppress the Lorentz structure throughout) is given by

$$A_{min} = g_1 d_{min} + g_2 d_{8ik} d_{kmn} + g_3(d_{8mk}d_{kin} + d_{8nk}d_{kim}) + g_4\delta_{8i}\delta_{mn} + g_5(\delta_{8m}\delta_{in} + \delta_{8n}\delta_{im}) + g_6 d_{15\,ik}d_{kmn} + g_7(d_{15\,mk}d_{kin} + d_{15\,nk}d_{kim}) + g_8\delta_{15\,i}\delta_{mn} + g_9(\delta_{15\,m}\delta_{in} + \delta_{15\,n}\delta_{im}), \quad (1)$$

where, m, i, and n are the internal-symmetry labels, and the repeated label k is summed from 0 to 15.

To keep our arguments general, and in particular to be able to make contact with the work of Ref. 3, we write the electromagnetic current as

$$V_\mu^{\text{em}} = V_\mu^{(3)} + (1/\sqrt{3})V_\mu^{(8)} - (\sqrt{2}/\sqrt{3})gV_\mu^{(15)} + (\sqrt{2}/3)fV_\mu^{(0)}, \quad (2)$$

where, at this stage, g and f are independent parameters. The $V^n\to\gamma$ vertex is then

$$A(V^n\to\gamma) = (e/g_\rho)[\delta_{n3} + (1/\sqrt{3})\delta_{n8} - (\sqrt{2}/\sqrt{3})g\delta_{n\,15} + (\sqrt{2}/3)f\delta_{n0}]. \quad (3)$$

We assume that ψ and η_c are pure $c\bar{c}$ states. φ is a pure $s\bar{s}$ state and ω is a pure $(u\bar{u}+d\bar{d})/\sqrt{2}$ state. η and η' have the same quark content as in SU(3), i.e.,

$$|\eta\rangle = \cos45°|\eta_\sigma\rangle - \sin45°|\eta_s\rangle,$$
$$|\eta'\rangle = \sin45°|\eta_\sigma\rangle + \cos45°|\eta_s\rangle, \quad (4)$$

where the angle 45° is strictly $(54.7° + \theta_p)$ with θ_p the pseudoscalar mixing angle (− 10°) and $\eta_\sigma = (1/\sqrt{2})|u\bar{u}+d\bar{d}\rangle$ and $|\eta_s\rangle = |s\bar{s}\rangle$.

If one assumes U(4) symmetry ($g_1\neq 0$, $g_i = 0$, $i = 2$ to 9), then one can show the following:

$$A(\omega\to\pi\gamma) = (e/g_\rho)g_1,$$
$$A(\rho\to\pi\gamma) = \tfrac{1}{3}(e/g_\rho)g_1[1-(g-f)]. \quad (5)$$

It would, therefore, seem that one could fit both $\Gamma(\omega\to\pi\gamma)$ and $\Gamma(\rho\to\pi\gamma)$. All of the amplitudes written in Ref. 3 can be derived through VMD with $g = 1$ and f arbitrary. Clearly, if we choose $f\approx 1/\sqrt{3}$ we can fit both of these rates. *This, however, cannot be done.* The charge operator corresponding to the current of Eq. (2) has diagonal elements $\frac{2}{3}+\frac{1}{6}(f-g)$, $-\frac{1}{3}+\frac{1}{6}(f-g)$, $-\frac{1}{3}+\frac{1}{6}(f-g)$, and $\frac{1}{6}(f+3g)$. Only for certain values of f and g does this charge operator have the correct expectation values for the baryon and meson states. In particular $\langle p|Q|p\rangle=1$ and $\langle n|Q|n\rangle=0$, at $q^2=0$, cannot be satisfied unless $f=g$. With $f=g$, the charge of the charmed quark is $\frac{2}{3}f$. The charges of D^0 and D^+ then imply that $f=1$. We shall use $f=g=1$ throughout. If $f=g$, then one is inevitably led, via Eq. (5), to $\frac{1}{3}A(\omega\to\pi\gamma)=A(\rho\to\pi\gamma)$.

In our model, processes which are forbidden by the Okubo-Zweig-Iizuka (OZI) rule[8] may proceed through the symmetry-breaking terms of Eq. (1). A new sum rule is now obtained relating $A(\rho\to\pi\gamma)$ to other radiative-decay amplitudes:

$$A(\rho\to\pi\gamma)=\tfrac{1}{3}A(\omega\to\pi\gamma)-\tfrac{1}{3}\sqrt{2}A(\varphi\to\pi\gamma)+(4/3\sqrt{2})A(\psi\to\pi\gamma), \tag{6}$$

However, the OZI-rule–violating amplitudes are small and the U(4) relation between $A(\rho\to\pi\gamma)$ and $A(\omega\to\pi\gamma)$ still holds approximately. One *cannot* fit both $\Gamma(\omega\to\pi\gamma)$ and $\Gamma(\rho\to\pi\gamma)$ simultaneously.

