Temperature Dependence of $1/f$ Noise in Silver and Copper

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We report the first systematic measurements of the temperature dependence of excess noise in metals. The magnitude of the noise in thin films of both Cu and Ag shows a rapid increase with increasing temperature between 90 and 350 K. In Ag, the noise reaches a maximum at $T_p \cong 410$ K, then decreases slowly. In Cu, the peak occurs at $T_p \cong 490$ K. The strong observed temperature dependences is in sharp disagreement with the predictions of current models for the origin of the noise.

The origin of excess (or $1/f$) noise is one of the oldest unsolved problems in the theory of solids.[1] This effect has been observed in a tremendous variety of physical systems[2-4]; however, it has only recently been possible to observe it in bulk-like metals.[5] Nevertheless, there is widespread agreement[5,6] among various workers concerning the magnitude and frequency dependence of the noise in metal films at room temperature. Indeed, most metals at room temperature have a voltage noise power spectrum in order-of-magnitude agreement with the phenomenological formula due to Hooge and Hoppenbrouwers (HH),[5]

$$\frac{S_V(f)}{V^2} = \frac{\gamma}{N_c f^{\alpha}}, \tag{1}$$

where $S_V(f)$ is the frequency-dependent noise power spectral density, V is the average voltage across the sample, N_c is number of charge carriers in the sample, and $0.9 \lesssim \alpha \lesssim 1.4$. The phenomenological constant γ has been assumed to be temperature independent[2] and to have a value (independent of material) of about 2×10^{-3}. The physical mechanism responsible for generating the noise, however, is far from clear. In this Letter, we present the first systematic measurements of the temperature dependence of $1/f$ noise in metals.

Thin films of Ag and Cu, 400–1600 Å thick, were prepared by thermal evaporation in a vacuum of 1×10^{-7} Torr onto sapphire substrates and scribed to dimensions typically 500×10 μm^2. The standard four-probe measurement technique was used. Contacts were made of various materials for different temperature ranges. Pressed indium, 60–40 Sn-Pb solder, and Dupont No. 8032 conductive silver composition were used. All gave identical results in the temperature regions where comparison could be made. In addition, measurements of the noise in air, helium, and vacuum gave identical results.

The dc biasing circuit used for these measurements consists of a battery pack in series with the sample and a limiting resistor. The noise signal is passed to an impedance-matching transformer (PAR 190), to a low-noise preamplifier (PAR 113), and finally to a frequency spectrum analyzer (SAICOR 51B). The Nyquist noise background is measured in the absence of current and then subtracted from the total noise power to arrive at the excess-noise spectrum. Measurements with a wire-wound resistor in place of the sample assured that noise was not generated in the measuring circuitry.

In order to maximize the ratio of excess noise to Nyquist noise, current densities in excess of 2×10^6 A/cm^2 are commonly used.[6] Current densities this large cause substantial local heating of the film over the ambient substrate temperature, precluding the use of conventional thermometry. To circumvent this problem, we first determine the resistance of the sample vs T at low current densities. The noise is then measured (in the presence of a large driving current) as a function of sample resistance. The sample is, in effect, used as its own thermometer.

A typical frequency spectrum is shown in Fig. 1. α, defined by Eq. (1), is given as $\alpha = 1.03 \pm 0.06$. Note especially the absence of any knees or bends in the spectrum. Small changes in α with temperature have been observed, however. α increases slowly from its value of 1.03 at 400 K to approximately 1.20 ± 0.08 at 150 K. This 20% increase in α with decreasing temperature occurred in samples of varying thickness, in silver and in copper. At a given temperature, the value of α appears to be independent of film thickness but is approximately 10% larger in Cu than in Ag.

The temperature dependence of the magnitude of the $1/f$ noise at 20 Hz in an 800-Å Ag film is shown in Fig. 2(b). The absence of any bends or knees in the frequency spectra at all tempera-

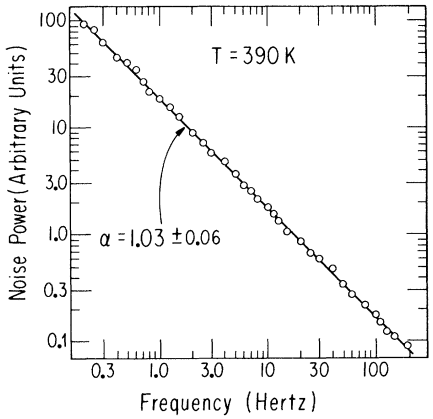

FIG. 1. Noise power frequency spectrum in Ag at 390 K.

tures and the small change in α with temperature guarantee that this curve is characteristic of the temperature dependence of the magnitude of the noise at all frequencies (at least between 0.2 and 200 Hz). The hatched box at room temperature represents a summary of measurements on Ag films by Voss and Clarke[6] (VC). The room temperature agreement between the two results is clear. The absolute magnitude of the noise at 410 K for this Ag film with a volume of 7×10^{-10} cm^3 is approximately 8.4×10^{-17} V^2/Hz. We should stress, however, the absolute magnitude of the noise in different films when compared via Eq. (1) is uncertain by roughly a factor of 3. The dashed curve represents the results of a model calculation to be discussed below. The striking features of the temperature dependence are the rapid decrease of the noise below room temperature and the peak occurring at $T_p \cong 410$ K. This rather unexpected structure in the noise was confirmed in a variety of Ag films. In particular, qualitatively similar structure was seen in films nearly twice as thick (1400 Å). In these thicker films, the peak occurred at a slightly higher temperature (~430 K), and the decrease in the noise below room temperature was slightly faster. In general, however, the temperature-dependence curves for films of different thicknesses fall virtually on top of one another when properly scaled according to Eq. (1) for differences in absolute magnitude.

Films for which data are presented were annealed for 1–2 h at 50 to 100 K above the highest temperature at which data are reported. Room-temperature noise decreases by a factor of 6 or 7 after annealing. The resistivity and temperature coefficient of resistivity also changed upon annealing by as much as 20% in the thinnest films. After the anneal, these quantities were regularly within 10% of bulk values. In addition, the noise is independent of temperature cycling after annealing.

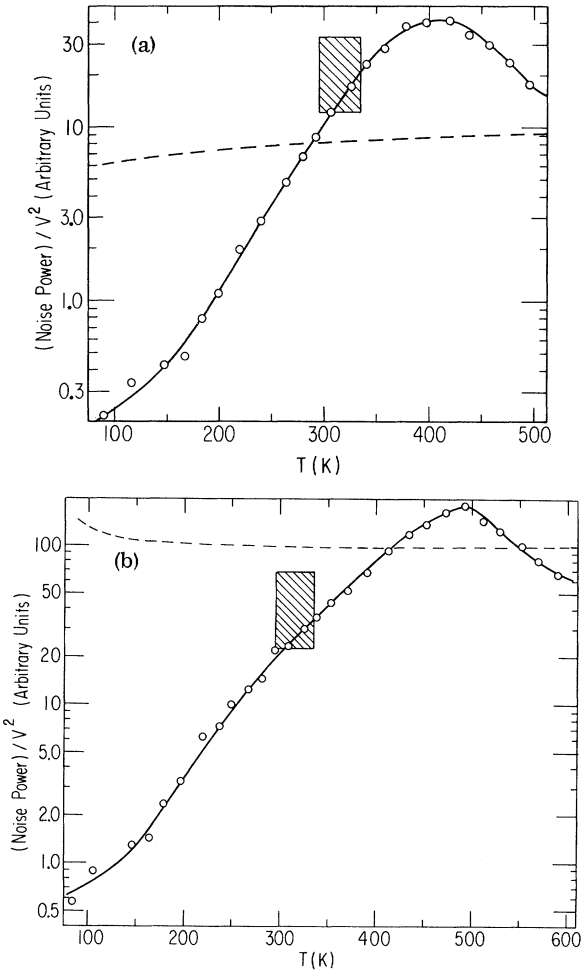

FIG. 2. $S_V(20)/V^2$ vs temperature for (a) 800-Å Ag film and (b) 800-Å Cu film.