One can also derive the sum rule,

$$A(\rho\to\eta\gamma)=(1/\sqrt{2})A(\omega\to\pi\gamma)+\tfrac{1}{2}A(\varphi\to\pi\gamma)+(3\sqrt{2}/4)A(\psi\to\eta\gamma)-\tfrac{1}{4}(\sqrt{2}-1)A(\psi\to\pi\gamma). \tag{7}$$

Again, if we ignore all OZI-rule–violating amplitudes we get $\Gamma(\rho\to\eta\gamma)\approx 50$ keV, in agreement with one of the solutions obtained recently by Andrews *et al.*[9] The value of $\Gamma(\rho\to\eta\gamma)$ obtained in Ref. 3 is an order of magnitude too low.

To fit our model parameters we use the rates for $K^{*0}\to K^0\gamma$, $\omega\to\pi\gamma$, $\varphi\to\pi\gamma$, $\varphi\to\eta\gamma$, $\psi\to\pi\gamma$, $\psi\to\eta\gamma$, and $\psi\to\eta'\gamma$. These rates depend on $[g_1+(g_6/\sqrt{6})+(2g_7/\sqrt{6})]$, (g_2+2g_3), g_4, g_5, g_8, and g_9. In particular, $\Gamma(\psi\to\pi\gamma)$ depends on g_9 alone, and $\Gamma(\psi\to\eta\gamma)$ and $\Gamma(\psi\to\eta'\gamma)$ on g_4 and g_8. Therefore, the last two rates can be *fitted independently of* $\Gamma(\psi\to\pi\gamma)$. The $\psi\to\eta_c\gamma$ amplitude depends on $(\frac{1}{3}g_1-g_6/\sqrt{6}-2g_7/\sqrt{6})$, g_5, g_8, and g_9 and thus can be controlled by a suitable choice of (g_6+2g_7). On the other hand, $K^{*+}\to K^+\gamma$ and $D^{*0}\to D^0\gamma$ require, in addition, the knowledge of g_3; $D^{*+}\to D^+\gamma$ and $F^{*+}\to F^+\gamma$ require the knowledge of both g_3 and g_7. In Table I we have listed our fit and predictions. The rates used in the fit are put in parentheses. We have used the same charmed-meson masses as in Ref. 3. We stress that the rates for K^{*+}, D^{*0}, D^{*+}, *and* F^{*+} *have been calculated with* $g_3=g_7=0$. Information on these rates will help determine these model parameters.

In the U(4)-symmetric limit of the VVP vertex, one finds

$$A(D^{*0}\to D^0\gamma)=\tfrac{4}{3}A(\omega\to\pi\gamma),$$
$$A(D^{*+}\to D^+\gamma)=A(F^{*+}\to F^+\gamma)=\tfrac{1}{3}A(\omega\to\pi\gamma). \tag{8}$$

These would produce $\Gamma(D^{*0}\to D^0\gamma)\simeq 70$ keV and $\Gamma(D^{*+}\to D^+\gamma)\simeq 4$ keV. The rates predicted in Ref. 3 for the charmed mesons are two orders of magnitude lower due to their factor $\Phi(m_v,m_p)$ which, of course, has no effect on the ratio $\Gamma(\omega\to\pi\gamma)/\Gamma(\rho\to\pi\gamma)$.

The hadronic VVP vertex of Eq. (1) also yields

TABLE I. Decay rates. (See text for explanations.)

Mode	Rate (KeV)	Exp. (KeV)
$K^{*0}\to K^0\gamma$	(100)	74 ± 35[a]
$\phi\to\pi\gamma$	(5.9)	5.7 ± 2.1[a]
$\phi\to\eta\gamma$	(63)	64 ± 10[b]
$\omega\to\pi\gamma$	(870)	880 ± 60[a]
$\psi\to\pi\gamma$	(0.005)	0.005 ± 0.0032[c]
$\psi\to\eta\gamma$	(0.055)	0.055 ± 0.012[c]
$\psi\to\eta'\gamma$	(0.150)	0.152 ± 0.117[c]
$\rho\to\pi\gamma$	78	35 ± 10[d]
$\rho\to\eta\gamma$	58	50 ± 13 or 76 ± 15[e]
$\eta'\to\rho\gamma$	130	<300[a]
$\phi\to\eta'\gamma$	0.24	-
$\omega\to\eta\gamma$	4.9	$3.0^{+2.5}_{-1.8}$ or 29 ± 7[e]
$\eta'\to\omega\gamma$	6.5	<50[a]
$\eta_c\to\rho\gamma$	1.0	-
$\eta_c\to\phi\gamma$	250	-
$\eta_c\to\omega\gamma$	150	-
$\psi\to\eta_c\gamma$	0.0[g]	<3.5[f]
$K^{*+}\to K^+\gamma$	21[h]	<80[a]
$D^{*0}\to D^0\gamma$	18[g,h]	-
$D^{*+}\to D^+\gamma$	0.76[g,i]	-
$F^{*+}\to F^+\gamma$	0.090[g,i]	-

[a]T. G. Trippe *et al.*, Rev. Mod. Phys. 48, No. 2, Pt. 2, S51 (1976).