The T dependence of the noise in an 800-Å Cu film is shown in Fig. 2(b). Note the qualitative similarities to the behavior in Ag. The peak, however, occurs at a rather higher temperature, $T_p \cong 490$ K, significantly above the peak in even the thickest silver films. The absolute magnitude of the noise at 490 K for this 1×10^{-9}-cm^3 Cu film is 2.6×10^{-17} V^2/Hz. Again, we see agreement in magnitude at room temperature with the work of VC and also a rapid decrease of the noise below room temperature. The noise in this film changes by nearly three orders of magnitude between 100 and 490 K.

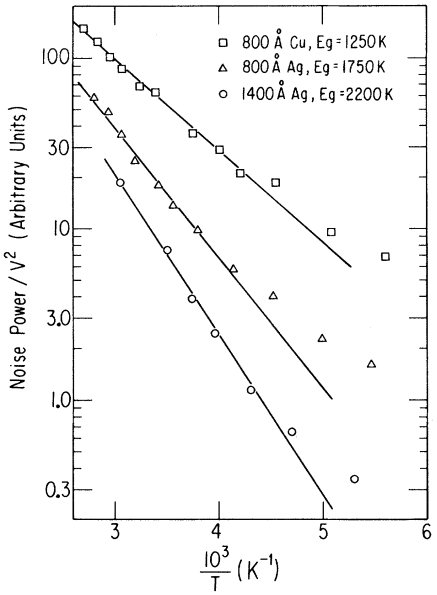

FIG. 3. $S_V(20)/V^2$ vs $1/T$ for T less than the peak temperature.

The low-temperature behavior of the noise is illustrated in Fig. 3, where we plot the logarithm of the noise at 20 Hz vs the inverse temperature. To a good approximation, the temperature dependence of the noise for $T \lesssim 350$ K can be described by

$$S_V(20) = N_0 + N' \exp(-E_g/k_B T), \tag{2}$$

where the parameters E_g, N_0, and N' depend on the detailed characteristics of the particular film including its thickness, with E_g increasing as the properties of the film become more "bulklike." It is important to point out, in this regard, that only the noise shows activation. The sample resistance is characteristic of a metal and shows the expected linear R vs T behavior at all temperatures examined.

We now compare our results with current theoretical ideas about the origin of $1/f$ noise. The phenomenological HH formula [Eq. (1)], which is in qualitative agreement with our results at room temperature, neglects temperature dependence completely. Furthermore, the observation of a strong dependence of the noise on material parameters implies that the HH formula cannot simply be corrected by allowing γ in Eq. (1) to be a universal function of temperature.

Perhaps the most successful model for the origin of $1/f$ noise in metals is the diffusion model suggested by VC.[6] That model, based on equilibrium temperature fluctuations, gives the following expression for the spectral density:

$$\frac{S_V(f)}{V^2} = \frac{\beta^2 k_B T^2}{C_V[3 + 2\ln(l/w)]f}, \tag{3}$$

where $\beta = (1/R)dR/dT$ is the temperature coefficient of resistance, C_v is the heat capacity of the sample, l and w are the sample length and width, respectively. The $1/f$ dependence does not arise naturally from the diffusion equation, however, and is put in phenomenologically. Nevertheless, the model provides reasonable agreement with experiment for the room-temperature magnitude of the noise in various metal films and has several other attractive features, especially its prediction of the experimentally observed spatial correlations of the noise.[6] It also predicts a temperature dependence for the magnitude of the noise, plotted as the dashed line in Fig. 2. As can be seen, the agreement at room temperature is reasonable in both Cu and Ag (within a factor of 3) but the structure in the experimentally observed temperature dependence is completely different from the model. Indeed, at low temperatures, theory and experiment disagree by more than two orders of magnitude. Furthermore, the model is inconsistent with our annealing experiments since $\beta = (1/R)dR/dT$ increases with annealing while the noise decreases. Nevertheless, we should point out that the failure of the VC theory to describe our experimental data does not completely rule out the possibility that $1/f$ noise arises from temperature fluctuations. Incorporation into the theory of the effects of the thermal-boundary resistance between the sample and the supporting substrate and/or the effects of phonon traps might rectify the disagreement between experiment and theory. It does appear, however, that the observed agreement at room temperature is fortuitous and that the temperature fluctuation model in its present form is incapable of describing films on substrates.

It is therefore useful to speculate about other possible origins of noise in metals. One possibility is vacancy and interstitial diffusion. The idea is similar to the one suggested by McWhorter[7] for $1/f$ noise in semiconductors. The experimentally observed $1/f$ spectrum obtains if the barrier heights, E_D, for vacancy-interstitial diffusion are uniformly distributed in the energy range $0.15\ \text{eV} \lesssim E_D \lesssim 1.1\ \text{eV}$. Typical values for bulk metals occur in this range, and in films it is easy to imagine that crystal imperfections might smear the energy distribution. Then, exactly as in the McWhorter model, we get a range

of time constants from an activated process, in this case the *diffusion* of vacancies, and a $1/f$ spectrum. The temperature dependence of the magnitude of the noise, on the other hand, could arise from the creation of vacancies, the magnitude of the noise being proportional to the *number* of diffusing vacancies. In bulk metals, the activation energy E_v/R_B for creation of a vacancy or interstitial is roughly 12 000 K. Over the range of temperatures in Fig. 3, the noise also appears activated with $E_g/k_B \cong 2000$ K for Ag. But vacancies are created preferentially at surfaces and deformations (e.g., at grain boundaries) and so a decrease in the bulk activation energy by roughly a factor of 6 is not unrealistic for films. In addition, E_g becomes somewhat larger as the sample is annealed, consistent with our expectations that a thicker annealed film should be somewhat closer to bulk. The speculation, then, is that creation of vacancies could explain the temperature dependence of the noise (at low temperatures), while diffusion of vacancies characterized by a distribution of activation energies could account for the $1/f$ nature of the frequency spectrum. Unfortunately, this analysis does not relate directly to perhaps the most interesting feature of the data, the peak at high temperatures.

In conclusion, the rapid temperature dependence measured in both Ag and Cu films provides a stringent test for models of $1/f$ noise. A model based on the two-step process of creation and diffusion of vacancies can provide rapid T dependence, but remains quite speculative. The temperature-fluctuation model of VC, on the other hand, has various appealing features, but in its present form, at least, fails in its prediction for the temperature dependence of the noise. Experiments in progress on the temperature dependence of $1/f$ noise in other metals hopefully will help clarify the situation.

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Ballistic Propagation of Near-Gap Phonons in Bulk Superconducting Tin

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Phonon transport in β-Sn crystals was studied by heat-pulse techniques using superconducting tunnel-junction detectors. The temperature-dependent attenuation by the quasiparticles was measured at fixed frequency ($\simeq$ 160 GHz), while, at lower temperature (0.6 K), the ballistic transmission was monitored up to the onset of pair-breaking frequencies. A new method of determining the energy gap follows, which is shown to apply to nonequilibrium states of the bulk.

The main source of phonon scattering in metals drops out below the superconducting transition temperature (T_c) as the electrons condense into the pair states. Since this fact was first discovered in ultrasonic attenuation,[1] much information about the electron-phonon interactions was gained by extending the measurements into the gigahertz range and, in particular, the pair-breaking mechanisms could be directly observed[2] in the vicinity of T_c. However, the phonon energies $h\nu$ (h is the Planck constant; ν the frequency) fitting the ground-state parameter 2Δ of the BCS theory[3] fall