[b]This is the weighted average of figures in the reference above (T. G. Trippe *et al.*) and Ref. 9.

[c]W. Braunschweig *et al.*, Phys. Lett. 67B, 243 (1977).

[d]Ref. 1.

[e]Ref. 9.

[f]Ref. 4.

[g](g_6+2g_7) was chosen to suppress $\psi\to\eta_c\gamma$. $\Gamma(\psi\to\eta_c\gamma)<3.5$ keV (Ref. 4) results in the following constraints: 15 keV $<\Gamma(D^{*0}\to D^0\gamma)<21$ keV, 0.61 keV $<\Gamma(D^{*+}\to D^+\gamma)<0.92$ keV, 0.04 keV $<\Gamma(F^{*+}\to\gamma)<0.15$ keV.

[h]$g_3=0$.

[i]$g_3=g_7=0$.

relations among the decay amplitudes of the ψ into a vector and a pseudoscalar meson,

$$A(\psi\to\rho^0\pi^0)=A(\psi\to K^0\bar{K}^0)=-\sqrt{2}A(\psi\to\eta\varphi)=\sqrt{2}A(\psi\to\eta'\varphi)=-\tfrac{1}{2}\sqrt{3}g_9. \tag{9}$$

The following ratios then result [as compared to experimental data (Ref. 4)]:

	This work	Experiment
$\frac{\Gamma(\psi\to K\bar{K}^*,\text{all})}{\Gamma(\psi\to\pi\rho)}$	1.1	1.1 ± 0.2
$\frac{\Gamma(\psi\to\eta\varphi)}{\Gamma(\psi\to\pi\rho,\text{all})}$	0.13	0.064 ± 0.038
$\frac{\Gamma(\psi\to\eta'\varphi)}{\Gamma(\psi\to\pi\rho,\text{all})}$	0.093	0.045 ± 0.037

VMD relates $\Gamma(\psi\to\pi\gamma)$ to $\Gamma(\psi\to\pi\rho)$ by $\Gamma(\psi\to\pi\gamma)=10^{-3}\Gamma(\psi\to\pi\rho,\text{ all})$.[10] Using $\Gamma(\psi\to\pi\rho,\text{ all})=0.76\pm0.19$ keV,[4] we find that $\Gamma(\psi\to\pi\gamma)\sim 1$ eV. However, the experimental rate[4] is 5 ± 3.2 eV. *VMD, contrary to expectations, does not appear to be working well.* Thus if we fit $\Gamma(\psi\to\pi\gamma)$ to 5 eV, we produce $\Gamma(\psi\to\pi\rho)$, $\Gamma(\psi\to KK^*)$, $\Gamma(\psi\to\eta\varphi)$, and $\Gamma(\psi\to\eta'\varphi)$ too large roughly by a factor of 5.

VMD also allows us to predict $P\to2\gamma$ rates once the $V\to P\gamma$ rates are fitted. We find $\Gamma(\pi\to2\gamma)=7.04$ eV, $\Gamma(\eta\to2\gamma)=0.56$ keV, $\Gamma(\eta'\to2\gamma)=5.72$ keV, and $\Gamma(\eta_c\to2\gamma)=280$ eV. The last rate is determined with parameters which forbid $\psi\to\eta_c\gamma$. If we impose a constraint $\Gamma(\psi\to\eta_c\gamma)<3.5$ keV, we get $\Gamma(\eta_c\to2\gamma)<3.7$ keV.

Lastly, a straightforward use of VMD with $f=g=1$ for off-shell photon yields for the squares of the coupling constants of ρ, ω, φ, and ψ decaying into a lepton pair the ratio 9:1:2:8. This results in a $\psi\to e^+e^-$ rate which is too large by a factor of 4. For off-shell photons ($q^2=m_\psi^2$) one might expect a q^2 dependence in the coupling constants to play a role. In fact, if we set $f=g=\pm\frac{1}{2}$ at $q^2=m_\psi^2$ we get 9:1:2:2 (for the above ratio), in agreement with experiments.[4]

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Systematics of Neutron-Induced–Fission Cross Sections in the MeV Range[(a)]

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Evidence of systematic behavior in neutron-induced–fission cross sections has been observed in the actinide elements. Recently measured fission cross-section ratios show this behavior over the incident-neutron–energy range from 3 to 5 MeV.

Among the earliest studies of fission systematics were those of Huizenga[1,2] and Batzel,[3] who plotted σ_f/σ_T and Γ_n/Γ_f ratios (derived from cross-section measurements) as a function of the liquid-drop–model fissionability parameter Z^2/A. Other early studies of systematic behavior include the systematics of spontaneous fission half-lives[4,5] and the systematics of fission thresholds.[6,7] All were studied as a function of Z^2/A. In 1955 Barschall[8] showed an apparent correlation between measured fission cross sections at an incident-neutron energy of 3 MeV and the parameter $Z^{4/3}/A$; however, no significance to this parameter was claimed. Smith, Smith, and Henk-